CAN QUANTUM THEORY HELP EXPLAIN DARK MATTER?

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What is this poster about?

That the universe contains *gravitationally* bound, macroscopic stationary quantum states that can be used to explain the origin and nature of dark matter.

Gravity bound quantum systems

- Although quantum gravity is poorly developed, applying Schrödinger's traditional equation to gravity should be valid in all but the strongest gravitational regions, i.e. if $r >> r_s$ ($r_s \equiv$ Schwartzschild radius). Recent experiments[(1)] have indeed demonstrated the quantised nature of gravity.
- Most gravitationally bound systems give nonsense:
  - 2 neutrons: for $n = 1$, $E_n \sim -10^{-65}$ eV
  - 2 x 1kg: $E_1 \sim -10^{+65}$ eV but r_1 ($\equiv$ radius) $\sim 10^{-58}$ m.
- Suitable systems incorporate low mass eigenstate particles (p, n, e^-, H etc.) surrounding a large central potential.
- In principle, a massive 'eigenstructure' could exist around a large central black hole surrounded by a plethora of eigen-states, size r_n limited by $E_n \rightarrow E_{\text{thermal}}$ ($\equiv$ 'background' energy).

Astronomical eigenstates!

Concept sounds incredible, however:

- Nothing in quantum theory forbids the existence of such states
- The classical electron radius is ~ 3 x 10^{-15} m, yet experiments now regularly demonstrate a variety of macroscopic electron quantum effects.[2,3,4,5,6]
- The latest record for the size of a quantum entangled photon pair is 67 km[7] with measured connectivity speeds between the two photons far exceeding $10^6 c$[8] - i.e. demonstrates the reality of superluminal quantum collapse across light horizons.
- Quantum theory predicts the potential existence of large scale eigenstates some of which have the required properties to render them ideal dark matter candidates.[12]

Eigenstructure formation

Primordial black holes (PBH) were produced at the various phase transitions in the early universe.(9,10) If supermassive PBHs were formed, it is extremely likely that eigenstates would have formed around them.

Primordial black holes:

- PBH mass M_{pbh} ~ horizon mass, M_{H}. M_{H} given by:
  $M_{\mathrm{H}} = M_{\odot}(T/100\ \mathrm{MeV})^{-2}(g_{\mathrm{eff}}/10.75)^{-1/2}$, ($g_{\mathrm{eff}}\equiv$degrees freedom)
- The largest PBHs (with $M_{\mathrm{pbh}} \sim 10^5$ or 10^6 $M_{\odot}$) occur with possibly enhanced production at $t \sim 1$s.(9,10)
- Expect ≤ 1 PBH formed per 215 Hubble horizons(11) although present galactic density of 10^{11} per present Hubble volume extrapolates back to 1 such supermassive PBH per 3 x 10^4 horizons.

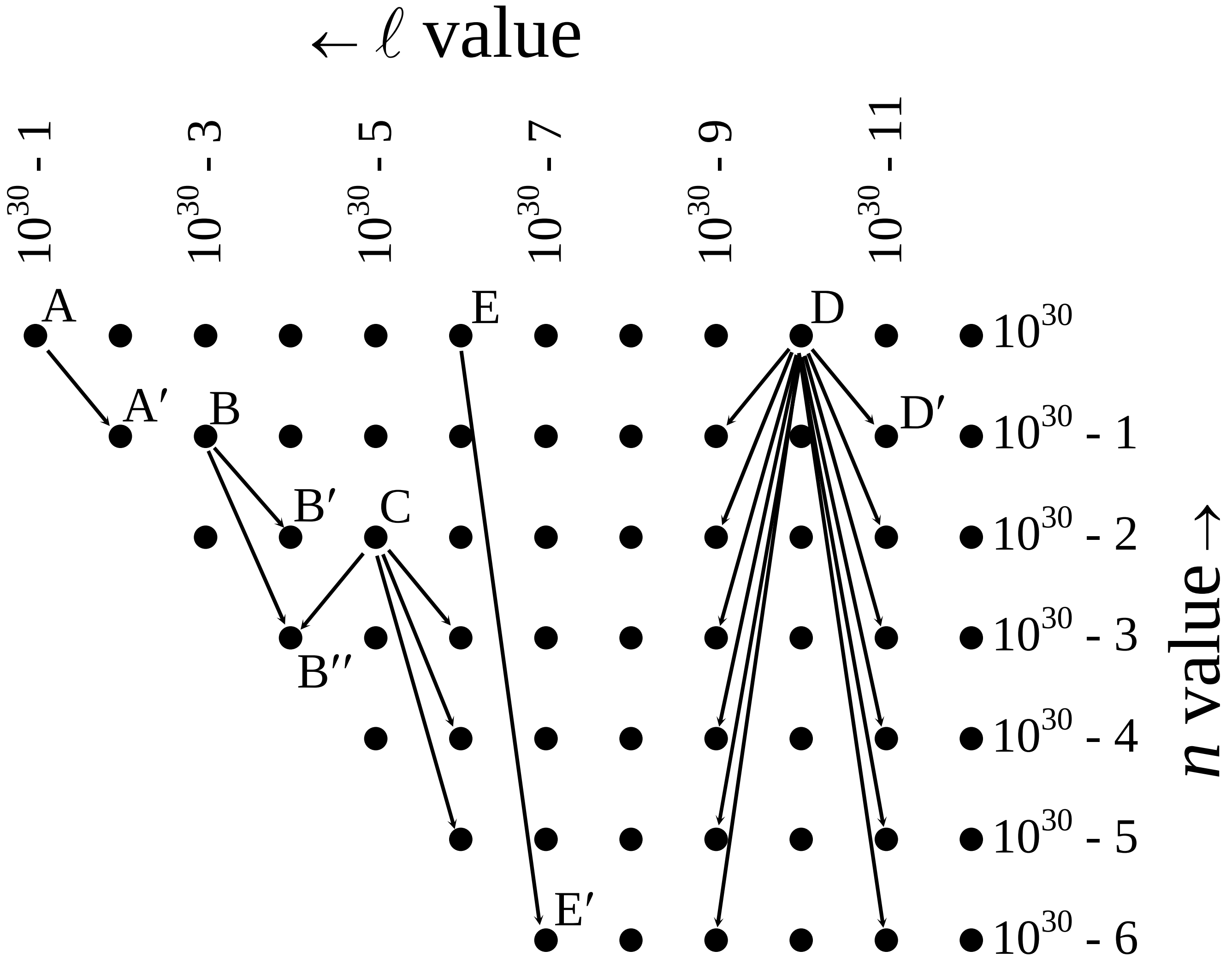
Eigenstructure formation:

