CRITICAL MATHEMATICS EDUCATION
IN THE NEOLIBERAL ERA

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Certificate of Authorship

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person nor material which to a substantial extent has been accepted for the award of any other degree or diploma at Charles Sturt University or any other educational institution. I agree that this thesis be accessible for the purpose of study and research in accordance with the normal conditions established by the Executive Director, Library Services, for the care, loan, and reproduction of thesis.

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Abstract

Public schools in the United States are currently facing the consequences of neoliberal educational implementations. Neoliberalism aims to privatise and redefine public education to satisfy the needs of the labour market; it imposes a standardised and ‘teacher-proof’ curriculum, which is considered by many to reduce the quality of education to meet the goal of improving test scores. On a broad conceptualisation, its influence has replaced the idea of the citizen with that of the consumer. Some research has suggested that this market-driven process strips public education of its potential to help students develop the skills, attitudes, and values needed to become critical citizens. Given this climate, the present research offers a case study. Drawing on a critical participatory action research approach, it investigates how critical mathematics education (CME) responds to the tension between the needs of a neoliberal system and the needs of students to fulfil their potential as citizens and as human beings.

The original contribution of this dissertation is that despite obstructive implications of market-driven changes, a practice of CME to promote critical citizenship can be implemented through open-ended projects that resonate with inquiry-based collaborative learning and dialogic pedagogy. This practice necessitates transforming the classroom into a community of mathematics learners to democratise classroom life and create opportunities to promote participatory and social justice–based citizenship. The study also identified two main limitations of CME resulting from: (a) being a counterhegemonic practice enacted within an educational (neoliberal) system, while simultaneously criticising that same system; and (b) a lack of adequate learning materials and professional support to enact a CME program.
Chapter 1: Introduction

1.1 Background and Rationale

Over the last three decades, public education in the U.S., and in many other countries, has been undergoing a transformation. It has been largely reshaped by top-down neoliberal policies, according to which the success of schools, teachers, and students is measured by quantitative, standardised test results. In this view, education is a personal commodity, and it is suggested that schools should be run like businesses (De Lissovoy, 2015; Giroux, 2012; Hursh, 2007b; Hyslop-Margison & Thayer, 2009; Kohn, 2000; Schneider, 2016; Smyth, 2011).

Neoliberal ideology makes the positivist assumption that knowledge is independent of human subjectivity; it therefore imposes on students an externally generated, standardised curriculum (Gandin & Apple, 2002; Hyslop-Margison & Naseem, 2007; Schneider, 2015). In this view, the goal of education is to transmit knowledge to students. Evidence for successful transmission is provided by test scores. Raising test scores thus becomes the primary focus; teachers are relegated to the role of transmitting an externally prepared, ‘teacher-proof’ curriculum1 with the goal of preparing students for standardised tests (De Lissovoy, 2015; Hursh, 2007b; Leistyna, 2007; Sacks, 2009). This phenomenon is experienced especially in working-class, radicalised communities in the U.S. (Darder, 2002, 2012). Students are framed as passive consumers of knowledge: there is little need for dialogue, active participation, collaboration, or inquiry oriented towards new possibilities.

1 The ‘teacher-proof’ curriculum is a fully scripted, narrow curriculum that does not allow teachers to make adjustments.
This neoliberal transformation redefines the connection between democracy and education in economic terms and promotes consumer-based, individually responsible citizenship. Nevertheless, it has been presented to the field as the only way to solve the problems in public education.

In contrast to the neoliberal view is a view that may be called humanising. In this approach, public education has a democratic mission: to provide students with opportunities to develop skills, attitudes, and values to be loving, lovable, and caring individuals, as well as critical citizens (Darder, 2002, 2015; Giroux, 1988; Kohn, 1999; Leistyna, 2007; Noddings, 2003; Stanley, 2007; Westheimer, 2015). Education that aims to meet the needs of students as human beings and young citizens should be dialogic and open to possibilities; it should promote participatory and social justice-oriented citizenship in order to establish and sustain a ‘thick’ version of democracy initiated as a bottom-up movement (Orlowski, 2012; Westheimer, 2015).

There is thus a sharp divergence between neoliberal education that meets the needs of the market and humanising education\(^2\) that meets the needs of students as human beings and citizens. Humanising education necessitates a bottom-up, critical approach to teaching in order to create channels for dialogue in classrooms.

A small but increasing number of scholars in mathematics education are focusing on the sociopolitical and socioeconomic aspects of teaching and learning (Frankenstein, 1983; Gutstein, 2006; Skovsmose, 2011). These studies are united in positing a radical critique of traditional perspectives, and are generally framed under the umbrella of ‘critical mathematics education’ (CME). CME aims to foster critical citizenship and catalyse transformative social changes (Frankenstein, 2010; Gutstein, 2006; Skovsmose, 1994; Valero & Zevenbergen, 2004). CME is concerned with

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\(^2\) The term is used in this thesis in the sense of Freire (2000), who states that dialogue is an ontological necessity for humanising education.
issues such as socioeconomic diversity, equity and justice, student and teacher autonomy, and the socioeconomic functions of education (Skovsmose, 1994, 2011; Skovsmose & Borba, 2004). From this vantage point, CME can be seen as a response to the neoliberal agenda in mathematics education.

The works of Skovsmose (1994, 2011; Skovsmose & Alrø, 2004) provide a coherent foundation from which to define a practice of CME. There is some classroom-based research that draws on CME in the high school context (Brantlinger, 2014; Gutstein, 2006). However, currently no classroom-based research exists in the CME literature that frames a practice of mathematics education within the high school context to challenge neoliberal imperatives. This deficit motivates the present research.

1.2 Aims and Research Questions

Guided by CME, in this research I enact a critical stance towards mathematics education. It is situated in my own classroom teaching as a means of investigating the ways in which high school mathematics can be taught and learned in the neoliberal era. An underlying purpose of this approach is to promote a thick version of democracy and critical citizenship. As noted, a practice of CME aimed at promoting participatory and social justice–oriented citizenship radically conflicts with a market-driven education, which is primarily designed to produce consumer-based, individually responsible citizens. Therefore, this approach is generally not welcomed in school settings fundamentally shaped by neoliberal ideology (Hyslop-Margison & Thayer, 2009; Smyth, 2011; Westheimer, 2015). As already mentioned, neoliberal educational policies impose a narrow, ‘teacher-proof’ curriculum, thereby reducing the teaching profession to a merely clerical endeavour (Giroux, 1988; Leistyna, 2007).
Recent theoretical studies (De Lissovoy, 2015; Giroux, 2012; Skovsmose, 2011; Skovsmose & Greer, 2012), consistent with critical pedagogy and CME, generate two main conclusions. First, with its top-down imposed policies and implementations, contemporary market-driven educational changes curtail the potential of educational practice to promote democratic values and critical citizenship. Second, it is, nevertheless, important for students and teachers to be engaged with an educational practice that enacts humanising education and that promotes participatory democracy. These contradictory stances cannot be reconciled without evidence produced via classroom-based research. Therefore, it is worthwhile investigating whether it is possible to implement CME in the presence of neoliberal restrictions.

As background to this dissertation, my Master’s thesis focused on teachers and the teaching profession in relation to neoliberal educational transformation in the U.S. Results indicated that although teachers’ values and attitudes shaped their responses to neoliberal changes, top-down, market-driven changes diminish teachers’ academic freedom and professional authority (Giroux, 1988). Additionally, reflecting on my own journey of conducting the research for my Master’s thesis and my own classroom experience as a high school mathematics teacher, I concluded that willingness is a necessary but not a sufficient qualification for being a critical educator. As a public school teacher, I was also engaged with the question posed by Giroux (2012): Can democratic education survive in a neoliberal education system? The research questions of the current study emerged out of my strong and consistent desire to make ‘small openings’ in my classroom (described in detail in later sections) to help my students develop both communicative competencies and critical mathematical literacy to become critical citizens. The overarching research question

3 The term ‘small openings’ is due to J. C. Scott (2008), a political theorist and former professor at the University of Wisconsin-Madison.
assumes that the teacher is willing to practice CME. Therefore, emergent questions for this study are as follows:

1. While facing top-down restrictions imposed by neoliberal educational policies and pedagogies on a daily basis, is it still possible for teachers to create small openings for humanising education through CME?

2. How can collaborative and dialogic mathematics education be facilitated to help students to become critical citizens?

3. How can CME be practised without disrupting the process of preparing students for standards-based assessment?

These emergent questions have oriented my thinking towards the central question of my research:

What are the potentials and limitations of CME in terms of classroom teaching in the neoliberal era?

An overarching concern of this investigation was to provide evidence that CME can counter neoliberal hegemony in education.

1.3 Significance of the Study

Although there is a growing body of theoretical studies in the CME literature, there are very few classroom-based studies conducted in high school mathematics contexts that support the applicability of CME. For example, the importance of dialogic pedagogy is emphasised in the existing CME literature (Skovsmose & Alrø, 2004). However, the research suggests that authoritative (anti-dialogic) teaching is the dominant approach in most mathematics classrooms (Alexander, 2005; P. Scott, Mortimer, & Aguiar, 2006). Perhaps, as a relatively new domain of research, the CME literature has yet to offer more helpful answers to the following specific
question: Can a mathematical formula, concept, or axiom be taught in a dialogic form within a neoliberal education system? (Answers to this question—as discussed in Section 4.2.5, Section 5.1, and Chapter 6—constitute original contributions to the CME literature.)

Similarly, although theoretical studies exist—reviewed in Section 2.8—which emphasise the importance of inquiry-based collaborative learning, the CME literature lacks examples of identified classroom-based research that provide distinctive insights into the dynamics necessary to promote participatory and social justice-based citizenship. My research aims to address this gap in the current literature. Findings will be presented that may bridge the existing gap between theory and practice in the CME literature.

The present research is empirically significant in its potential provision of classroom-based data that provides new insights into definitions of dialogue, collaboration, and inquiry as aspects of CME. Moreover, existing studies offer limited insight into pedagogical practices. Therefore, it is crucial to generate research in mathematics classrooms to establish a dialectical relationship between theory and practice (Aguilar & Zavaleta, 2012; Almeida, 2010; Hannaford, 1998; Vithal, 1999).

As indicated in Chapter 5 and Chapter 6, the present research has the potential to make a significant contribution to professional development of teaching mathematics in relation to democracy and justice.

The prominent studies in the existing CME literature draw either on Freire or Habermas. Instead, to capitalise on these important scholars, the philosophical perspective of the current study is built on the ideas of both Freire (1997, 2000, 2013) and Habermas (1972, 1984, 1987, 2005). From this point of departure, an emphasis on the complementary ground of Freire (embodying a Latin American perspective)
and Habermas (embodying a European perspective) collectively empowers the theory of critical education to create a counterhegemonic force against neoliberal educational policies, therefore greatly contributing to scholarly studies in CME.

1.4 Nature of the Study

My research questions could be most appropriately investigated through classroom-based research. Therefore, an action research methodology was well suited for my project. Resonating with the natural flows of classroom teaching, action research methodology allows the cycle of plan-act-observe-reflect (W. Carr & Kemmis, 1986). The research methodology adopted here enables students to democratically participate in classroom activities and the process of knowledge construction. The methodology can be considered an adaptation of critical participatory action research (CPAR) as conceptualised by Kemmis, McTaggart, and Nixon (2014), in which research participants are seen as active agents of change as opposed to passive objects of the process. Therefore, the methodology is conceptually consistent with both critical and participatory praxis.

This study was conducted in a high school mathematics classroom where I teach full time. It involved a year-long mathematics class with 28 students, aged 14 to 17. In this sense, my classroom was a ‘bounded system’ (Merriam, 2009), and my research was a case study. The cycle of plan-act-observe-reflect ‘finds comfortable embrace in the case study, a basic methodology for action research’ (Pine, 2009, p. 213). In brief, the current project can be seen as a case study that draws on CPAR methodology.

Included in this study is data collected from student journals and presentations, as well as my field notes and reflective journal. I analysed the data from a dialectical perspective (Winter, 1989), which includes three basic premises: (a)
a phenomenon, although it is a unified whole, is structured in relation with other phenomena; (b) phenomena exist in a context that contains opposite (uncoupling) forces; (c) phenomena are in constant change: the change is the result of tensions between uncoupling forces and the unity of the phenomenon. To do justice to these dialectical premises, I used the ‘versus’ coding method developed by Saldaña (2013). Saldaña’s system adequately captures the tensions between CME phenomena in the midst of the uncoupling forces of neoliberal ideology.

1.5 Outline of the Dissertation

In the following chapters, the research problem framed above is presented in detail. Chapter Two provides an expanded literature review that integrates the studies of critical pedagogy into CME and identifies neoliberalism as an oppressive dynamic in and out of the classroom.

In Sections 2.2–2.5, I elaborate on neoliberalism as the dominant discourse in the current U.S. socioeconomic and sociopolitical environment. This discourse pervades everyday life, colonising young citizens’ life-world (Habermas, 1996). It is often claimed that neoliberalism prevents problems from being debated without distortion and manipulation. I review implications of the standardisation and accountability movement in the U.S. and the impacts of neoliberalism in education in a broader sense. This analysis will reveal the tension between the imperatives of the system and the needs of students as human beings and citizens. These sections converge to frame a theory of critical education as a bottom-up response to neoliberal hegemony.

Sections 2.6 and 2.7 review the historically evolving nature of critical theory and critical pedagogy. I will then elaborate on the ideas of Habermas (1972, 1984, 1987, 1996, 2005) and Freire (1997, 2000, 2013) in order to establish the theoretical
background of this study in the domains of epistemology, ontology, and research methodology. (These sections are also important for the methodology section in Chapter 3, as this study constitutes ‘critical’ research, differing considerably from positivist and interpretive research.)

In Section 2.8, a literature review is presented focusing on educational concepts such as dialogic pedagogy, collaborative learning and inquiry, as well as distinguishing communicative rationality from technical (or instrumental) rationality. This section reviews dialogic pedagogy as considered by Freire (1997, 1998, 2000, 2013), communicative rationality and the ‘ideal speech situation’ as outlined by Habermas (1972, 1973, 1975, 1979, 1984, 1987, 1990, 1996, 2005), and the zone of proximal development theory developed by Vygotsky (1978). Other work on dialogic pedagogy, collaborative learning, and inquiry-driven education is also presented. The discussion here centres on how dialogue, collaboration, and inquiry are relevant for CME.

