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Continuous interactive simulation: Engaging the human sensory-motor system in understanding dynamical systems

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Abstract

Direct and continuous interaction with a simulated dynamical system enables the full human sensory-motor loop to be applied to the problem of understanding the behaviour of dynamical systems. A virtual environment consisting of a continuous interaction device and a multi-sensory display allows a user to interact with an arbitrary dynamical system in real time. The resulting tight coupling between the user and the dynamical system allows the user to explore the system through active participation in the dynamics of the system. An example drawn from population dynamics demonstrates that users with no specialist skills are able to discover a range of properties of an arbitrary dynamical system through sensory-motor exploration alone.

Keywords: dynamical systems, simulation, interactive visualisation, continuous interaction, human movement

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1. Introduction

Human movement is, essentially, a problem in understanding dynamical systems. The human sensory-motor system must interact with a wide variety of environmental dynamics in order to produce desired movement outcomes [1]. Doing so requires the solution of difficult problems such as prediction, optimisation and control in the face of delayed and incomplete sensory information, time varying nonlinear dynamics, and constant disturbance [2]. In the course of any well-practised movement most people are completely unaware of the difficulties involved and “just do it”. For example, a skilled pilot is able to draw on an implicit sensory-motor understanding of aerodynamics in order to produce the movements needed to execute a particular manoeuvre given the current state of the aircraft and the prevailing conditions. Furthermore, there is an essentially creative, improvised character to human behaviour that allows movement to unfold and adapt to contingencies in the environment as they arise. In a very real sense we are all, naturally, experts in dynamics.

The need to understand the behaviour of dynamical systems is, of course, relevant to many disciplines. Dynamical systems models feature prominently in fields as diverse as physics, ecology, economics, sociology, and so on. Developing an understanding of how nonlinear dynamical systems behave and how they can be controlled is a challenging problem requiring specialist techniques that often apply to only limited classes of systems [3] . Given the human

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capacity to learn new physical skills involving unfamiliar, often complex dynamics it might be possible to develop an understanding of the behaviour of a dynamical system based on sensory-motor learning rather than on mathematical theory or specialist domain knowledge. This paper describes a simple approach to enabling direct sensory-motor interaction with a real-time simulation of an arbitrary dynamical system. Continuous movement actions are mapped onto system parameters and the continuously evolving state of the system is mapped onto a sensory environment to complete the sensory-motor loop. The tight coupling between a user and the system that results allows the user to explore the behaviour of the system through active participation in the dynamics of the system, in much the same way they would explore any unfamiliar physical situation. With practice, a user's initial fumblings may develop into skilled manipulations that demonstrate how the system behaves and the various ways in which it can be controlled.

This paper is organised as follows. Section 2 describes related work, section 3 describes the general approach to engaging human sensory-motor capabilities in exploring dynamical systems, section 4 provides a simple example drawn from the study of population dynamics that demonstrates how a purely sensory-motor approach to exploring dynamical systems can yield important properties of a system together with insight into the ways in which the system can be manipulated, and section 5 provides a discussion of the key issues that arise from the proposal described in this paper.

2. Related work

The use of computers to facilitate the engagement of human sensory capabilities in understanding dynamical systems is well established. In particular, visual representations of the behaviour of a dynamical system allows human perception to be applied to the problem of identifying key properties of the system. For example, fixed points, limit cycles, attractors and sepatrices can be recognised in a visual representation of the trajectory of a continuous dynamical system [4]. Virtual and augmented reality environments have been used to provide a more immersive visual experience to better exploit human perceptual capabilities (e.g., [5]). Other sensory modalities such as hearing and touch have also been used in the presentation of a broad range of scientific data including the behavior of dynamical systems (e.g., [6, 7]).

Interactive visualisation aims to enhance the ability of an investigator to explore and understand a system through interaction with a visualisation of a simulated system [8]. This interaction may be directed at the visualisation of the system (rendering techniques, user point of view etc) or at the simulated system itself. A particular form of the latter, known as computational steering [9], allows users to modify the parameters of a simulation in order to explore the behaviour of a system under different conditions. Such parameter modifications may be used to establish new initial conditions for the simulation or to change the behaviour of the simulation as it executes. While computational steering emphasises user interaction, the realities of simulating large systems means that direct manual interaction with the simulation may not be practical [10]. Instead there have been suggestions that much of the direct manual control of such environments be automated with the user assuming a more supervisory role in which they set goals, monitor progress, and examine results as they become available [10, 11]. While there are good reasons for taking humans "out of the loop" there is a risk that increased levels of automation mean that opportunities for applying human creativity and improvisation, often in situations where they can be of most value, are lost [12, pg 4].