- Populated via gravitational recombination within the radius ($\equiv r_{\text{thermal}}$) of an eigenstate with E_n = thermal background energy.
- As levels populate, potential deepens, background density and temperature drop and r_{thermal} increases. The increasing r_{thermal} sphere and depleted free particle background density within it strongly favours formation of high angular momentum states.
- Limiting factors are the rate at which particles can enter the r_{thermal} sphere and the potential well formation rate.
- For an initial PBH mass of 10^6 $M_\odot$, estimate a final eigenstructure mass ~10^{42} kg in 15 s; for 3 x10^5 $M_\odot$ the figure is 90 s, provided "over horizon" quantum collapse can occur.[8]
- Final state density determined by the many radiation/collision processes; only a small fraction of eigenstates being occupied.
- May occur pre-BBN, or possibly affect elemental abundances.

Why eigenstructures might be dark matter candidates

- High angular momentum (n, $\ell >> 1$) gravitational eigenstates appear to exhibit almost all of the properties of dark matter.
- They:
  (a) interact gravitationally.
  (b) have suitable mass capacity and energies.
  (c) have lifetimes far longer than the age of the universe.
  (d) will not gravitationally collapse
  (e) are weakly interacting with photons and particles.
  (f) visible matter could, in part, follow their distribution.
  (g) unlikely to have cusp-like mass distributions.

Figure 1 - High (n, ℓ) states, plotted with ℓ $(=n-j)$ increasing to the left. Each dot represents $(2\ell+1)$ z-projection sublevels.



Mass capacity and energy

- The number of states from $n = 1$ to $n = n_{max}$ is ~ 2/3 n^3 for large n, giving ~10^{100} for an n_{max} value appropriate to the extent of the galactic halo.
- There are sufficient states (n, ℓ) in even the range of $10^{29} \leq n \leq 10^{30}$ and $n - 10^9 \leq \ell \leq n - 1$, to easily accommodate the expected halo mass. Furthermore, using a central potential, total galactic mass of 10^{42} kg, an energy of -250 eV occurs for $n \sim 10^{33}$, at a radius of about 5 x 10^{21} m, the halo radius.[(12)]
- As seen in figure 1, any decaying transitions beginning from an arbitrarily ‘deep’ state D will, except for D/D′, always terminate relatively closer to the left diagonal, implying migration towards the higher, longer-lived ‘ℓ relative to n’ states and greater stability, in early eigenstructure evolution.

State lifetimes

- Radiative lifetime is $\tau = \frac{3\varepsilon_0 \pi \hbar c^3}{\omega^3 {p_{if}}^2}$ where $p_{if} \equiv$dipole matrix element $\langle i | e\,\mathbf{r} | f \rangle$ for the transition from initial state $\langle i |$ to final state $\langle f |$, $\omega \equiv$ frequency, and the other symbols have their usual meanings. Critical parameters are ω and p_{if}, and rates vary enormously, most too complex to calculate.
- For sufficiently large *n*, levels such as A/A′, B/B″ etc. in figure 1 are very close and ω $\left(= \frac{\mu G^2 m^2 M^2 \Delta n}{2\hbar^3 n^3} \right)$ becomes extremely small, often resulting in transition times ***far longer*** than the age of the universe. For $j < 10^9$ say, there are only a "small" number of decay channels available so that the total

lifetime of any state is still exceedingly long. For transitions between states with larger j, the value of ω grows quickly, but for certain states, p_{if} becomes so exceedingly small that, even with the enormous number of decay routes available, the lifetimes of many states still remain >> universal age.