Sections 2.9 and 2.10 focus on the existing CME literature. Through works of leading figures, such as Skovsmose (1994, 2011), Gutstein (2006), Frankenstein (1990, 1994, 2010), and Ernest (2010), I review the basic premises of critical mathematical literacy and CME. In the final section, I focus on connections between (mathematics) education and democracy through the work of Dewey (1916), Hannaford (1998), Hyslop-Margison and Thayer (2009), Almeida (2010), Aguilar and Zavaleta (2012), and Thésée (2013). This section also reviews work by Orlowski (2012), Westheimer (2015), and Gandin and Apple (2002) to distinguish justice-oriented, participatory (‘thick’) democracy from market-driven, neoliberal (‘thin’) democracy.
In Chapter 3, I first outline a critical qualitative research perspective, which differs from an objective and interpretive perspective. Specifically, I describe the epistemological and ontological premises of critical educational research (Darder, Baltodano, & Torres, 2009; McLaren & Giarelli, 1995; McLaren, Kincheloe, & Steinberg, 2013). I then present my research as an example of critical participatory action research (CPAR), as implemented through the case study method. As outlined by Kemmis, McTaggart, et al. (2014), CPAR aims to help research participants understand and transform their practices. Because CPAR focuses on a single case, it resonates with the case study approach (Pine, 2009; Stake, 1995). The chapter substantiates the claim that the aims of the research and the methodology are consistent with each other.

Chapter 3 also presents a detailed description of the participants and the site at which the research takes place. Next, it outlines the action plan, beginning with the reconnaissance stage. A key concept here is the end-of-unit project (EUP). Each EUP is a lesson plan integrated into a standards-based curriculum but aimed at implementing CME. The process of collecting and analysing qualitative data is reviewed (Denzin & Lincoln, 2008; Miles & Huberman, 1994). The chapter also describes ‘versus’ coding, an important part of the data analysis processes (Saldaña, 2013). The chapter concludes with a discussion of the ethics, validity, and generalizability of action research.

Chapter 4 presents the details of the method. Section 4.1 describes the research setting and the preparation stage. Section 4.2 provides a detailed description of five EUPs. For each EUP, this initial thick account describes the process of developing the lesson, organising group work, facilitating whole-class discussions, and recording reflective journals. I then analyse each EUP in light of the specific
goals of CME, based on my observation notes, my reflective journal, and students’ journals. This section provides a grounded picture of the empirical practices as they occurred in the classroom during the EUPs. The chapter concludes with my overall reflection on EUPs.

Triangulating data from all five EUPs, Chapter 5 analyses the findings with a view to determining the scope and limitations of CME in the neoliberal era. CME is discussed in relation to dialogue (Section 5.1), collaboration (Section 5.2), inquiry (Section 5.3), and citizenship (Section 5.4).

In Chapter 6, conclusions are presented that offer answers to the research questions posed here. The original contribution of my research to the existing literature is explicitly stated. Chapter 6 acknowledges the gap between the theory and practice of CME and the limitations of the study. Suggestions are made for further research in CME in the secondary school context. The chapter concludes with a discussion of how the study contributes to research methodology in secondary mathematics education.
Chapter 2: Literature Review

2.1 Introduction

This chapter reviews literature concerning three main domains relevant for this thesis: neoliberalism, theories of critical education, and critical mathematics education (CME) as it is situated within the broader body of the mathematics education literature. First, I review neoliberalism through the lens of critical theory. After providing a concise account of neoliberalism and its hegemonic power, I focus on its educational implications for high school mathematics and the connection between education and democracy. I next examine the evolving nature of critical theory and the principles of critical pedagogy (CP). Then I elaborate the theoretical background of the study, which is built on the complementary ideas of Habermas and Freire. I probe theories of dialogue in education as they pertain to CP within mathematics, and then outline CME as a response to the tension resulting from contemporary top-down neoliberal educational implementations. Finally, I review studies addressing socioeconomic and sociopolitical aspects of mathematics education that can be framed under the title of CME.

2.2 Neoliberalism

Liberalism was a political ideology mobilised for individual rights against the monarchs in the eighteenth century. The main goal was to liberate people from oppression. While the methods of liberation, as well as the source of the oppression may be quite different depending on the time and the place in question, liberation is inevitably the fundamental purpose of liberal political thinking and liberal political movements. (Riley, 1990, p. 2)

As Riley suggests, in its historical context, liberalism was a progressive step towards emancipatory approaches. The term liberalism still evokes a set of concepts—
including freedom, democracy, and tolerance—that can be related to a variety of domains, such as culture, economics, and politics, and is generally regarded as a ‘leftist’ world view in the United States and other Western countries (Hedges, 2009, 2010). For this reason, one may mistake neoliberalism as a progressive ideology or at least a non-conservative approach. However, in the United States, as Ross and Gibson (2007) point out, ‘It is important to understand that political conservatives and liberals (in mainstream U.S. politics) both support economic (neo)liberalism’ (p. 2). In this broader sense, neoliberalism might be considered as neither a leftist nor a progressive world view.

The philosophical roots of neoliberalism date back to the Walter Lippmann colloquium, which took place over five days in 1938 in Paris. In this colloquium, ideas, strategies, and ideological and political issues were debated with the goal to restore capitalism, which had run into crisis during the Great Depression of the 1930s (Dardot & Laval, 2009). As Hursh (2007b) points out, ‘Neoliberal economic policies arose as a corporate and political response to the previous Keynesian economic accommodation’ (p. 19). The Keynesian approach suggested a welfare state and social net as a corrective to capitalism and to prevent future economic crises. As such, neoliberalism can be viewed as a historical stage of capitalism.

However, implementation of neoliberal policies did not begin until the early 1980s: the Regan era in the U.S. and simultaneously the Thatcher era in the UK. Since then, neoliberalism has been a dominant discourse around the world (Cannella & Lincoln, 2013; Chomsky, 1999; Luxton & Braedley, 2010; Macrine, McLaren, & Hill, 2010). The neoliberal hegemony has been so strong that it cultivates corporate discourses in everyday life, as Cannella and Lincoln (2013) articulate:

Interpretations of knowledge and literally all human activity have been judged as valid and reliable if they fit the entrepreneurial imperatives; if they foster
privatization, competition, corporatization, and profiteering….The free market illusion, over everything from definitions of public and higher education as benchmarked and measurable to privatization of services for the public good. (p. 174)

Neoliberalism is a global project of the transnational \textit{capitalist ruling classes}, which strategically aims to increase capital accumulation (Chomsky, 1999; De Lissovoy, Means, & Saltman, 2015; Harvey, 2005). The imperative is to privatise public institutions, including education, turning them into sites of profit, and eliminating social welfare programs, from which the poor and working class benefit, replacing the concept of public or community good with individual interest (De Lissovoy et al., 2015; Harvey, 2005). According to a neoliberal ideology, capital should be totally free from regulation. In this view, the free market is considered a self-corrective driving force that creates the best outcome for everyone. Beder (2009) articulates this idea:

The new right argued that competition and unrestrained selfishness was of benefit to the whole society in capitalist societies….As a nation gets wealthier the wealth will ‘trickle down’ to the poor because it is invested and spent thereby creating jobs and prosperity. [But] the global financial crisis has shown that financial market provides opportunities for investment that provide relatively few extra jobs and feed an ephemeral prosperity that can be wiped out in days. (p. 18)

In many ways, these outcomes contradict the neoliberal argument, according to which bottom-up capital accumulation process would eventually benefit the whole society. The intense capital accumulation in a few hands has not ultimately created prosperity for all (De Lissovoy et al., 2015). In contrast, over the last 35 years, income inequality and poverty have increased both nationally and globally. For example, Torbat (2008) found that in the U.S., the top 20\% of the population receives
50% of the national income, but the share of the bottom 20% is just 3.4%. The top 1% of American population owns 40% of the national wealth; indeed, 400 people in the U.S. own 50% of national wealth (Torbat, 2008).

Research has also shown that neoliberalism has created inequality on a global scale: ‘As neoliberal policies were implemented around the world disparities in wealth and income increased and poverty increased, contradicting neoliberal theories that by increasing the wealth at the top everyone would be better off’ (Beder, 2009, p. 20). For instance, as Hayase (2013) noted, less than 1% of the world’s transnational corporations, mostly banks, control 40% of global business. Approximately 0.001% of the world population owns assets worth $14.6 trillion—or over 20% of the world’s annual GDP (Hayase, 2013, para. 4). Therefore, as West and Smiley (2012) suggest, neoliberal economic policies have divided the society into two: the rich elites and the rest of society.

During the global financial crisis in 2008 in the U.S., the Obama government subsidised the corporations with taxpayers’ money, an act that sharply contradicts the very premises of free-market ideology, which claims that government should not interfere with the market. Drawing on neoliberal arguments as presented earlier, if the free market was a self-corrective device, the U.S. government should not have rescued bankrupted corporations (Hedges, 2009). According to Hedges (2009), people were persuaded that what was good for corporations was also good for ordinary people. The concept of hegemony, as introduced by Gramsci (1971), helps us to understand this situation: the hegemonic ruling class controls the cultural means of production (i.e., media), turning their perspective into common sense in order to manufacture the consent of the masses.
Neoliberal policies have been well orchestrated and implemented on a global scale depending on the local contexts of nation states. Educational theorists—for instance, Apple, Kenway, and Singh (2005)—relate neoliberalism to the process of globalisation, conceptualising it as ‘globalisation from above’ (p. 3). In their view, neoliberalism, through transnational companies, envisions the world as a single market and aims to restructure social, economic, and political life of people almost all over the word. Accordingly, this vision creates a (new) class division on a global scale: the global elite and the global working class (McLaren & Farahmandpur, 2005). In this sense, ‘globalised neoliberalism’ can be regarded as an aspect of the globalisation process that specifically refers to the last 35 years. However, globalisation in itself is not a new concept. According to Wallerstein (2004), globalisation is a multi-dimensional phenomenon whose historical roots go back to the fourteenth century. Wood (1995) has argued that the internationalisation of capital has been a cornerstone of capitalism since its inception.

2.3 Neoliberalism and Everyday Life in the U.S.

Research has shown that as a policy movement, neoliberalism has had many negative impacts on the lives of citizens through the redistribution of wealth away from public resources and also away from the private wealth and income of the middle and working classes. As Ventura (2012) confirms, the average standard of life and income of the working class decreased: ‘Anxiety over the prospect of outsourcing resulted in job insecurity, depressed wages, the growth of poverty, a widening income gap between middle class and wealthy, and longer working hours for everyone from college grads to high school drop-outs’ (p. 7). However, citizens have not witnessed any serious and consistent uprising or political movement with a tangible alternative to neoliberalism. Therefore, it has been considered worthwhile to engage more
closely with the mechanism through which neoliberalism manufactures consent from people who are disempowered (Chomsky & Herman, 2008; Haiven, 2007; Lasn, 1999; Ventura, 2012).

Such engagement requires a clear discussion about how neoliberal ideology impacts young citizens’ life outside of school, shapes everyday life, and maintains its cultural hegemony. However, there is a very limited number of works that consider neoliberalism as everyday culture (Haiven, 2007). In Neoliberal Culture, Ventura (2012) analyses neoliberalism in terms of everyday life. She writes, ‘Neoliberalism, as a structure of feeling, is not merely an ideology, not merely an economic perspective, not merely rationality, but is the concatenation of them’ (p. 2). She argues that market-driven ideology shapes almost every aspect of life:

Neoliberal culture as a structure of feeling impels us to extend the market, its technology, approaches and mindsets into all spheres of human life, to move the ideology of consumer choice to the centre of individual existence, and to look to ourselves rather than larger social-welfare structures or society as the source of our success or the blame for our failure—indeed, to define success and failure in market terms. (p. 2)

Within the individualist culture of a neoliberal ideology, citizens are for themselves only. It is a posture that suggests people can help themselves, but have no right to expect any assistance from society. In fact, as Ventura (2012) indicates, there is no such thing as ‘society’ in the neoliberal view: ‘Neoliberal culture undermines the validity of thinking and working on the level of society’ (p. 3). She concludes that individuals who internalise this perspective become atomised and alienated. Ventura notes that neoliberalism does not directly posit itself against any world view; it ‘avoids operating in antagonistic relation to any other ideologies or to formal structures of power’ (p. 12). In this view, one’s failure, success, problems, misery, or
happiness are merely an individual matter rather than a social, political, or structural problem. This way of thinking and feeling does not leave any space to look at one’s life in connection with the existing sociopolitical structure of the system, and thus tends to depoliticise the everyday life of individuals.

The mechanism through which neoliberalism manufactures consent operates strongly at the workplace. Hedges (2009) explains how this process of depoliticisation happens within this context, which parallels Ventura’s approach. Hedges argues that the ‘positive psychology’ movement in the U.S. is at the service of legitimising neoliberal culture by promoting an illusion:

[Positive psychology] sanctions interpersonal and psychological attacks and lavish praise to destabilize an individual’s sense of self and promote compliance. It uses the coercive pressure, including attacks on individuality and criticism as a form of negativity, to ensure conformity….Positive psychology, like celebrity culture, the relentless drive to consume, and the diversionary appeals of mass entertainment, feeds off the unhappiness that comes from isolation and the loss of community. (p. 137)

According to Hedges (2009), people’s passive acceptance of neoliberal corporate culture is not spontaneous—it is a well-orchestrated process to manufacture mass consent. Hedges articulates the ways in which individuals at the workplace are turned into passive objects:

There is a dark, insidious quality to the ideology promoted by the positive psychologist. They condemn all social critics and iconoclasts, the dissidents and individualists, for failing to surrender and seek fulfilment in the collective lowing of the corporate herd. They strangle creativity and moral autonomy. They seek to meld and shape individual human beings into a compliant collective. The primary teaching of this movement, which reflects the ideology of the corporate state [neoliberalism], is that fulfilment is to be found
in complete and total conformity, a conformity that all totalitarian and authoritarian structures seek to impose on those they dominate. (p. 138)

Hedges (2009) further indicates how neoliberal hegemony is produced and reproduced at the workplace by turning individuals into a compliant collective. He describes the consequences of this process in the following way:

The loneliness of a work life where self-presentation is valued over authenticity and one must always be upbeat and positive, no matter what one’s actual mood or situation, is disorienting and stressful….Here, in the land of happy thoughts, there are no gross injustices, no abuses of authority, no economic and political system to challenge, and no reason to complain. Here, we are all happy. (p. 139)

Haiven (2007) expands this argument by considering neoliberalism as a pedagogical force in everyday life, which produces and reproduces a culture of fear, individualisation, hopelessness, cynicism, and depoliticisation. This corporate pedagogy promotes obsessive consumerism and atrophies critical and creative skills of young people by creating desires towards and through electronic gadgets such as smartphones, video games, hand-size computers in and out of classroom. Haiven points out that these are complex problems that do not have universal solutions. That is, it is not easy to create counterhegemonic reactions. Responses to these hegemonic dynamics are

more likely to be evolving, collaborative, and unpredictable except in their particular contexts….It seems more fruitful to examine the ways in which radical politics can enable just and lasting forms of collective struggle against the atomising and depoliticising tide of neoliberalism and its attendant militarism, cynicism, and hopelessness. (pp. 91–92)

From this point of departure, it can be concluded that any attempt to counter neoliberal hegemony should aim to create a collective response rather than merely individual efforts.
As public life is becoming more colonised by neoliberal imperatives, children have become one of the most noticeable victims of the process. Bakan (2011, p. 5) provides a detailed account of ways in which neoliberal ideology impacts young people’s lives. He concludes that corporations directly or indirectly influence young people in a variety of ways:

- There is a youth-culture industry, which is ‘is targeting children with increasingly callous and devious methods to manipulate their forming and vulnerable emotions, cultivate compulsive behaviour, and addle their psyches with violence, sex, and obsessive consumerism’ (p. 5).
- Pharmaceutical companies, for profit, produce ‘dangerous psychotropic drugs’ and try to sell them through ‘dubious and often illegal marketing tactics to boost sales’ (p. 5).
- As corporations ‘dump thousands of new chemicals’ into environment, chronic child illness such as asthma, cancer, autism, and birth defects have been increasing.
- Children as young as five years old are working illegally on farms owned by big corporations.
- America’s public schools are becoming lucrative private-sector markets.