Within the human computer interaction (HCI) community, a recognition of the fundamental importance of physical engagement with the environment to our experience of the world has led to the development of methods of interacting with computer systems based on human movement [13]. If movement can be used as input to computer systems, then those systems might become easier and more natural to use. At the heart of movement-based interaction is some form of continuous rather than discrete user interface. In contrast to the discrete interactions afforded by conventional user interfaces consisting of text boxes, menus, push buttons, selection lists, and so on, continuous interaction involves a tight coupling between the user and the system in which there is an intensive exchange of information over an extended period of time [14]. A range of sensing devices can be used for continuous input such as joysticks, haptic devices, data gloves, infra-red sensing, and video-based motion detection. The key requirement is to map a continuous range of movement onto a continuous range of input to the system over an extended period of time.

Most interactive simulation platforms use conventional discrete user interface devices for controlling a simulation [9]. In cases where continuous interaction is used it is most often used for controlling the visualisation of the system rather than controlling the parameters of the simulation itself. This form of interaction is common in virtual and augmented reality environments and in computer generated animation where continuous interaction is used to navigate

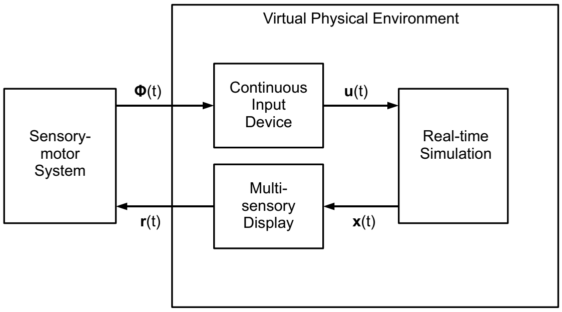

Figure 1: Virtual environment for engaging the human sensory-motor system in the dynamics of a simulated dynamical system. A continuous interaction device maps movement actions, $\phi(t)$, onto the control variables of the simulated system, $\mathbf{u}(t)$, to produce a time-varying system response, $\mathbf{x}(t)$, according to the dynamics of the system, which is then mapped onto an environmental response, $\mathbf{r}(t)$, by a multi-sensory display. The complete interaction loop is exercised at every time step of the simulation.

a scene or manipulate an object representing some aspect of the visualised system (e.g., [15, 16, 17]). An obvious exception can be found in flight, vehicle and equipment simulators. These simulators capture the essential dynamics of a system and the control relationship between the human operator and the relevant parameters of the simulated system. For example, continuous movement of a joystick produces continuous changes in the parameters representing the deflection of the control surfaces of a simulated aircraft. Providing a visual representation of the resulting state of the aircraft, such as a simulated view out of the cockpit window, completes the sensory-motor loop and allows a pilot to practice the skills needed to fly a plane.

More general application of human sensory-motor capabilities to the understanding and control of simulated dynamical systems has been developed in the context of manipulating constrained physical systems such as architectural structures [18] and molecular docking problems [19]. Continuous interaction with the parameters of a simulated dynamical system has also been applied to the manipulation and control of unstable rigid body systems [20]. The motivation for this work was to create more realistic animation of rigid body systems by exploiting an animator's intuitive motor learning and motion planning skills. This idea has since been applied extensively to the animation of human and animal figures in particular (e.g., [21]). The proposal put forward in this paper generalises these ideas to continuous interaction with *arbitrary* dynamical systems that may have no physical basis whatsoever.

3. Continuous interactive simulation

Consider a continuous dynamical system of the form

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}(t), \mathbf{u}(t), t\right) \tag{1}$$

where $\mathbf{x}$ is a vector of variables that represent the state of the system, $\mathbf{u}$ represents a vector of control variables, and $\mathbf{f}$ is a function of the state of the system, the control variables, and time. For models of real world systems this function is generally nonlinear. The behaviour of such models can readily be simulated in real-time using numerical techniques to yield a trajectory for the system's state over time. The course of this trajectory depends on the form of $\mathbf{f}$, the initial conditions of the system, and the continuously changing values of the control variables, $\mathbf{u}$.