- For example, provided $ni, ji >> 1$ and $ni >> ji$ ($ni \equiv$ initial, $nf \equiv$ final), transitions starting from some arbitrary state E of figure 1 and ending on the left diagonal at E′ (i.e. $\langle ni, \ell i = ni - ji| \rightarrow \langle nf = ni - ji, \ell f = nf - 1 = ni - ji - 1|$), have the radial part of their p_{if} function $\approx e\, b_0\, ni \left(\frac{2}{\pi ji}\right)^{\frac{1}{4}} \left(\frac{e}{2}\right)^{\frac{ji}{4}} \left(\frac{ji}{ni}\right)^{\frac{ji-3}{2}}$, where $b_0 = \frac{\hbar^2}{\mu G m M}$ (≡ "Bohr radius" parameter counterpart). For $ni = 10^{30}, ji = 10^{16}$ this is $\sim 10^{-10^{17}}$! High ℓ states are therefore effectively frozen in time with the eigenstructure unable to gravitationally collapse.

Physically, p_{if} values can be infinitesimal in two ways:

- When $ni >> nf$ the rapidity of the spatial oscillation of the higher ni eigenfunction can result in almost complete cancellation over each cycle of the lower nf eigenfunction.
- Laguerre polynomials decrease so rapidly outside their 'range', any substantial difference in ni and nf resulting in negligible overlap. The zeros γ_k of any Laguerre polynomial appear empirically related via $\sqrt{\gamma_k} - \sqrt{\gamma_1} \propto (k-1)$. This enables estimation of their position, extent, and the overlap integrals.
- Rough calculations indicate that for the high angular momentum states where ℓ is within 10^9 of n ($\sim 10^{30}$), there is no overlap unless ni and nf differ by no more than 1 part in 10^{11} corresponding to wavelengths of over 1km at the emission epoch.

Photon and particle interactions

- The Einstein A and B coefficients are related by $B\rho(\omega)=\frac{A}{\exp(\hbar\omega/kT)-1}$, directly proportional to the spontaneous rates and photon densities. CBR is one source of photons. Rates for transitions that correspond to CBR energies and involve high ℓ states ($\sim n$) are typically $\sim 10^{-10^{28}}$ s^{-1}. Even allowing for high branching densities and degeneracy factors the rate is clearly negligible. Similar arguments apply to galactic background radiation from stars etc., more so in fact because the $\langle i|$ and $\langle f|$ states are even more widely separated.
- The differential cross section for bound inelastic Compton scattering is solvable using the lowest order two photon

perturbation Hamiltonian. Here the overlap integral involves the combined atomic state/photon vectors with the vector potential **A** of the field and the momentum operator $(\hbar/i)\nabla$.[13] Jung[13] shows that, when photon energy is high, the overlap integral becomes $\langle f | e^{i(\mathbf{k}_i - \mathbf{k}_f)\cdot\mathbf{r}} | i \rangle$ where $\mathbf{k}_i$ and $\mathbf{k}_f$ are the photon wave vectors. Clearly for the reasons given previously this integral will be negligible when $|i\rangle$ and $\langle f|$ do not overlap.

- Most collisional phenomena involving atomic transitions depend on some type of overlap integral involving the initial and final atomic states. Certainly excitation depends directly on the oscillator strength which in turn also depends on the dipole matrix element.[14,15] Significant photon scattering or level shift for overlapping large (n,ℓ) states however then becomes insignificant because then the levels are so close.

Summary

Significant features of work so far:

- The existence of eigenstates on a macroscopic scale is a direct prediction of quantum theory.
- Many of these states will have properties which make them ideal candidates for dark matter:
  - Binding energies approaching background energies coincident with the expected galactic halo radius
  - Extremely long lifetimes, resulting in negligible emission and inherent inability to collapse.
  - Extremely weak interactions with photons and particles.
- A viable scenario exists for their formation and diffuse structure.

Problems, uncertainties and future direction:

- Effect of massive eigenstructures on large scale structure and galactic evolution - consistency with WMAP observations etc.
- Accurate modelling of eigenstructure formation is needed to assess its degree of credibility.
- Can eigenstates really be that big? Quantum collapse and relativity problems - how does the apparent superluminal collapse of quantum systems occur on macroscopic scales and how does it integrate with gravity and general relativity theory?
- Formation time of eigenstructure and effect on BBN.
- How might we detect the predicted long wavelength radio wave background?

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