Bakan (2011) further elaborates on market-driven changes such as privatisation and standardisation of curricula in public education in the U.S. He argues that the needs of the neoliberal system and the needs of children as human beings are incompatible. As the motivation of a corporation is to produce profit, the neoliberal assumption that privatisation will serve students better is highly questionable.

In response, Bakan (2011) posed the following questions: How can one connect a motivation of increasing profit and meeting the needs of students? What if meeting students’ needs is costly? Within a reified neoliberalism, then, such a ‘cost’ to ‘service consumers’ decreases profit. Would any corporation compromise and put
students before profit? Bakan argues that corporate motivation is becoming increasingly self-centred. By definition, corporations’ interest is productivity, no matter what it means for the rest of the society (Bakan, 2011). Similarly, Chomsky (1999) points out that neoliberalism as the dominant discourse has always and will always put profit over people.

2.4 Education, Democracy, and Neoliberalism

Educational theorists such as Giroux (2014), Abdi and Carr (2013) and Apple (2005, 2014) tend to consider market-driven changes in education as part of the neoliberal project. In this view, market-driven changes imply a certain political and ideological stance to redefine education in market terms. For example, Giroux (2014) argues that recent changes in education cannot be understood without insight into the tenets of neoliberalism:

It is impossible to understand the current assault on public education without coming to grips with the project of neoliberalism and its devaluation of the social, critical agency and informed thinking as part of its attempt to consolidate class power in the hands of a largely white financial and corporate elite. (para. 17)

As Giroux (2014) points out, education in public institutions is targeted by neoliberal globalisation. From a neoliberal view, education is an individual commodity rather than a public service or a human right, and thus it can be sold and bought like any other product in the marketplace (Coco, 2013). By this premise, schools would be more effective if they were privatised and run like businesses.4 Curriculum should be standardised for management and control purposes, focused only on preparing students to enter the future labour force and consumer class (De

4 As the neoliberal approach reshaped higher education in U.S., funding higher education was switched from a collective responsibility to an individual one. As a consequence, college tuition drastically increased over the last 30 years and the student loan debt issue turned into a crisis.
Lissovoy et al., 2015). Apple (2005) elucidates what recent neoliberal educational changes mean in terms of democracy: ‘Consumer choice is the guarantor of democracy’; ‘the ideal of the citizen is that of the purchaser’; and ‘rather than democracy being a political concept, it is transformed into a wholly economic concept’ (Apple, 2005, p. 273). From the neoliberal perspective, education is valued only in terms of its contribution to economy in market terms, which means that anything irrelevant to market growth in education ‘may be legitimately cast off from the ship’ (Brown, 2015, p. 211).

Considering the market as a self-regulating force and reducing citizens to economic beings results in shrinking public space for democracy (Blacker, 2013; Cornelissen, 2014). Therefore, when neoliberal educational changes attempt to redefine education on an economic basis, they undermine democracy as a political concept and obliterate education for critical citizenship. Giroux (2012) elaborates:

The role of public education, if we are to believe the Heritage Foundation and the likes of Bill Gates–type billionaires, is to produce students who laud conformity, believe job training is more important than education, and view public values as irrelevant. Students in this view are no longer educated for democratic citizenship. On the contrary, they are now being trained to fulfil the need for human capital. (para. 5)

In parallel with Giroux’s approach, Ravitch (2011)—who once was a fierce supporter of neoliberal educational changes but later became a strong opponent—points out that such market-driven changes promote more testing, more charter schools (publicly funded, privately run U.S. schools), and less critical thinking. Ravitch suggests that market-driven educational changes can be seen as a project that promotes anti-intellectualism. Therefore, these changes cannot be regarded as reforms. Giroux (2012) again elaborates:
They [neoliberal advocacy groups] are reactionaries and financial mercenaries who are turning teaching into the practice of conformity and creating curricula driven by an anti-intellectual obsession with student test scores, while simultaneously turning students into compliant subjects, increasingly unable to think critically about themselves and their relationship to the larger world. This poisonous virus of repression, conformity and instrumentalism is turning public education into a repressive site of containment, a site devoid of poetry, critical learning and soaring acts of curiosity and imagination. (para. 7)

As a result of market-driven changes, public schools in the U.S. are gradually moving away from being public spheres where students-as-citizens can negotiate, learn, and experiment with skills, democratic values, and knowledge to be able to be active and critical citizens to keep authorities accountable. They have, instead, become places that are colonised by corporations, where students are seen as objects for economic growth from which the ruling class benefits the most.

Neoliberal advocacy groups in the U.S. operate mainly in three domains to legitimise, implement, and institutionalise market-driven educational changes. First, through lobbying, they have already persuaded the federal government to force each state to accept, implement, and institutionalise market-driven changes in public education (Bakan, 2011; Ross & Gibson, 2007; Schneider, 2016). The implementation of common core state standards (CCSS) can be seen as the most recent example of this process (Schneider, 2015). Second, they have been running campaigns to manage public perception in order to manufacture general consent for privatization of public schools.

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5 In the U.S., for example, the Bill & Melinda Gates Foundation, the Heritage Foundation, the Thomas B. Fordham Foundation, the Manhattan Institute, and the Milton and Rose D. Friedman Foundation are some of major organisations that sponsor private think tanks that are fierce supporters of the neoliberal movement in education and initiate a variety of activities in academic, political, and cultural domains to influence public opinion for privatization of public schools.

6 By 2014, some 49 states had adopted a standardized curriculum; the website http://www.corestandards.org/ provides information on standards for each content area.

7 The documentary Waiting for Superman is a good example of the efforts to create the impression that public education is failing, and that ‘bad’ teachers (and the unions that protect them) are responsible for the failure.
neoliberal educational changes (T. Scott, 2011). Third, they aim to influence educational literature, promoting empirical research and scientism in order to reinforce the hegemony of neoliberal ideology in the academic world (Hyslop-Margison & Naseem, 2007). In order to achieve these ends, they impose, from the top down, ideas that deny the linkage between students’ academic success and their socioeconomic background:

Blaming individuals rather than society for student academic failure and generally poor educational quality. Rather than viewing the social structure of opportunity as a primary unit of analysis in education, the focus of empirical research implies that the problems of educational achievement and attainment are individual, technical, and scientific rather than economic, social, and structural ones. (Hyslop-Margison & Naseem, 2007, p. 112)

Advocacy groups for market-driven changes in education argue that students’ family background is irrelevant for students’ academic success. If schools are run like businesses and effective management is in place, they will be successful regardless of anything else (Chubb & Moe, 1990; Schneider, 2016).

Although neoliberals have led and are still leading this educational transformation process, they are not alone. Apple (2000a) considers this educational change as a conservative restoration process: ‘The conservative restoration [that is] guided by a tense coalition of forces, some of whose aims partly contradict others’ (p. 72). Apple argues that conservatives are the second driving force in this coalition: he suggests that while neoliberals are for weak state power, neoconservatives stand for a strong state; cooperation between these parts might appear unlikely. However, as Apple points out, the two factions of the ruling class overcame their differences when their interests overlapped:

The conservative modernization implied in this alliance can overcome its own internal contradictions and can succeed in radically transforming educational
policy and practice. Thus, while neoliberals call for a weak state and neoconservatives demand a strong state, these apparently contradictory impulses can come together in creative ways. The emerging focus on centralized standards, content and tighter control paradoxically can be the first and most essential step on the path to marketization through voucher and choice plan [charter school movement at national level]. (p. 72)

From the perspective of critical theory, this alliance can be understood to be between neoliberals and neoconservatives, two capitalist ruling classes in the U.S. Their class interests are usually in harmony, while their political approaches may differ. Privatisation of public schools benefits both parties as they belong to the same capitalist class. In that sense, neoliberals’ coalition with neoconservatives is not contradictory, as Apple (2000a) points out: ‘As odd as it may seem, neoliberal and neoconservative policies that are seemingly contradictory may mutually reinforce each other in the long run’ (p. 73). Cooperation between the Democratic Party and the Republican Party for market-driven changes in education can be better understood in this context.8

Neoliberal educational policies have been framed and implemented differently, depending on local contexts of countries. In the U.S., both Democrats and Republicans carried out these educational changes. The No Child Left Behind (NCLB) Act, which was signed by former president George W. Bush in 2001, can be seen as a milestone in the process. This federal policy initiative was then carried on by the Obama administration under the title of Race to the Top (RTT). These educational policies have had drastic consequences for public education and the everyday classroom life, both for students and teachers.

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8 For example, the standardisation movement created a big market regarded as ‘edu-business’ for publishing and testing companies. At the same time, this process enables conservatives to prevent educational movements like bilingual education initiatives.
By embracing neoliberal perspective, NCLB and its extension RTT neglect the correlation between students’ academic success and socioeconomic background. They deliberately ignore any perspective that draws attention to how the socioeconomic structure of a society reproduces inequality, ‘because [this correlation] invalidates a conservative and corporate-driven ideological agenda based largely on social Darwinian principles and micro-level accountability’ (Hyslop-Margison & Naseem, 2007, p. 106). That is, advocates for market-driven educational policies claim that scientific research can identify the ‘best practices’ of teaching, which can then be formalised, standardised, and carried out in every classroom, regardless of the class or cultural context that informs students’ daily lives (Darder, 2012).

However, education is a highly context-dependent activity. Hyslop-Margison and Naseem (2007) argue that the market-driven standardisation approach denies the fact that students have wills of their own, their behaviour usually spontaneous and unpredictable:

What works with one student or group of students, or within any given particular classroom, may prove to be a near or complete disaster in another context. Finally, the subject matter itself impacts on the classroom context since one would not teach philosophy in the same manner as grammar or spelling. How are we to make any reasonable sense, then, of generic claims regarding ‘best practice’ given the seemingly infinite variables involved? (p. 109)

The standardisation movement implies a one-size-fits-all approach. It imposes a narrowed and ‘teacher-proof’ curriculum, aiming to replace education by training (Boyles, 1998). It considers teachers as technicians who deliver pre-packaged knowledge to students, rather than public intellectuals who interact with students through a dialogic approach (Giroux, 1988; Sacks, 2009). The RTT initiative ties teachers’ job security and students’ graduation to standardised test scores. Schools
receive less government money if test scores are not high enough (P. R. Carr & Porfilio, 2015; Schneider, 2016). Under this regime, teachers who are concerned about their job security would be tempted to ignore important pedagogical aspects of education (such as education for critical citizenship), and focus instead on having students perform skill-drill type of work to prepare them to pass standardised tests (Apple, 2000b; Darder, 2002; Hursh, 2007a; Hyslop-Margison & Thayer, 2009; Leistyna, 2007; McNeil, 2009).

Education has a vital role to play in developing and sustaining a democratic society (Chomsky, 2003; Dewey, 1916). Therefore, the overall objective of public education is to provide students with opportunities of becoming critical citizens who can actively participate in the process of decision making that affects their individual and social lives, and who are competent to hold authorities accountable. Curriculum and pedagogy should be integrated to foster certain skills, attitudes, and democratic values (Giroux, 1989; Hyslop-Margison & Thayer, 2009; Ravitch, 2011; Skovsmose, 2011). However, neither NCLB nor RTT seems to recognise democratic values and the role that education plays in cultivating them. Hyslop-Margison and Naseem (2007) explain:

An education system designed to respond to the needs of the marketplace predictably appears radically different from one focused on preparing students for the responsibilities of democratic citizenship. NCLB [and RTT] [do] not contain a single reference to either democracy or democratic citizenship. (p. 121)

As neoliberal changes in education attempt to shift away from democratic ideals of education and focus on increasing test scores by portraying students as consumers and future members of the labour force, administrators and principals in public schools are covertly forced to value educational activities as long as they are
oriented towards increasing test scores. Kurt Stemhagen of Virginia Commonwealth University conducted research on the connection between mathematics and democracy. As he was presenting his findings to the school district, he had a conversation with one superintendent. He paraphrased the superintendent’s comments:

‘Can you link specific beliefs to improving [math] test scores? At the end of the day, that is what we have to address….We use MERC\(^9\) [this research] as a way to justify good [math] teaching; can you put something in the report that links beliefs to empirical outcomes so that we can justify acting on the recommendations?’ (Stemhagen, 2011, p. 11)

The superintendent’s perspective cannot be seen as personal opinion; rather, it indicates how the mainstream ideology of education is colonised by top-down neoliberal policies and how difficult teaching for democracy is in the neoliberal era, where education is reduced to raising test scores. This point underlines the importance of the current study, which investigates potentials and limitations of CME in relation to top-down, imposed, market-driven educational policies and implementations.