In order to engage human sensory-motor capabilities in exploring and understanding such a system a physical realisation of the system is needed with which a user can interact. This can be achieved using a virtual environment consisting of a real-time simulation of the system, a multi-sensory display, and a continuous interaction device, as illustrated in fig 1. The multi-sensory display provides a perceptual environment consisting of visual, auditory, and haptic elements onto which the state of the simulated system, $\mathbf{x}$, can be mapped. As the simulation of the system proceeds the display is updated to provide a representation of the system's dynamics as they unfold. The continuous interaction device allows a user's continuous movement actions to be mapped onto the control variables, $\mathbf{u}$, of the system. The complete interaction loop is exercised at every time step of the simulation. As the user moves, the behaviour of the system changes, which is then reflected in the sensory representation of the system. This effectively closes the sensory-motor loop around the dynamics of the system so that it becomes the subject of the central nervous system mechanisms underlying human sensory-motor learning. The user is now free to explore the behaviour of the system in purely sensory-motor terms. Observing the consequences of various movement actions (or inaction) allows the user to develop an understanding of what can and can't be done with the system given the controls available.

An important detail concerns the meaning of "real-time" in this context. Time scales for problems in dynamics can vary tremendously from the sub-second realm up to days, months or many years. However, the human sensory-motor loop requires about 150ms to produce a motor response to a sensory stimulus [22]. In order to effectively engage human movement capabilities in controlling an arbitrary dynamical system, the time scale of the system under study may need to be scaled up or down so that a user can respond to changes in the state of the system. This can be achieved by substituting t in equation 1 with $T = at$, where a is a suitable scaling factor.

There are many ways in which the multi-dimensional state of a simulated dynamical system could be presented in a multi-sensory display [23]. A straightforward way to do this is to adopt the commonly used phase space representation of a dynamical system in which the state of the system is depicted as a point in a space defined by the state variables, $\mathbf{x}$, of the system. If an object is placed at the initial state of the system and its position updated as the simulation proceeds the behaviour of the system will be represented by the motion of the object in phase space[2]. In this case, the mapping of the system's state onto the visual environment is a simple scaling of the state variables onto the axes of the phase space representation. The visual representation can be further enhanced using stereoscopic projection and three dimensional doppler sound effects to help reinforce the sense of position and motion in space. Similarly, there are many ways in which continuous movement actions could be mapped onto the control variables of a simulated system. A simple form of continuous input is provided by multi-axis joysticks with each axis of movement mapped to a different control variable. In this case the mapping of hand movements onto control variables is a simple scaling of joystick position onto control variables. Fig 2 shows an arrangement in which the motion of a ball in three dimensions is controlled with a hand-held controller that allows a user to explore the dynamics of systems of up to three state variables and four control variables.

While the visualisation techniques outlined above are relatively simple, direct continuous engagement with the user's sensory-motor loop allows them to explore the behaviour of a system by literally "playing" with it to see what it does. This exploration does not require any domain knowledge or mathematical skill, just the normal sensory-motor curiosity that we all seem to share.

4. Population dynamics

This section provides a simple example of how human sensory-motor capabilities can be used to explore the behaviour of an arbitrary dynamical system. The study of population dynamics is central to many fields including ecology and economics. However, the dynamics are complicated due to nonlinear interactions between populations. One of the first models to take interactions between populations of interdependent species into account is the Lotka-Volterra model of predator-prey systems originally developed in the 1920s [24].

A common form of the Lotka-Volterra model is given by the following pair of first order nonlinear differential equations.

$$\dot{H} = \alpha H - \beta HP - h_H \tag{2}$$

$$\dot{P} = \delta HP - \gamma P - h_P \tag{3}$$

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[2] The velocity of this motion is given directly by equation 1.

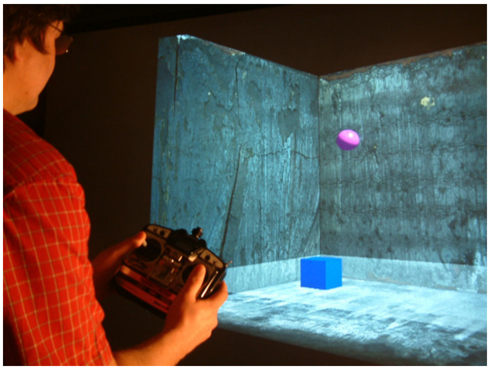

Figure 2: Continuous interactive simulation. The motion of a ball in three-dimensional space represents the dynamics of an arbitrary dynamical system. Two 2-axis joysticks on a hand-held controller are used to map hand movements onto the control variables of the system. The box represents a target state and the plane on which it sits represents a constraint on the state of the system.