### 2.5 Positive Aspects of Neoliberalism?

Having discussed neoliberal educational ideologies, it is important to evaluate the potentials and opportunities that such a market imperative enables and facilitates. Considering an educational policy as positive or negative necessitates asking fundamental questions: Is the policy initiative positive (or negative) for whom, from what and whose point of view, and by what measure? As indicated previously, the present study, as critical educational research, aims to review market-driven educational changes through fundamental questions such as these: Whose interests are

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\(^9\) The project was carried out under the auspices of Metropolitan Educational Research Consortium (MERC), a partnership between a particular school of education and seven school districts.
served? What are the consequences of neoliberal educational changes to human emancipation, social justice, equality, and democracy?

Keeping these questions in mind, researchers can try to understand neoliberal educational policies and implementations by comparing recent outcomes and the stated neoliberal promises. As neoliberal educational reforms have been enacted through the NCLB policy, we can review its logic. According to NCLB, public schools in the U.S. are failing, and this failure will negatively affect American dominance in the global market. Therefore, free-market reforms need to be applied to save public schools and produce the future labour force (Bakan, 2011; Kovacs & Christie, 2008; Schneider, 2015).

In Politics Markets & American Schools, Chubb and Moe (1990), proponents of neoliberal approaches to education, frame a neoliberal perspective to point out the failure of public schools, putting forth privatisation as the only viable alternative to fix education. In their view, democratic governance of American public schools is the main source of their failure:

The key to understand why America’s public schools are failing is to be found in a deeper understanding of how its traditional institutions of democratic control actually work. The nation is experiencing a crisis in public education not because these democratic institutions have functioned perversely or improperly or unwisely, but because they have functioned quite normally. Democratic control normally produces ineffective schools. (p. 227)

In this neoliberal view, school choice takes centre stage. Standardised (allegedly objective) test scores inform parents which schools are best (i.e., have the highest test scores). Then parents have a choice: They shop around for the best schools and send their children to these successful schools wherever they are located (the cost of transportation belongs to parents). From this market-based logic, Chubb
and Moe (1990) argue, parental choice will trigger competition among schools; hence, good schools will survive and bad ones will close down. As a result, schools, regardless of their surrounding socioeconomic and cultural conditions, will get better across the country, and the gap between advantaged and disadvantaged students will disappear. Every student would receive a high quality education; every student would go on to graduate from college; the U.S. would continue to dominate the global economy (Chubb & Moe, 1990).

Neoliberal advocacy groups have appealed to some parents and the wider public in general at a surface level, promising to provide the best education for their children and promoting the U.S. dominance in the global market. These proposals might be well-intentioned; however, in contrast to neoliberal advocacy claims, since the NCLB Act was initiated in 2001, there has been only limited school-based anecdotal evidence\(^\text{10}\) to suggest that public schools have responded positively to market-driven initiatives (P. R. Carr & Porfilio, 2015; Hill, 2009; Hursh, 2007a; Kohn, 2004; Kovacs & Christie, 2008; Ravitch, 2016). In fact, research on the impact of market-driven educational changes indicates that ‘low-income students are less able to take advantage of school choice programs due to costs of transportation and information’ (Lincove, 2009, p. 3)—a condition also reflected in the poor educational achievement of working class students of colour (Darder, 2012). Similarly, studies undertaken by S. Ball (1993), Orlowski (2012), and Abrams (2016) indicate that market-driven changes in education have benefitted privileged groups of society and victimised poor and low-income groups. This outcome clearly contradicts the neoliberal argument, which promises to bridge the gap between poor and well-off students (Apple, 2014; Kohn & Shannon, 2002; Kozol, 2005; Schneider, 2016).

\(^{10}\) Such evidence comes from a couple of charter schools that are well funded by advocacy groups and used as showcases.
Another inconsistency in the neoliberal promise for the labour market is the argument that market-driven changes will prepare American students for jobs. In reality, ‘the jobs of future workers [current students] are being relentlessly exported overseas to places with lower labour cost’; thus the claim of preparing the future work force is no more than empty rhetoric or ideological manipulation (Smyth, 2011, p. 15). Besides, there is no direct connection between education and the economy such that improvement in education would improve the economy (Spring, 2014). For example, the great financial crisis in 2008 did not occur because of bad education, but because of the failure of big corporations that were run by highly educated managers and CEOs (Kovacs & Christie, 2008).

Ensconced in a neoliberal perspective, Chubb and Moe (1990) have suggested that an accountability system would increase the quality of teachers and teaching. According to this proposal, teachers’ evaluations would be based on their students’ standardised test results; teachers whose students score higher would receive a little more pay than those whose students score lower. Competitive payment would eventually produce better teachers. Behind this proposal lies a neoliberal ideology that would generate mistrust towards public school teachers. It makes an unethical assumption (De Lissovoy, 2015; Hursh, 2015; Orlowski, 2012) that teachers have the capacity to perform quality teaching, but they do not work to their full capacity, and that competitive salaries would motivate them to work to their full capacity and thus they would be better teachers. At the same time, the proposal suggests removing low-scoring teachers, but offers no explanation about who would replace them.

While none of the neoliberal arguments for educational change has yet been verified by research (Ravitch, 2016), there is evidence that market-driven changes have created a new sector, ‘edu-business’, from which multinational publishing-
testing companies and educational consultancy companies genuinely benefit (Abrams, 2016; Boyles, 1998; De Lissovoy et al., 2015; Hursh, 2007a; Kohn, 2004; Kovacs & Christie, 2008; Schneider, 2015). One may critically assess these outcomes and implications to arrive at possible answers to the question, ‘Market-driven changes are positive (or negative) for whom, from what point of view, and by what measure?’

Taken together, since 1980, neoliberal policies have been legitimised and institutionalised; they have become a dominant discourse. However, research has shown that over the last 35 years, social, political, and economic outcomes contradict the neoliberal arguments that capital accumulation and the market are self-regulating forces that would eventually benefit the whole society. On the contrary, these outcomes indicate that neoliberalism has worked in favour of the ruling classes, at the expense of the rest of the society; it has resulted in increased inequality and poverty at the national and the global scale. In other words, as Habermas (1996) articulates, citizens’ life-world has been largely colonised by neoliberal imperatives; as a result, public spaces have been shrinking.

The shrinkage that Habermas alludes to includes life in classrooms, as public schools are strategically targeted by neoliberal policies. Neoliberal educational changes are instantiated in (mathematics) classrooms as rote memorisation, nondialogic teaching and learning, repetitive skill drill exercises, and standardised multiple-choice assessments, as well as ‘toxic practices [associated with] outcomes, benchmarking, standards, competencies, accountability, performance management, [and] performance pay’ (Smyth, 2011, p. 15). From a neoliberal perspective, as Hyslop-Margison and Naseem (2007) note, ‘Students are portrayed as mere objects in history and ideologically inculcated with a consumer-driven world view devoid of imagination, hope, or alternative social visions’; they are expected to adapt
themselves to existing conditions rather than ‘engaging with or democratically transforming their political, economic, and social landscape’ (p. 121).

Neoliberalism has also been said to influence students’ lives outside of school through the youth-culture industry. Young people are kept from hearing about exploitation, child labour, and corporate colonisation of the life-world. For example, ‘The corporate curriculum teaches no lessons about how consumption works; how consuming is equated with the good life; how advertising constructs their desire, identities, and values; how other values of citizenship are pushed to one side’ (Kenway & Bullen, 2005, p. 40). Market-driven educational practices may serve the needs of the neoliberal system, which tends to consider students as the future labour force and consumers-in-waiting. But educational changes are far from helping students develop critical (mathematical) literacy and the democratic values to become critical citizens (Giroux, 2012; Kohn, 2004; Ravitch, 2011; Skovsmose, 1994).

The neoliberal perspective seemingly conflicts with the needs of individual students to fulfil their potential; it ignores the democratic orientation of education. This situation may lead to an educational crisis and necessitates a timely and critical response oriented towards considering alternative ways of teaching and learning in mathematics classrooms. Being critical of neoliberal educational changes, the current study embraces pedagogical approaches such as critical pedagogy that view humans and society as unfinished, subject to continual evaluation and transformation. As dynamic subjects in history, students, respected as lifelong learners, have a right to influence economic conditions and in the process, create a more just, stable and caring social experience. (Hyslop-Margison & Naseem, 2007, p. 122)

With this in mind, the theory of critical education as a bottom-up response to top-down neoliberal educational changes is presented in the following section.
2.6 A Response: Critical Educational Theory

The concept of critique in the philosophical context has an evolving history dating back to ancient Greek civilisation. Descartes (2008) developed a concept of critique through universal doubt, whereas Kant engaged with it in terms of epistemology in his seminal book, *Critique of Pure Reason* (Kant, 1934). Later, in his critique of Kant, Hegel pointed out the transformative power of self-reflection in his book *Phenomenology of Spirit* (Hegel, 1977). Following this, Hegel’s doctoral student Marx revised Hegel’s dialectical approach, reframing the concept of critique in the context of political economy. The Frankfurt school (FS) then transformed the concept of critique into a school of thought: critical theory (Rasmussen, 1996).

Critical theory was developed by the members of the FS in the 1930s, which ‘was animated by interests in freedom, happiness and justice, and attempted to systematise the political components of Marxism with a critique of political economy and socialist politics’ (Kellner, 1989, p. 44). The members of the FS presented their work under the title of critical theory (CT) due to the politically hostile ambiance against Marxism in the U.S. at that time (Kellner, 1989).

There are diverse lines of thought among the members of the FS. But, as Giroux (2009) points out, the common objectives of the FS were to reassess ‘the newly emerging forms of capitalism, along with the changing forms of domination’ as well as ‘the meaning of human emancipation’ (p. 27). Horkheimer (2002) distinguished CT from traditional theory: Traditional theory, through ‘logical empiricism’ (p. 182) and quantitatively calculable laws of mathematics and science, aims to bring ‘unity and harmony’ (p. 182) into the capitalist system to justify market rules, such as the relationship between supply and demand. Therefore, researchers
who draw on traditional theory ultimately favour the interests of the ruling class and reproduce the status quo.

CT aims to be an interdisciplinary and a holistic theory to detect and challenge social, political, economic, and cultural factors that lead to an oppressive and unjust society. As Kellner (1989) writes,

Critical theory has refused to situate itself within an arbitrary or conventional academic division of labour. It thus traverses and undermines boundaries between competing disciplines, and stresses interconnectedness between philosophy, economics and politics, and culture and society. Critical theory is distinguished from traditional, mainstream social theory through its multidisciplinary perspective and its attempt to develop a dialectical and materialist social theory. (p. 7)

It is important to distinguish, as McLaren et al. (2013) elucidate, the difference between traditional theory and CT in the domain of academic research:

Traditional researchers see their task as the description, interpretation, or reanimation of a slice of reality; [however,] critical pedagogical researchers often regard their work as a first step towards forms of political action that address the injustices found in the field site or constructed in the very act of research. (p. 348)

According to Kellner (1989), the reason that critical theorists focus on the economy and adopt a Marxian critique of political economy is that the economy is a defining element in all domains of social life; the economy ‘provides the framework for developing a theory of society’ (p. 47). However, CT is critical of vulgar interpretations of Marxism, such as ‘economic reductionism and determinism’ (p. 46). More importantly, critical theorists and researchers ‘enter into an investigation with their assumptions on the table, so no one is confused concerning the epistemological and political baggage they bring with them to the research site’ (McLaren et al., 2013,
In this sense, CT implies certain epistemological and ontological stances that are different from both positivist and interpretive research paradigms.\textsuperscript{11}

Some critical educational theorists are inspired by well-known thinkers such as Michel Foucault, Antonio Gramsci, and Luis Althusser. Although not directly associated with the FS, these thinkers are still regarded as critical theorists because of their direct or indirect connection to Marxism. Their ideas have shaped contemporary critical education theory. As McLaren et al. (2013) put it,

Critical theory should not be treated as a universal grammar of revolutionary thought objectified and reduced to discrete formulaic pronouncements or strategies….We are defining a criticalist as a researcher, teacher, or theorist who attempts to use her or his work as a form of social or cultural criticism and who accepts certain basic assumptions. (p. 341)

McLaren et al. (2013) frame these assumptions—the common driving forces of critical theorists—as follows:

- Thought is mediated by power relations that are historically constituted.
- Facts cannot be separated from values; there is no objective knowledge.
- The concept of hegemony as it functions to manufacture consent.
- The role the socioeconomic relationship of capitalist production and consumption plays to form everyday life in a given society.
- Oppression has many faces and an interconnected approach is necessary.
- Mainstream research practices are generally, although most often unwittingly, implicated in the reproduction of class, and race, and gender oppression. (p. 341)

Following the spirit of CT, this study is built on a philosophical frame informed by the complementary ground of the theories of Freire and Habermas. Freire embraced these objectives in his educational work (Freire, 2000, 2013; McLaren &

\textsuperscript{11}This point will be elaborated in the methodology chapter.
Lankshear, 1994); Habermas (1975; 1984;1987;1972) contributed to these common objectives as a member of the FS.

Freire’s educational philosophy has three fundamental roots: Marxism, radical Catholicism, and John Dewey’s progressive philosophy. It is characteristic of Freire’s work that he often refers to Marxian thinkers associated with the FS. Accordingly, CT has a strong influence on his educational theory. Freire uniquely synthesised Latin American liberation theology and Dewey’s progressive approach into his emancipatory project. Among his many books, his seminal book, Pedagogy of the Oppressed, is a classic in the education literature. Freire’s work focuses on the oppressive and liberating potential of education; it aims to frame an emancipatory pedagogy oriented towards transformative social changes (Darder, 2002, 2015; Gadotti, 1996; McLaren & Leonard, 1993; Morrow & Torres, 2002; Roberts, 2010).

Habermas is directly associated with the FS, as Theodor Adorno, a well-known member of the school, was his PhD supervisor. Habermas was a professor of both philosophy and sociology, and his work is wide-ranging. In a consistent way, he dealt with issues of modernism, democracy, and Marxism without rejecting the ideals of the Enlightenment. Habermas argued that modernism is an unfinished (ongoing) process, and so is his work. That work focuses mainly on non-dominating (dialogic) communication to frame an emancipatory project. Habermas distinguishes between the life-world and the systems world, which parallels Marx’s notion of base vs. superstructure (Habermas, 1987; Timur, 2008). Habermas (1987) defines two types of action: strategic and communicative. Strategic action is used by those with power and money (‘the system’) to impose certain ideas, in order to colonise the public’s cultural life (‘life-world’), whereas communicative action aims to reach a consensus through equal contribution of all participants. Habermas (1990) argues that non-dominating
and unrestricted communication requires presuppositions conceptualised as the ‘ideal speech situation’, which can be summed up as follows:

- Every subject with the competence to speak and act is allowed to take part in a discourse.
- Everyone is allowed to question any assertion.
- Everyone is allowed to introduce any assertion into the discourse.
- Everyone is allowed to express their attitudes, desires, and needs.
- No speaker may be prevented, by internal or external coercion, from exercising their rights as laid down above. (p. 89)

This notion of the ideal speech situation can be applied to different settings. When applied to the classroom, what Habermas (1990, 2005) proposes as ideal conditions for undistorted communication can be framed as the following:

- All students should be welcomed. Inclusiveness should be non-coercive.
- Each student should have equal power and opportunity to speak and act.
- Mutual trust, honesty, and respect should be in place. No strategic action or secret agenda should exist.
- Conclusions arrived at should be based on better argument only. No any other coercive forces should play any role in the process of knowledge construction and decision making.

While the life-world can be structured by communicative rationality and claims of validity, the system world is built on distorted communication and claims of power. It follows that transformative changes can be initiated only from the bottom up—from life-world to system. This point of departure inspires critical educational theorists to establish a vital connection between democracy and education in the context of transformative social changes (Darder et al., 2009; Morrow & Torres, 2002; Murphy & Fleming, 2010; Outhwaite, 2009; Young, 1990).
2.7 Habermas and Freire on Communication

As Morrow and Torres (2002) explain, Freire’s and Habermas’s theories are not the same, but rather complementary in their intention towards a more just, equal, and free society:

The distinctive form of critical theory in question here ultimately presupposes a critical realist ontology that rejects essentialism; a constructivist and pragmatist conception of epistemology that is anti-positivist but not antiscientific or purely subjectivist; a pluralist conception of methodology within which participatory dialogue provides a regulative ideal, but it is not an exclusive model of social inquiry. (p. 33)

The ideas of Freire and Habermas have epistemological, ontological, and methodological implications.

2.7.1 Nature of Reality

As noted earlier, Habermas (1987) puts forward two types of action: communicative and strategic. Through communicative action, people may reach consensus and mutual understanding, which may result in a more democratised society. Habermas considers strategic action as the origin of domination. Strategic action is carried out through distorted communication; it is used for control and management. In Freire’s work, similarly, the source of domination comes from anti-dialogical action (Freire, 2000). Morrow and Torres (2002) suggest that whereas Freire defines praxis on the ground of ‘action-reflection,’ for Habermas, praxis is established through ‘symbolic interaction’ (p. 42).\(^\text{12}\)

In Freirean pedagogy, humans are unfinished: the humanisation process is their raison d’être. Through praxis, individuals can transform their society to make it better and become better human beings; it is a perpetual process towards a better future (Freire, 2000). According to Habermas (1979), validity claims and

\(^{12}\) Symbolic interaction means communication among people.
developmental competencies are the keys to future possibility: People need communicative competencies at a certain level to be able to take part in communicative action and develop critical literacy.

2.7.2 Theory of Knowledge

Both Freire and Habermas reject ‘positivist naturalism and classical hermeneutics’ and consider ‘knowledge as communicative process’ (Morrow & Torres, 2002, p. 43). That is, they reject the notion of objective knowledge. Objective conditions may exist, but they can only be appropriated through dialogical social interactions. Knowledge production is a social process and therefore knowledge is not independent of human subjectivity. For example, students have different experiences with their school and therefore their knowledge about their school differs from each other. However, in spite of these differences, they would all agree on the physical existence of the school building.

Human subjectivity\(^\text{13}\) is evident in both Freire and Habermas. Freire’s understanding of knowledge is dialectical: it is established through dialogue (Freire, 2013). Habermas (1984) defines the knowledge construction process through the theory of argumentation and communicative action. In Freire’s view, the outside world can be comprehended via ‘subject-subject dialogue’, whereas, in Habermas’s view, it is through ‘intersubjective consensus within discursive communities’ (Morrow & Torres, 2002, p. 54).

Habermas argues that knowledge is constituted by three types of human interest: empirical-analytical, historical-hermeneutic, and critical-emancipatory (Habermas, 1972). Critical and emancipatory interests enable us to become aware of

\(^{13}\) It is important to emphasise that this subjectivity does not imply absolute relativism as it does in postmodernism.
oppression and domination in a given society, and thereby transform them (W. Carr & Kemmis, 1986). Morrow and Torres (2002) conclude that Freire envisions critical consciousness as necessary for transformative social change. The process of becoming conscious can be materialised through ‘a reflective reappropriation of reality’ (p. 54). However, in the Freirean sense, consciousness can only develop and evolve through a communal process—no individual alone can become socially conscious. It is through dialogical relationship with others and the world that consciousness emerges and unfolds (Darder, 2015). Thus, emancipatory knowledge interests (Habermas) form critical consciousness (Freire).

Types of knowledge reveal different forms of rationality. Freire argues that communicative rationality is oriented towards freedom and emancipation, while technical rationality aims at control, and therefore results in repression (Freire, 2013). Technical rationality is embedded in the banking concept of education in which students are seen as passive receivers and teachers as transmitters (Freire, 2000). Freire considers this technical rationality as the root cause of alienation in modern society. Habermas discusses instrumental rationality, which is similar to Freire’s notion of technical rationality. Both Freire and Habermas put forward communicative rationality as an alternative to technical or instrumental rationality (Morrow & Torres, 2002).

2.7.3 How to Seek Knowledge

In order to generate different forms of knowledge, Freire advocates participatory action research and contextual plurality (Morrow & Torres, 2002). Habermas (1973) argues that research methods should resonate with the context of the research and be developed through participation. Freire advocates an ‘engaged conception of inquiry’ in which human subjectivity plays an important role. However,
Freire is against ‘reducing research to mere ideological struggle’ (Morrow & Torres, 2002, pp. 57-61). From this point of departure, one can conclude that both Habermas and Freire embrace communicative (dialogical) action as a way of doing inquiry to seek knowledge.

It is also important to note that dialogue takes a central place in both Freirean pedagogy and in educational studies inspired by Habermas’s theory. But in educational contexts, there are other approaches to dialogue besides those of Freire and Habermas. In the next section, I elaborate on qualities of communication in the classroom and their epistemological, social, and political implications.

2.8 Dialogue, Collaboration, and Inquiry in Mathematics Classrooms

There has been an increasing emphasis on the importance of dialogue for quality of education ‘due to growing frustration at the dehumanising effects of the “standardisation and achievement” approach’ driven by neoliberal policies (Aloni, 2013 , para. 6). The term dialogue is widely used in a variety of contexts across the education literature, and may lead to confusion and misconception about its implications. Therefore, it is necessary to distinguish the concept of dialogue in the context of critical (mathematics) education from traditional approaches to dialogue. Dialogue as a pedagogy takes centre stage in Freire’s work and provides a solid foundation for its implications for education. Habermas’s theory of communicative action—oriented towards non-dominating communication—provides a complementary framework to consider dialogue as an essential component of emancipatory pedagogy.

Empirical research indicates that in mathematics and science education, teachers tend to use authoritative (nondialogical) talk, as it makes it easy to transmit axioms and theorems (Alexander, 2005; Mortimer & Scott, 2003; P. Scott et al.,
Although this may be justified in the context of instrumental rationality, it raises an important question from the standpoint of CME; namely, to what extent is authoritative (nondialogical) teaching acceptable in the mathematics classroom? In the CME literature, *dialogical* teaching is strongly emphasised, but CME writing is mostly theoretical and lacks classroom-based examples. Edwards-Groves, Anstey, and Bull (2014), working in the primary school, go some way to address this gap by providing empirical examples of the interactional influence of more dialogic approaches in classroom lessons. Nevertheless, there is a dearth of empirical research investigating dialogue as pedagogy in secondary classrooms.

Approaches to dialogue in education come from two main perspectives. As Nicholas and Bertram (2001) explain,

The contemporary vision of dialogue as a pedagogy that is egalitarian, open-ended, politically empowering, and based on the co-construction of knowledge, reflects only certain strands of its history. Contrasting accounts see dialogue as a way of leading others to pre-formed conclusions; or as a way for a master teacher to guide the explorations of a novice; or as a set of ground rules and procedures for debating the merits of alternative views. (p. 1102)

This statement divides dialogical approaches into two main lines of thought. Dialogue as communicative rationality (the contemporary vision) and dialogue as instrumental rationality (the contrasting account). The theoretical underpinnings of the present study, complementary with Freire and Habermas’s ideas, are inspired by the former strand, which embraces dialogue as communicative rationality. From this point of departure, dialogue as pedagogy is endorsed because it democratically promotes horizontal teacher-student relationships; it opens up possibilities by promoting critical and creative thinking; it is liberating by helping students to develop communicative competency and critical consciousness; it is a collaborative process through which students can learn *with* and *from* each other. The essence of this approach is that it
seeks ways of establishing non-dominating communications and relationships in an educational context oriented towards critical literacy and humanising education. Dialogue, hence, is not only an effective way of learning, but also an end in itself. As such, Freire considers dialogue to be an existential necessity (Freire, 2000).

However, the ‘contrasting account’ sees dialogue through a lens of technical (instrumental) rationality, considering it to be an effective way to transmit pre-packaged knowledge to students. Habermas (1984) considers this form of communication, in which a pre-set agenda is imposed on learners, as distorted; similarly, Freire defines it as ‘banking’ education—an oppressive pedagogy in which an anti-dialogical process disseminates dehumanising education (Darder, 2002; Freire, 2013; McLaren & Kincheloe, 2007; Shor, 1987).

Rogers (1995) advocates that teachers adopt a facilitative attitude, as opposed to an authoritarian one, and thereby create a liberating classroom environment. He distinguishes between dialogic and traditional classrooms. According to Rogers, in traditional education,

- The teachers are the possessors of knowledge, and the students the expected recipients.
- The lecture, or some means of verbal instruction, is the major means of getting knowledge into the recipient. The examination measures the extent to which the students have received it. These are central elements of this kind of education.
- The teachers are the possessors of power, the students the ones who obey.
- Rule by authority is the accepted policy in the classroom.
- Trust is at a minimum.
- The subjects [students] are best governed by being kept in an intermittent or constant state of fear.
- Democracy and its values are ignored and scorned in practice.
• There is no place for whole persons in the educational system, only for their intellects. (pp. 295–297)

However, Rogers (1995) also articulates a ‘person-centred’ education which draws on a dialogic approach, as follows:

• Teacher as authority figure should be ‘sufficiently secure’ within herself/himself ‘in relationship to others’ to generate trust.
• The teacher, as facilitator, shares ‘responsibility of learning’ with students.
• The teacher designs inquiry-based projects derived from students’ life.
• The students organise their own learning project [collectively or individually].
• An atmosphere that is conducive to learning from and with each other.
• Learning how to learn is the main focus.
• A facilitative classroom environment that promotes self-discipline, reflection, and learning which tends to be deeper. (pp. 299–301)

Although these clarifications draw a thick line between two interpretations of dialogue, they raise an important question in context of CME. In mathematics classrooms, can direct teaching or lecturing be done through a liberating dialogue, when students are introduced to a new concept or subject? More specifically, how can a mathematics teacher avoid ‘banking education’ as she teaches a mathematical axiom or theorem, for which the lecture is often a preferred teaching method? To date, the CME literature does not have a comprehensive answer to this question. The present research aims to provide one.

Nicholas and Bertram (2001) argued that a lecture (direct teaching) could be structured dialogically:

On the one side, lecturing or more ‘monological’ approaches can actually take a range of forms. Some lectures do indeed ask only to be heard and remembered (although even doing this with some success at gaining
understanding requires a much more active response by the student than simply hearing and recording the data). Other lectures invite a high degree of thoughtfulness, scepticism, and imaginative response by the audience; some could even be considered ‘dialogical’ in the sense that the speaker frames issues and questions in a way that invites an active reinterpretation of meaning from multiple standpoints among the listeners. (p. 1108)

This theoretical argument, which claims that even lecturing can be dialogical, does not counter the argument that nondialogical teaching is still the dominant approach in most mathematics classrooms.

In a comparative international project concerned with the dimensions of classroom talk, Alexander (2001, 2015) found that dialogical teaching is rare in classroom interactions. Instead, teacher practices mostly involved strategies such as rote memorisation, repetition, recitation, and direct instruction. These types of talk usually lead to instructional moves formalised as initiate-respond-evaluate (IRE) and initiate-respond-feedback (IRF). This finding replicated research conducted in the U.S. by Mehan (1979). In a comprehensive analysis of classroom talk, Freiberg and Freebody (1995) demonstrated the limitations that an IRE interactive structure places on the interactive potential of students. Alexander (2015) argued that although IRE and IRF might teach students some analytical skills, they do not foster critical literacy.

Dialogic interaction has been shown to be a key in children’s development of thinking and understanding. Its implications extend across at least five domains: neuroscientific, psychological, sociocultural, political, and communicative (Alexander, 2001). From this point of view, Alexander (2015) argues that

Rote, recitation, instruction and exposition are frequently used: indeed, worldwide they are probably the default modes of teaching talk. There is

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14 The Five Nation Study was conducted in Russia, France, the U.S., England, and India.
always a place for them, but discussion and dialogue are less common and children need to experience them much more frequently. Discussion and dialogue require learners not merely to listen and answer, but also to think, engage and take decisions about their learning. By using discussion and dialogue we seek to empower learners both cognitively and socially, not merely to tell them things or test what they know already. (p. 3)

Nystrand (1997) and Edwards-Groves et al. (2014) agree that a more dialogic approach to instruction motivates and engages students since it values students’ contribution to the learning process, evokes interest in students’ thoughts, and makes them feel included. Alexander (2015) acknowledges pedagogic implications of dialogic teaching, arguing that the dialogical approach is much better than the ‘best-practices’ paradigm imposed by the neoliberal educational movement. But he considers nondialogical (authoritative) approaches as a necessary part of dialogical classrooms.

A similar approach is put forward by Mortimer and Scott (2003). In their view, authoritative classroom discourse is appropriate when mathematics and science teachers introduce new concepts or topics, but when students’ ideas and views are being exposed informally, a dialogic approach is more appropriate. This perspective is based on instrumental rationality and can therefore produce functional literacy only. However, when Mortimer and Scott (2003) suggest that authoritative talk is inevitable in dialogic classrooms, they pose a conundrum that CME has to deal with, both theoretically and practically: Researchers in CME, taking standardisation movements in education into account, need to do much more study on the scope and limitations of dialogue in mathematics classrooms. In this sense, the original contribution of my research may be significant.