Equations 2 and 3 describe the rate of growth of prey and predator population densities, H and P respectively. H and P are the state variables. The parameter α is the natural rate of prey population growth, β is the rate at which predator-prey interactions result in prey mortality, δ is the reproduction rate of predators per prey eaten, γ is the intrinsic mortality rate of predators in the absence of prey. The parameters h_H and h_P represent the rate at which the predator and prey populations are harvested. h_H and h_P are the control variables. Negative values of these parameters correspond to populations being stocked with new individuals.

Following the approach described in the previous section a 2-dimensional phase space representation of the Lotka-Volterra dynamics was constructed in which the motion of a ball represented the changing state of the system. The horizontal axis of the phase space represented the prey population density and the vertical axis represented the predator population density. A hand-held controller with two joysticks was used for interaction with the system. One axis of the right-hand joystick was mapped to positive values of the prey harvest rate, h_H. One axis of the left-hand joystick was mapped to negative values of the predator harvest rate, h_P. Constraining input in this way corresponds to a scenario where a population is controlled through direct harvesting and the introduction of a predatory species. The values for the parameters α, β, δ, and γ were set to 0.216, 0.0108, 0.0029, 0.173 respectively according to a model of soybean caterpillars controlled through the application of pesticide and the introduction of predatory bugs [25]. The system was simulated with a Runge-Kutta solver with a time scale factor of 1 second of real time to 6 days of population time. The presentation of the system made no mention of its domain origins. Users were simply presented with a ball that behaved rather strangely and responded in unfamiliar ways to joystick input. As a result users were forced to deal with the system in purely sensory-motor terms.

In a series of short, loosely structured sessions five users were invited to explore the dynamics of the system by experimenting with the ways in which the system behaved and responded to their movement actions. Each participant spent about 30 minutes in self-guided exploration. The system state and all control variable changes were continuously recorded during these sessions. On the basis of these scant directions the users discovered that they could steer the ball through a number of "moves", some of which are illustrated in fig 3. Interestingly, users spontaneously adopted the language of movement to describe what they had discovered using words such as "nudge", "hold", "drop", "catch", and so on, suggesting that they were dealing with the system in physical terms. Furthermore, users reported that they found the experience of exploring this highly contrived unfamiliar physical situation to be "fun" and gained some satisfaction from learning how to manipulate the system.

By examining the recorded data for each of these moves they can be interpreted in domain-level terms, as summarised in table 1.

Table 1: Moves discovered by users and their domain-level interpretation.

Move	Description	Domain-level interpretation
No action	Left to its own devices the ball travels in a continuous counter-clockwise orbit	The familiar boom and bust cycle characteristic of Lotka-Volterra systems
Nudging the ball	Nudging the ball with either the right or left hand changes the size of the orbit	Small impulses of either pesticide application or predator introduction will either increase or decrease the size of the boom-bust cycle depending on where in the cycle they are applied.
Stopping the ball	Using the right hand, the left hand or both hands the system can be brought to rest at the point about which the system orbits	The system has a stable equilibrium at $H = 60.0, P = 20.0$. The system can be stabilised at this point under the action of predator introduction alone, prey harvesting alone, or a combination of both
Losing the ball	Excessive use of the right hand causes the ball to hit an invisible barrier to the left from where it sinks to a stationary position	Excessive use of pesticide reduces the caterpillar population to zero after which the predatory bug population also collapses
Holding the ball with the right hand	The ball can be held still anywhere along the vertical line extending below the stable equilibrium using only the right hand	The system can be stabilised anywhere along the line segment $H = 60.0, P < 20.0$ using a constant level of pesticide application. This line corresponds to the P nullcline of the system.
Holding the ball with the left hand	The ball can be held still anywhere along the horizontal line extending to the left of the stable equilibrium using only the left hand	The system can be stabilised anywhere along the line segment $H < 60.0, P = 20.0$ using a constant level of predator introduction. This line corresponds to the H nullcline of the system
Dropping the ball from one hand to the other	The ball is held in the left hand, released, and allowed to “fall” into the other hand	The system is allowed to transition from a regime of predator only stabilisation to pesticide only stabilisation under the action of its intrinsic dynamics
Bouncing the ball with the left hand	The ball can be bounced along the horizontal line extending to the left of the stable equilibrium	The system, under a regime of predator only stablisation, can be moved to lower levels of prey population density using small impulses of predator introduction followed by an adjusted stabilising level of predator introduction