Researchers suggest different ways to assess whether teaching is dialogic or
not. Burbules (1993) proposes two criteria for dialogic teaching: (a) Is the lesson critical and inclusive? (b) Is the lesson oriented towards a single answer, or does it allow students to make their own interpretation? Alexander (2015) outlines a more comprehensive perspective, suggesting five criteria for dialogic teaching:

- Collective: Participants address learning tasks together.
- Reciprocal: Participants listen to each other, share ideas, and consider alternative viewpoints.
- Supportive: Pupils express their ideas freely, without fear of embarrassment over ‘wrong’ answers, and they help each other to reach common understandings.
- Cumulative: Participants build on answers and other oral contributions and chain them into coherent lines of thinking and understanding.
- Purposeful: Classroom talk, though open and dialogic, is also planned and structured with specific learning goals in view. (p. 4)

Alexander (2015) does not provide any consistent philosophical background for his criteria, but combining these five conditions, it can still be concluded that dialogue in classroom is not simply a conversation. Dialogue is therefore considered a much more comprehensive approach to teaching and learning. Even informed by instrumental rationality, classroom talk should have certain characteristics to be considered dialogue. Even though Alexander (2015) does not address asymmetrical power relations between students and teacher, and does not think of critique as an educational task, a theory of critical education would not necessarily disagree with his proposal, but it would pose a fundamental question: What if students reach a common understanding through a free discussion and that common understanding is in contrast with the official or dominant understanding in a given society? There is no glimmer of an answer to this question in Alexander’s approach. Perhaps it is because he thinks that education is not a political domain.
Drawing on theories of Habermas, Freire, and Vygotsky, Flecha (2000) proposes seven criteria for an interaction to be a dialogic learning: egalitarian dialogue, cultural intelligence, transformation, instrumental dimension, creation of meaning, solidarity, and equality of differences. From this standpoint, dialogue as pedagogy generally aims to transform individuals and society, rather than helping students to passively adapt to existing conditions. This perspective differentiates dialogue based on communicative rationality from dialogue based on technical or instrumental rationality. However, Flecha (2000) indicates that dialogical learning does not reject the instrumental dimension of knowledge. For example, students in algebra class need the quadratic formula to solve second-degree equations. A dialogical stance is not against learning this formula; instead, it problematises ways in which the formula is taught and learned.

Along these lines, Darder (2002), in Reinventing Paulo Freire, engages Freire’s notion of teacher directivity or the directive nature of education. She quotes Freire as follows:

All educational practice is directive by its very nature, the question the coherent progressive educators must deal with is what do they need to do to diminish the distance between what they say and they do as not to allow directivity into authoritarianism or manipulation. (Freire, 1993, p.116)

Elaborating on Freire’s assertion, Darder (2012) writes:

Rather than placing emphasis strictly on the directive quality of instructional methods (e.g., lecture, worksheets, vocabulary list, science manuals, etc.) that may be employed for the introduction of required content in different subject areas—an absolutely legitimate and necessary component of teaching and learning—a revolutionary practice is concerned with the underlying intent and purpose of knowledge that is being presented and the quality of dialogical opportunities by which students can appropriate the material to affirm,
challenge, and reinvent its meaning in the process of knowledge production. (p. 114)

A dialogic classroom implies interactions not only between the teacher and students, but also among peers. This brings us to a point at which inquiry-driven and collaborative learning approaches need to be incorporated into the dialogic classroom. In critical pedagogy, according to Shor (1987, p. 95), dialogue is both ‘the form of study’ and ‘a democratic model’ of teacher-student and peer relationships. Dialogic teaching and learning are closely associated with collaborative and inquiry-based education, in which dialogue takes place between students and teacher and among students (Shor, 1987). Depending on epistemological perspective, collaborative and inquiry-based learning may take different shapes and be framed in different ways.

Some theoretical studies suggest that dialogic and collaborative learning improves the quality of students’ mathematics learning (Horn, 2014; National Council of Teachers of Mathematics, 2000; Pietsch, 2009). However, there is very limited empirical study of students’ learning in collaborative classrooms (Plaza, 2010). The theoretical studies elucidate the pedagogical challenges of implementing collaborative learning in a traditional classroom setting—where the classroom is ruled by authority, the teacher is knowledge transmitter, students are passive receivers, and learning is a competitive process (Flecha, 2000; Kohn, 2006; Rogers & Freiberg, 1994; Smith & MacGregor, 1992).

There are, however, theoretical approaches—such as Vygotsky’s social constructivist learning theory—that combine dialogue, collaboration, and inquiry in a classroom. Vygotsky (1978) introduced the concept of the zone of proximal development (ZPD), which is crucial to understand the dynamic in classrooms where students are engaged in inquiry-based learning, and knowledge is constructed
collaboratively (Skovsmose & Alrø, 2004). According to ZPD theory, a student’s intellectual abilities drastically differ when performing alone compared to when performing with the assistance of the teacher or a more competent classmate in small-group work. The concept of ZPD has evolved with contributions of many educators, thinkers, and researchers. Specifically referring to peer interaction in mathematics classrooms, Cesar (1998) suggests that ZPD can happen even without a more competent peer in the group:

Recent studies showed that peer interactions are much more powerful in themselves than what Vygotsky conceived, as both in asymmetric and symmetric dyads pupils are able to progress and, more important still, in asymmetric dyads they both progress. This means that there is no need for a more competent peer in order to facilitate better performances, interaction itself is enough. (p. 3)

Working in each other’s ZPD, students learn with and from each other. The relationship between students and teacher is similar. As Wells (1999) indicates, ZPD shifts the teacher from being ‘primarily a dispenser of knowledge and assigner of grades’ to ‘a fellow learner whose primary responsibility is to act as leader of a community committed to co-construction of knowledge’; in short, the teacher also learns with and from their students (p. 331). This aligns with Freire’s notion of horizontal student-teacher relations. Wells (1999) summarises the significance of ZPD:

- ‘It is constructed in the interaction between participants in the course of their joint engagement in a particular activity’.
- It creates an educational ambiance in which each can learn with and from others.
- ZPD ‘leads to development of identity as well as of skills and knowledge’.

The quality of the interaction between students is crucial. ‘Learning will
be successful when it is mediated by interaction that expresses mutual respect, trust and concern’.

- Learning in the ZPD leads to both individual and societal transformation as each one is dialectically connected to the other one. ZPD does not aim at a fixed or pre-determined end. ‘The ZPD is thus a site of conflict and contradiction as well as of unanimity; the transformation it engenders leads to diversity of outcome which may radically change as well as reproduce existing practices and values’ (p. 333).

These dimensions lead to the conclusion that ZPD provides a solid framework for critical mathematics teachers as they develop and teach lessons that are inspired by a more dialogic approach, engage in collaborative study to co-construct knowledge, and use inquiry as a way of dealing with uncertainty. Wells (1999) considers communities of inquiry as ‘joint-will’ to find answers to shared questions: ‘The aim of inquiry is not knowledge for its own sake, but disposition and ability to use the understandings so gained to act informally and responsibly’ (p. 121). In this sense, a community of inquiry fosters individual and social agency as they are dialectically connected. Inspired by Vygotsky’s theory, Kennedy (2009) argues that through dialogue, inquiry, and collaboration, a mathematics classroom can become a community of inquiry:

Community of inquiry theory and practice offer new ways of understanding and rethinking the teaching and learning of mathematics, and new insights into how school mathematics might be reconstructed as collaborative dialogic inquiry. Its emphasis on communal dialogue makes it an ideal medium for the interplay between individual and collective cognitive and psychodynamic processes in the development of mathematical concepts, and in the development of the skills and dispositions of argumentation. In addition, it offers a promise for the transformation of mathematics teaching and learning from a rigid, transmissional model to one which is student-centered, self-regulatory, and inquiry-driven. (p. 76)
In this approach, collaboration and dialogue are not seen as instrumental tools to improve learning—dialogue is the end in itself, as it is in Freire’s dialogic pedagogy. Kennedy (2009) elaborates on this inquiry system further, arguing that community of inquiry theory can help to create conditions for the ‘ideal speech situation’ as defined by Habermas (1990). From this perspective, as Flecha (2000) points out, dialogic learning requires egalitarian and reflexive dialogue between the teacher and students, as well as among students. When knowledge is constructed by a course of communication, it draws on a validity claim, not a power claim. Turning a mathematics classroom into a community is central to dialogic teaching. However, there is to date no classroom-based research in the CME literature to generate insight into creating a community of learners. My research aims to address this gap.

These ideas date back to the work of educationalist and philosopher John Dewey (1916). Dewey envisioned inquiry as an essential part of an education that would help students become competent democratic citizens. He explicitly criticised transmission styles of education: ‘Education is not an affair of “telling” or being told, but an active construction process’ (p. 38). Dewey envisioned inquiry as a collective activity and a process of becoming members of a community.

Staples (2007) also considered inquiry to be a collaborative activity. She framed inquiry-based lessons in high school mathematics into two categories:

Inquiry into mathematics and inquiry with mathematics. Inquiry into mathematics involves delving into mathematical ideas and concepts and trying to understand the structure, power, and limitations of mathematics. Inquiry with mathematics involves using mathematics as a tool to make sense of problem situations and come to some reasonable resolution. This type of work involves problem solving, modelling, and applications to business, physics or other ‘real-world’ phenomena. (p. 2)
Staples (2007) indicated that collaborative inquiry requires a particular classroom environment. In order to construct a classroom ambiance suitable for collaborative learning and dialogical inquiry, one has to carefully consider potential obstacles: What factors could keep a student from actively participating in the collaborative learning process?

Flecha (2000) anticipates three factors that may prevent dialogue: cultural, social, and personal. That leads to another question: How do we overcome these barriers? Unfortunately, there is not enough research to answer following practical questions: What if an individual student in a group self-assigns him- or herself as leader? What kind of skills, values, and knowledge do students need in this process? How do we consider power relations among students in the class? What consideration, if any, should be given to the physical organisation of the classroom? How should students be grouped, and by what criteria? Is peer tutoring regarded as collaboration or not? These questions require an intense focus on the dynamics of collaborative learning in order to map the scope and limitations of CME—an overarching aim of the present study.

Rogers (1995) argued that collaborative learning could occur in a community of learners when there is dialectical connection between individuals and the community:

The sense of community does not arise out of collective movement, nor from conforming to some group direction. Quite the contrary. Each individual tends to use the opportunity to become all that he or she can become. Separateness and diversity—the uniqueness of being ‘me’—are experienced. (p. 190)

The type of community Rogers (1995) described is one that differs from a herd, which is formed by collective compliance. In his view, a teacher’s facilitative
attitude—as opposed to an authoritarian attitude—plays a significant role in creating such a community of learners:

When teachers are empathically understanding, their students tend to like each other better. In an understanding classroom climate, every student tends to feel liked by all the others and has more positive attitudes towards self and school. This ripple aspect of the teacher’s attitude is proactive and significant. (p. 161)

Alexander (2006) classroom-based research showed that when learning activities were collaborative rather than competitive, whole-class discussions and dialogical teaching were most effective: A classroom ambiance that generates a feeling of collaboration nurtures dialogical interaction. Kohn (1992) elaborates this argument in his seminal book, *No Contest: The Case against Competition*. He explains that competitive learning undermines students’ self-esteem, confidence, and overall development. Competitive learning generates anxiety and encourages students see their peers as obstacles to their success. This eventually creates a toxic environment that is not conducive to humanising education. Kohn also acknowledges that competition in the U.S., in education and other domains of life, has become a kind of sacred belief imposed by the state. He argues that competitive learning does not serve the needs of students as human beings.

Similarly, hooks (2003) considers competitive learning as dehumanising. In her view, competitive learning creates an atmosphere where the sense of community and solidarity is lost:

Competition in the classroom disrupts connection, making closeness between teacher and students [and peer relationships] impossible….Competition rooted in dehumanising practices of shaming, of sadomasochistic rituals of power, preclude communalism and stand in the way of community. (p. 131)

Moreover, hooks concludes that competition in the classroom may result in atomised
individuals and selfish consumers—just as envisioned in neoliberal ideology.

Skovsmose and Alrø (2004) conducted classroom-based research in a secondary mathematics classroom in Denmark, as part of the research initiated by the Centre for Research in Learning Mathematics, to investigate connections between qualities of communication and qualities of mathematics learning. They concluded that dialogue cannot be separated from action, and therefore they put forward an inquiry community (IC) model—a critical approach to communication in the classroom that resonates with dialogic teaching. According to the IC model, dialogue as inquiry in a mathematics classroom aims to challenge the traditional, non-critical approach to mathematics education. The IC model (Skovsmose & Alrø, 2004) includes elements such as the following:

1. Getting in contact: as an entry stage of inquiry, students know and confirm each other.
2. Identifications of mathematical ideas.
3. Answering why-questions.
4. Reflections through thinking aloud.
5. Reformulation of collaborative understanding.
6. Challenging though hypothetical questions.

Skovsmose and Alrø (2004) stated that the empirical evidence did not prove their hypothesis, but their classroom observations still guided them to certain conclusions. The IC model represents a set of dialogical actions in the mathematics classroom: ‘Such an act involves making an inquiry, running a risk, and maintaining equality’ (Skovsmose & Alrø, 2004, p. 15).

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Skovsmose (1994, 2011) conceptualizes the non-critical approach as the ‘exercise’ paradigm. In the next sections, I will return to this concept.
Aulls and Shore (2008) state, ‘It is important to recognise that the experience of being an inquirer is often liberating, exciting, and sometimes transformative. To inquire entails some risk and ambiguity and, therefore, like all complex learning, it is also demanding’ (p. 1). Inquiry approaches challenge rote learning in mathematics and science education (Aulls & Shore, 2008; Manconi, Aulls, & Shore, 2008; Tsankova & Dobrynina, 2005). Barfurth and Shore (2008) argue that ‘Combining peer collaboration with inquiry learning into classroom-based activities opens the possibilities to witnessing and understanding new learning processes that might be quite distinct from traditional classroom behaviour and learning’ (p. 150). Through inquiry, as Manconi et al. (2008) indicate, students may develop collective reflections and transferable process skills; they may learn content matter for conceptual understanding and problem-solving strategies. Freire (1997) conceptualises inquiry-based education as activity oriented towards problem solving through a problem-posing approach.

Skovsmose and Alrø (2004) point out that the dialogic approach runs the risk of unpredictability of direction in the classroom; it ‘can stir up emotional problems as well as cause enjoyment’ (p. 122). Equality is an interpersonal relationship essential for dialogue, because ‘in a dialogue, there is no demonstration of power’ (p. 124). In this view, dialogical teaching and learning reconnect mathematics education with democracy, in the sense of Dewey: ‘the significance of dialogic teaching and learning in mathematics has to do with the critical relationship between mathematics education and democracy. *Dialogic teaching and learning is significant for classroom practice that supports a mathematics education for democracy*’ (Skovsmose & Alrø, 2004, p. 136; emphasis in original).
Based on the discussion above, a dialogic approach to mathematics education, in context of inquiry and collaboration and from the perspective of critical theory, can be framed as follows:

- Learning should be a collaborative act in the spirit of solidarity, not competitive.
- Communication in the classroom should be based on validity claims, not power claims.
- Inquiry should be open-ended and related to the life-world of students.
- Dialogue should be egalitarian, non-hierarchical, and not authoritative.
- Dialogue should be an end in itself, rather than instrument of something else.
- Dialogue should aim for transformation, not adaptation.

2.9 Critical Mathematics Education

One may question why—as there are no categories of study like critical English education, critical history education, or critical biology education—do we need a special category for critical approaches to mathematics education? Would not critical pedagogy or critical literacy be enough to cover all fields, including mathematics? This might be explained by unpacking the common perception that mathematics is something unto itself and is thus a value-free subject (Aslan Tutak, Bondy, & Adams, 2011). The logic behind this perception may be illustrated as follows: If two plus two equals four, then what difference would it make how this fact is taught? Based on a three-year-long, classroom-based study in a secondary mathematics class, Gutstein (2006) concluded that the difference can be understood by asking questions such as these: ‘What constitutes mathematical literacy, how does it relate to political economy, and how do different conceptions of it relate to the ways schools educate students?’ (p. 5). The first point of departure of critical mathematics
education is that mathematics education is a political domain (Frankenstein, 2010; Gutstein, 2006; Skovsmose, 2011; Yasukawa & Johnston, 2001).

2.9.1 What and Why Is Critical Mathematics Education?

There are different terms used to name critical studies in mathematics education: *radical mathematics* (Frankenstein, 1983), *mathemacy* (Skovsmose, 1994), *critical mathematics* (Ernest, 2002b), *mathematics-statistics for critical citizenship* (National Institute of Adult Continuing Education, 1997), *matheracy* and *ethnomathematics* (D'Ambrosio, 1999; Powell & Frankenstein, 1997), *mathematical literacy* (Gutstein, 2006), and *critical numeracy* (Yasukawa & Johnston, 2001).

Critical mathematics education (CME) can be seen as an umbrella term that captures all these sub-terms. CME can be defined through *concerns* that are related to concepts such as socioeconomic diversity, equity and justice, and students’ and teachers’ autonomy (Skovsmose, Nielsen, & Powell, 1995).

Skovsmose (2011) attempts to unpack CME through concepts such as uncertainty, diversity of situations, students’ foreground, landscape of investigation, critical conception of mathematics, reflection, and mathemacy. However, he emphasises that his theory is far from being a complete system. Since 1985, Ole Skovsmose has developed critical ideas about mathematics education, which can be framed under the title of CME. He was the first to link critical theory with mathematics education. His studies focus on ‘epistemological issues and social contexts and issues concerning mathematics, with special emphasis on educational and social critique/social justice’ (Ernest, 2010, p. 65). He is regarded as the father of CME.

Skovsmose (1994) proposed three concepts as integral to practising CME. The first is *critical distance*, which means keeping a distance from the mainstream
curriculum by filtering problems in textbooks through the lens of whose interests are served and what ideas exist behind them. Freitas (2008) states that ‘One way of practicing critical mathematics is to revise what is offered through the mainstream curricula using the concerns [of CME]’ (p. 82). The second key concept is the critical competence of teachers and students that can be developed through active participation and dialogical student-teacher relation. The third is critical engagement.

Skovsmose (2011) argues that word problems oriented towards CME should be based on two criteria: They should be relevant to students’ life-world, and they should address socially significant issues. Myriad aspects of our life-world are shaped by mathematics. Therefore mathematics can have empowering and disempowering effects both at the individual and societal level. Skovsmose (1994, 2011) explains this through the formatting power of mathematics: Mathematics is one of the defining elements that shape business life, technology, and the economy, as well as the daily life of people. In his view, a student needs critical mathematical literacy to be able to identify and challenge oppressive and non-democratic aspects of this formation. This leads to another question: What kind of learning nurtures critical literacy?

2.9.2 Critical Literacy in Mathematics Education

Critical literacy is an important concept in critical pedagogy. Freire distinguishes critical from functional literacy. From Freire’s perspective, critical literacy takes places when reading the word leads to reading the world (Freire & Freire, 2004). Shor (1993) describes critical literacy as follows:

Critical literacy is analytic habits of thinking, reading, writing, speaking or discussing which go beneath surface impressions, traditional myths, mere opinions, and routine clichés: understanding social contexts and consequences of any subject matter; discovering the deep meaning of any event, text,
Frankenstein (1990) advocates critical mathematical literacy. An educator in a community college in the U.S., she uses ‘statistical data to reveal and explode all the sentences and myths that are slipped into our lives in so many ways’ (p. 336). Frankenstein argues that critical mathematics literacy includes the ability to read numerical data in a way that allows citizens to ‘question taken for granted assumptions’ (p. 336). Frankenstein (2005) sets out four objectives of critical mathematical literacy:

- Understanding the mathematics.
- Understanding the mathematics of political knowledge.
- Understanding the politics of mathematical knowledge.
- Understanding the politics of knowledge. (p. 1)

In her statistics class, Frankenstein uses statistical reports from different sources to stimulate discussions on issues such as discrimination, inequality, and injustice to achieve the objectives listed above.

Gutstein (2006) conducted a comprehensive auto-ethnography of his classroom teaching from 1997 to 2003 in a Chicago public middle school, where most of the students were from a Latino immigrant community. In order to teach mechanical aspects of mathematics, Gutstein used percentage-increase problems, ratio problems, and some algebra problems. In order to link mathematics to students’ life-world, he used real-world problems about neighbourhood development, the cost of housing, average household income, and people’s buying power. Students kept journals in which they reflected on their learning experiences: ‘This [project] let me see how unfair the world really is and all the superiority white people feel above and beyond what they call minorities’ (p. 68). This example indicates that Gutstein’s
classroom practice successfully fostered critical mathematics literacy, as students were able to identify socioeconomic inequalities and racial discrimination affecting their life in and out of school.

However, there are three critical points about Gutstein’s research that require caution. First, he was a university professor who was implementing a new curriculum as ‘teacher’ in a middle school mathematics classroom (i.e., he was not the regular public school teacher). His status as a university professor may have provided him with a great advantage in terms of classroom management, changing the curriculum, and relations with parents. Second, he taught in a school where 98% of the students were from low-income Latino families, making the cohort relatively homogenous. Third, he focused on basic algebra content, which, although it fit the middle school mathematics curriculum, provides no perspective for advanced topics (e.g., trigonometry or solving higher-degree equations) in high school level mathematics.

Through a classroom-based study in a high school mathematics class, Skovsmose (1994) developed projects (lessons) to cultivate critical mathematical literacy (CML). One of his projects primarily aimed ‘to develop and analyse situations from daily life as well as facilitating the project to address some global problems concerning the use and supply of energy’ (p. 155). Skovsmose proposed two criteria for problems to ensure that students develop CML: relevance to the students’ life-world, and conducive to making generalisations.

A lack of curricular material for practising CME is a major challenge for mathematics teachers who are willing to foster critical literacy in mathematics. Freitas (2008) points out that these ‘real-life’ problems in traditional mathematics textbooks usually represent official knowledge, and thus they do not connect with critical citizenship or social justice. Instead, they are oriented towards functional literacy and
picture students as consumers. Revising problems from traditional textbooks in light of CME’s objectives, Freitas suggests that mathematics teachers can produce their own open-ended problems and incorporate them in daily classroom practice. Andersson (2012) also indicates that CME should develop a language to re-contextualise mathematics education. Teachers should be careful about the ethical and political dimensions of real-world problems, but that is not enough. D. L. Ball, Goffney, and Bass (2005) suggest that teachers should take the sociocultural background of students into consideration when revising or designing real-world problems:

The settings for many mathematics problems are most familiar to middle class white students. Plans for garden plots, mileage covered on family vacations…[these] contexts may be more familiar and engaging to some students that to others….The enthusiasm for ‘real-world’ problems, left unchecked, may disadvantage students for whom the chosen settings are not understood or valued. (p. 3)

Real-world problems can be a point of departure to create a classroom where students democratically negotiate and critically reflect on what they learn.

However, the work of Andersson (2012) and D. L. Ball et al. (2005) is theoretical, pointing to a gap between theory and practice. Freitas (2008), however, tested her idea: she had pre-service teachers in a teacher education faculty revise word problems from mainstream mathematics textbooks. Although interesting, this process does not provide insight into possible outcomes when such problems are used in classrooms. This kind of insight can come only from classroom-based research that covers preparation, application, and reflection stages of word problems.
2.9.3 The Mathematics Classroom as a Democratic Public Sphere

As noted, the connection between education and democracy has a long history, going back at least to Dewey (1916). Even though later theorists such as Freire (2000), Chomsky (2003), and Russell (2009) establish this linkage differently, education in terms of democracy is usually considered a way of life rather than simply citizens’ right to vote. One view of democracy—a ‘thick’ version—promotes participatory and justice-oriented citizenship, whereas a contrasting ‘thin’ view describes a market-driven and consumer-based citizenship, promoting personal responsibility (Gandin & Apple, 2002; Hyslop-Margison & Thayer, 2009; Lund & Carr, 2008; Westheimer, 2015). In the context of participatory and social justice-oriented citizenship, mathematics education can be connected with democracy in many ways (Almeida, 2010; Vithal, 1999; Wagner & Davis, 2010).

Indeed, democracy itself has been variously defined. For the purposes of the current study, the following definition will be used to capture the links between democracy and critical theory. Critical theory considers democracy to be a process of emancipation, through which ‘human beings consensually choose and control the conditions of their lives to the greatest extent possible’ while rejecting ‘liberal possessive individualism as the basis for a democratic polity’ (Bohman, 1996, p. 190). In other words, critical theory rejects the capitalist norm as an essential marker of democracy. This rejection houses a significant point. As Hyslop-Margison and Thayer (2009) point out, having a free election process is by itself not sufficient for a country or society to be democratic: ‘Electoral processes provide a necessary condition for democratic societies, but they also create the illusion of democratic legitimacy’ (p. 1). If the great majority of citizens do not take part in an election, it cannot be considered
legitimate. Most countries in the Western world face this legitimacy issue, whether they admit it or not (Hyslop-Margison & Thayer, 2009).

Gandin and Apple (2002) coined the terms mentioned earlier to distinguish between two versions of democracy and citizenship. The ‘thick’ version promotes participation and social justice–oriented citizenship that goes beyond the electoral process; it envisions participation in every domain of decision making in a society that affects public life. The ‘thin’ version, on the other hand, envisions personally responsible and law-abiding citizens (Westheimer, 2015). However, the neoliberal education philosophy underwritten with a corporate agenda and a market-driven perspective makes it difficult to be engaged in thick democracy (Hill, 2008; Sleeter, 2007). Using the same term, Lund and Carr (2008) argue that market-driven changes in education result in ‘a mild, somewhat superficial, and thin exposure to critical democracy and engagement, especially in relation to social justice’ (p. 8).

In his seminal book, *Towards a Philosophy of Critical Mathematics Education*, Skovsmose (1994) outlines four basic elements of democracy, which will serve as points of departure to connect mathematics education with democracy. The process of electing a government is the first element. Second is a fair distribution of goods and public services. Third is equality for all citizens as well as basic human rights and freedom for all citizens to express themselves and pursue a happy and satisfactory life. Fourth is a sociopolitical climate that allows citizens to be actively involved in the decision-making process and to keep track of the consequences.

Aguilar and Zavaleta (2012) conclude that connection between mathematics education and democracy can be analysed in three domains:

Firstly, mathematics education can provide students with mathematical *skills* to critically analyse their social environment, and also to identify and evaluate the uses and misuses of mathematics in society. The second link relates to the
fact that the mathematical education that students receive in a classroom can promote or inhibit values and attitudes that are essential to build and sustain democratic societies. The third link is the acknowledgment that mathematics education can function as a sort of social filter that restricts the opportunities for development and civic participation of some students. (p. 5)

The combination of values, skills, and attitudes converge into critical literacy in mathematics to read the world critically and reflectively, and to challenge it. It is necessary to realise that how students learn is as important as what they learn. For example, students can learn the concept of linear functions through an ‘exercise paradigm’ (Skovsmose, 1994). Skovsmose (2011) elsewhere points out that traditional school mathematics is largely built on curriculum materials that contain routine problems and exercises that are given in step-by-step instructions converging upon a single correct answer, and oriented towards functional literacy.

Allen (2011) describes a daily classroom routine that has been a dominant approach in the U.S., which illustrates how the exercise paradigm is formed:

- Teacher begins the lesson with a warm-up or launch activity.
- Class corrects homework from the previous lesson.
- Teacher presents new material.
- Class practices new idea or technique.
- Teacher assigns homework for the next class. (p. 2)

The five stages above were melded by the standardisation movement into one singular aim: to prepare students for standardised tests. Through this routine in mathematics classrooms, Allen (2011) points out, ‘Students became bored with their math classes and developed skewed perceptions of the discipline—as something static rather than dynamic, discovered rather than created, irrelevant to their lives, disconnected from human experience, and based on innate skills, rather than hard work’ (p. 2). Skovsmose (1994) concluded that the exercise paradigm results in
students developing a comfort zone and the learning process becoming repetitive and non-critical. In contrast to the standardisation movement, from a CME perspective, mathematics problems should be inquiry-based and open-ended\textsuperscript{16} to enable students collectively to challenge their comfort zone and reflectively open themselves to possibilities (Skovsmose & Alrø, 2004).

Thus, instead of the exercise paradigm, mathematics can be taught through an inquiry-based dialogic approach to education featuring horizontal student-teacher relationships (Freire, 2000; Skovsmose & Alrø, 2004). Dialogic approaches have multiple implications that are relevant for the enactment of a critical educational process. ‘Dialogue is not only a multiform approach to pedagogy; its different forms express deeper assumptions about the nature of knowledge, the nature of inquiry, the nature of communication, the roles of teacher and learner, and the mutual ethical obligations’ (Nicholas & Bertram, 2001, p. 1102). The ways in which students learn have defining impacts on their development. CME argues that learning activities, curricular materials, student-teacher relations, and student-student relations are not neutral: They rather have certain cultural, political, and social implications. Whereas the exercise paradigm would most likely produce obedient and authority-dependent individuals, the dialogic approach may help students develop critical literacy and democratic competencies (Ernest, 2002a; Gutstein, 2006; Skovsmose, 2011).