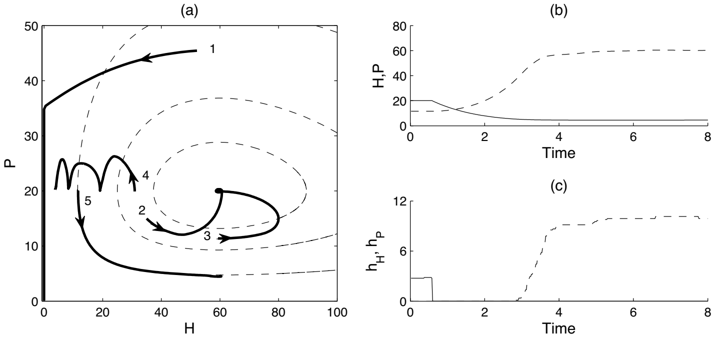

Figure 3: (a) Some of the "moves" users discovered while experimenting with the Lotka-Volterra system. 1. "Losing" the ball 2. "Stopping" the ball with the left hand 3. "Stopping" the ball with the right hand 4. "Bouncing" the ball with the left hand 5. "Dropping" the ball from the left hand and catching it in the right. The dashed lines show the uncontrolled trajectory of the system for various initial conditions. (b) State variable histories for dropping the ball from the left hand and catching it in the right (dashed line = H, solid line = P). (c) Control variable histories for dropping the ball from the left hand and catching it in the right (dashed line = h_H, solid line = h_P) .

5. Discussion

The global properties of the 2-dimensional Lotka-Volterra system discovered in this example can be determined analytically [26]. However, the example also demonstrates that these properties can be discovered by users with no specialist knowledge through sensory-motor exploration facilitated by continuous interaction with a simulation of the system. While the example was a relatively simple system, it was readily dealt with by the users. Work is underway to apply the approach outlined in this paper to more complex systems that pose a greater challenge to analytic techniques.

Continuous interactive simulation allows a user to explore the behaviour of a dynamical system through experimental manipulation of the system. As the example illustrates, this allows a user to discover not only global properties of a system such as cycles, equilibria, and so on, but also ways in which the system can be controlled. For example, users discovered not only that there is a stable equilibrium, but also how to steer the system to that state. In general, the problem of controlling a nonlinear dynamical system in order to achieve a particular outcome is a good deal more difficult than simply describing its global properties, even for relatively small systems. This suggests that continuous interactive simulation might form the basis of a novel dynamics problem solving environment. With an understanding of a system's dynamics gained through experimentation a user can be asked to perform a task that corresponds to a specific control problem. For example, a constrained minimum time optimal control problem might be presented to a user as "put the ball in the box as quickly as possible keeping the ball above the horizontal plane at all times" (see fig 2). A user's movement actions as they attempt such a task can be interpreted as a solution to the problem. As their performance improves with practice so will the quality of the solutions produced. Such a scheme may allow human capabilites to produce solutions to problems that are difficult to solve otherwise or to produce new, perhaps more creative, solutions to problems that can be solved using more conventional techniques.

There are clearly limits to the scale of dynamical system that can be explored using continuous interactive simulation. The human sensory-motor system is a limited resource and there will, inevitably, be many systems that are too large or too complex for humans to deal with directly. The requirement to be able to simulate a system in "real-time" also places limits on the scale of the systems that might be explored in this fashion. Key to maximising the effectiveness of this form of exploration will be engaging human sensory-motor capabilities to the fullest extent possible. The simple phase space representation described in this paper can be extended to support higher dimensional systems [4] and readily available 6-axis joysticks can allow control over more control variables than the simple 2-axis joysticks used in the example. While there are many interesting systems with several state and control variables, this scheme represents a somewhat impoverished way of engaging human movement capabilities. The normal physical environment in which movement occurs typically involves a great many variables across a wide range of sensory modalities. In terms of interaction, the human body is capable of about 110 independent joint rotations with associated muscle co-contractions [1]. Richer forms of physical engagement, perhaps using full-body motion capture with haptic feed-back, for example, may allow human sensory-motor capabilities to be applied to more complex systems.

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