Ernest (2002b) agrees with Skovsmose, but, in a different publication, Ernest (2002c) points out the need for balancing everyday reality and ideals, saying ‘[CME] must also honestly and openly address the instrumental life goals of the learners themselves, both in terms of needed skills and passing exams’ (p. 6). However, his suggestion is more of an intellectual abstraction rather than practical wisdom to guide

\textsuperscript{16} Open-ended problems usually lead to multiple answers depending on preferred perspectives toward solution.
daily classroom practices of a mathematics teacher. Ernest (2002c) further suggests that CME should include classroom discussions in which different opinions are welcomed and students have a say about the content and overall goals. He points out that through authentic learning materials and dialogic teaching approaches, CME should systematically encourage students to question what they learn: their teacher as a source of mathematical knowledge, their school as a source of restrictions, and their society as a source of some repressive elements (Ernest (2002a)).

Learning materials and classroom ambiance in a mathematics classroom should be in harmony with the dialogic teaching-learning process so that students can internalise democratic values. Ellis and Malloy (2007) identified four domains within which students can develop a sense of democratic culture:

- Problem-solving curriculum.
- Inclusivity and rights.
- Equal participation in decisions that affect students’ lives.
- Equal encouragement for success. (p. 161)

Hannaford (1998) concluded that if mathematics were taught through a dialogic process, students could develop and enjoy democratic values such as listening to others, respecting different opinions, and being tolerant. According to Hannaford, three principles must be followed to run a mathematics classroom in which students can practise and internalise basic democratic values:

- Teachers must treat pupils, and the pupils must learn to treat each other, as intellectual equals.
- All teachers’ arguments must be openly and completely explained.
- The teachers’ arguments are only confirmed as satisfactory by their pupils’ free understanding and assent. (p. 182)
Hannaford’s assumption is that once students internalise democratic values, they would demand democracy whenever authorities are being undemocratic or there is a lack of democracy in a given society.

While the connection between democracy and mathematics is made in terms of egalitarian dialogue between student and teacher and among students, Almeida (2010, p. 22) attempted to compare the principles in mathematics education with democracy as a political system. She compared teaching a mathematical proof in the classroom with dynamics of democracy as a political system (see Table 2.1).

<table>
<thead>
<tr>
<th>Mathematics</th>
<th>Democracy</th>
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<tbody>
<tr>
<td>Set of axioms and principles.</td>
<td>Set of moral axioms that provide each citizen with ‘defined and undeniable’ rights.</td>
</tr>
<tr>
<td>Teachers as authority figures ‘treat the students as equal partners’ in the classroom.</td>
<td>‘Elected representatives treat all citizens as equal partners in governance’.</td>
</tr>
<tr>
<td>Students question and negotiate mathematical arguments, axioms, and numeric and algebraic procedures. ‘Mathematics teachers’ arguments are only confirmed as satisfactory by the self-understanding and the consent of their students’.</td>
<td>‘All the policy statements of elected representatives are open to scrutiny and debate. The policies of elected representatives are confirmed as satisfactory only by the free understanding and majority consent of citizens’.</td>
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Almeida’s comparison implies that when a mathematics teacher develops an argument within a mathematical proof, it has to be based on given axioms, theorems, and procedures. In other words, teachers’ arguments are not based on power claims but on validity claims. However, that validity necessitates students’ consent as well. Before moving on to next stage in the proof process, teachers have to justify their argument based on mathematical theorems and axioms, just as elected representatives need people’s consent and (ideally) have to follow the Constitution.

CME emphasises the vital connection between dialogue as pedagogy and its impact on students’ developing agency to become critical citizens. However, it does not provide a detailed account of ways in which a dialogic teaching of mathematics
takes place in practice, especially when the standardisation movement is taken into account. In the next section, I will elaborate on the concept of dialogue in relation to collaboration and inquiry in the mathematics classroom and in relation to critical citizenship.

2.10 Young Citizens in the Mathematics Classroom

It is established in CP literature that critical literacy and dialogic classroom experiences are two essential elements for students to become critical citizens. When students study mathematics through a dialogic pedagogy, as Shor (1987) argues, dialogue becomes ‘a democratic model of social relations, used to problematise the undemocratic quality of life’ (p. 95), in and out of classroom. In this sense, a dialogic approach to teaching and learning mathematics has social, political, and cultural implications for citizenship. D’Ambrosio (2010) similarly argues that critical approaches to literacy, mathematics, and technology are necessary for education oriented towards critical citizenship. Literacy, matheracy, and technocracy ‘constitute what is essential for citizenship in a world moving swiftly towards a planetary civilization’ (p. 58). D’Ambrosio considers critical (mathematics) literacy and critical literacy in technology as necessary to be a world citizen and to live with dignity in a quickly changing world.

2.10.1 Critical Pedagogy and Critical Citizenship in the Mathematics Classroom

The intimate relationship between mathematics education and democracy necessitates an analysis of related concepts such as critique and citizenship, together with classroom practices that involve dialogue as pedagogy, inquiry, and collaboration. These concepts need to be interconnected in order to find answers to the question, How should mathematics classrooms be conceived such that students become active and critical citizens? This need comes from the fact that, as noted in
Section 2.4, neoliberal ideology, drawing on positivism, proposes its own definition of democracy: It aims to reshape the education system to produce ‘instrumental citizenship’ in support of a market-driven society. Critical citizenship must therefore counter the cultural hegemony of neoliberalism. Reviewing John Dewey’s ideas about the connection between democracy and education is a suitable point of departure to distinguish a critical theory of education from a progressive theory of education, and then to elucidate the standpoint of critical theory on neoliberalism.

Dewey’s (1916) emphasis on the connection between democracy and education may help us understand the notion of critique, within the historical tradition of critical theory. From Dewey’s perspective, democracy can be best built and maintained through the scientific method, as it rejects all dogmatic approaches as well as external authorities. In this view, the scientific method is the motor of progress both for individual and social life. Education should provide students with opportunities of inquiry-based learning to internalise scientific formation, which will eventually lead to unfettered thinking and progress in society; this process will eventually help students develop citizenship skills and integrate them into the society. This Dewey-inspired perspective uncritically embraces scientific rationality based on the ideals of the Enlightenment.

However, sociopolitical experiences such as rise of fascism in continental Europe and the Stalinist movement in Soviet Russia in 1930s obligated members of the Frankfurt school to critically review the ideals of the Enlightenment, including scientific rationality. A member of the Frankfurt school, Marcuse (2013), argued that the scientific method (positivism) may not always be progressive and lead to free thinking. In fact, it could become an oppressive force that reduces human beings to a single dimension; therefore it should be subject to critique. Habermas (1972)
addressed Marcuse’s criticism. He argued that modernism is an unfinished project that includes not only scientific-technical knowledge interests, but also practical and emancipatory knowledge interests.

Overall, critical theory advanced an interdisciplinary project that aims not only to explain existing conditions but also to change them. As Skovsmose and Alrø (2004) argue, Dewey’s view of education in relation to democracy is non-critical. Therefore, CME should transcend Dewey and show ‘how critique and learning mathematics can be connected and how the competence of mathemacy can be supported’ (p. 20) in order to help students develop and experience critical citizenship.

A citizen in a general sense is someone who is a legally recognised member of a state: This recognition entails certain rights and responsibilities. However, one must possess certain competencies and be willing to utilise them to move from being a passive-obedient citizen to an active, critical citizen. Citizenship in this sense has cultural implications extending far beyond its legal definition. As Stevenson (2011) frames it, ‘By citizenship, I mean our connection to particular social and cultural locations, the possibility of a participatory involvement in shaping our society, and our understanding of our rights and responsibilities’ (p. 5). In the U.S., this definition notably includes children of immigrants, whose parents have no legal ties to the government, and who are often regarded as ‘undocumented or illegal immigrants’.

Education in a given society usually aims to reproduce the status quo; schooling practices are designed accordingly. The curriculum in most countries contains citizenship or civic courses in which a good citizen is

Someone who possesses certain propositional knowledge, often including rather banal facts about national history and electoral/legislative process, and applies this information in a certain prescribed fashion. Such as citizen might
be involved in community service, obeys legal dictates, and feels compelled to cast a ballot when civic duty affords the opportunity to do so. (Hyslop-Margison & Thayer, 2009, p. 1)

This type of citizen can only reproduce the status quo. Critical citizenship goes far beyond. For example, a critical citizen may have skills of adaptation to society as well as competencies to participate in collective initiatives that challenge oppressive, exploitive, and undemocratic situations.

Doganay (2012) points out that citizenship education does not have to be delivered in a separate civics course. Instead, it should be integrated into all courses and at all grade levels. Any course can help students develop skills, values, attitudes, and knowledge to become active and democratic citizens. Beutel (2012) argues that lessons and projects designed as group work can promote some of these qualities: ‘Even the project itself is a democratic method by means of its basic elements: choosing and planning a topic jointly, realising it with all group members and recording as well as evaluating it together’ (p. 12). Simmt (2001) contrasts instructional practices that contradict citizenship and those that promote active citizenship. In this view, mathematics instruction that presents mathematics as ‘a set of facts, skills and procedures’ oriented towards a single right answer does not promote citizenship qualities at all. However, instruction that embraces a problem-posing approach and applies investigation and dialogue would potentially promote qualities of good citizenship.

Having reviewed the neoliberal threat, Hyslop-Margison and Thayer (2009) outline qualities of critical citizenship in the classroom from critical theory and pedagogy, including the views of Habermas and Freire. A critical citizen
• can form individual and collaborative initiatives and ‘participate in their community by volunteering or engaging in various forms of political activism and they educate others around them on the issues’;

• possesses communicative competency to be engaged in dialogue with others to reach consensus ‘rather than dismissal simply on the grounds of initial disagreement’;

• has intellectual and civic courage to listen to and review different ideas and revise his/her view not ‘on the basis of external pressure or political isolation but change their views only in the face of justifiable arguments and or evidence against them’;

• should be searching, with a ‘healthy scepticism’ and critical standpoint, ‘new knowledge, reflecting on that knowledge and translating it into immediate social action’;

• is critically literate not to be manipulated by hegemonic forces and able to distinguish things that can be changed such as social, economic, and political structures in a society from things that are given. (pp. 117–118)

While Hyslop-Margison and Thayer (2009) proposal is inspirational—and it coherently frames what a critical citizen should be able to do—it omits a very important question: How can these objectives be realised within neoliberal restrictions?

Thésée (2013) also indicates the connection between education and democracy in neoliberal context:

The contemporary neoliberal, globalized forces that frame the geopolitical environment further entrench, justify, mask, and perpetuate tyranny at infinite micro- and macro levels. The fundamental question is: What type of education do we want to inscribe society with a radical love and individual and collective emancipation aimed at a meaningful democracy? (p. 204)

Drawing on Freire’s ideas, Thésée (2013) considers democracy as a practice of overcoming oppression; she outlines five domains of action oriented towards individual and collective liberation from neoliberal oppression:
• Reclaiming: from Silence to Voice.
• Remembering: from Omission to Memory.
• Participation: from Passive to Active participant.
• Self and Collective emancipation: from Powerlessness to Empowerment.
• Radical love: from Hate to Radical love. (p. 202)

In these domains, the starting points are associated by neoliberal citizenship as
passive consumerism; the end points are oriented towards active critical citizenship.

Thésée’s proposal is theoretically strong and insightful. However, it lacks practicality.

Drawing on Habermas’s communicative action theory, Murphy and Fleming
(2010) proposed a classroom application that promotes critical and active citizenship.

A classroom should

provide [students with] an experience of democratic investigation, critique,
and collaborative action planning and implementation. It also involves the
ability to engage in discussions about important issues...while both accepting
the difference and utilizing these creative diversities. In this kind of education,
reflective practice becomes more akin to the critique of ideology and an
exercise in communicative action. (p. 203)

If learning mathematics is oriented towards critical citizenship, classroom dynamics
should help students not only learn but experience these character and intellectual
virtues.

Dialogue, collaboration, and inquiry are integral parts of the critical
mathematics classroom. Dialogic teacher-student and peer relations aim to turn a
classroom (life-world) into a public sphere where mathematical ideas and knowledge
can be negotiated and learnt. When this learning happens through inquiry, students
may learn and experience skills and values of collaboration, respectfully listening to
each other, revising their perspectives and ideas in order to form a collective
reflection. As the process of inquiry involves challenges, students push their comfort zone; they learn with and from each other through inquiry-based education.

Mathematics classrooms can be collaborative learning communities where knowledge is built democratically, where students negotiate and critically reflect on what they learn. Critical education thus transcends Dewey’s connection between democracy and education. Dewey’s standpoint uncritically relies on the scientific method; therefore it implies the domestication and adaptation of students. However, a theory of critical education is critical of both learning content and process; it considers students to be agents of change. It therefore aims for nothing less than social transformation.

2.11 Criticism

Research in mathematics education is mostly based on an assumption that educational settings are the same everywhere in the world, and the same as in developed nations. However, there are schools in the world that lack the basic living conditions of the developed world, let alone have access to decent educational settings and materials. Children aged 6 to 10 in developed countries comprise only 10% of the world’s child population; 86% of children within the same age group live in undeveloped countries, and 4% live in developing countries (Skovsmose, 1995, p. 13). This means that mainstream studies in mathematics education exclude nine students out of ten at the global level. This assumption of homogeneity should be addressed by CME as neoliberal globalisation brings about ghettoisation of the world (Skovsmose, 2011).

The assumption criticised by Skovsmose can be extended to the point that research in mathematics education must take neoliberal educational transformation into account before reaching any conclusion, as neoliberal transformation has
changed and is still changing the scope and concept of education, as we are most currently witnessing in the U.S. with initiatives of the newly elected president, Donald Trump. Pais’s critique of CME is relevant here. Pais (2012) argues that the research community in CME assumes that equity can be achieved within a capitalist society. He strongly disagrees with this assumption, arguing that ‘exclusion’ in education is ‘contingency’. In Pais’s view, ‘School exclusion is what is necessary to maintain school’ (p. 57). He insists that equity through education is impossible in capitalist societies, because capitalist economies and politics require inequality: ‘Inequality is a necessity of capitalist economics, while equality functions as the necessary ideological supplement concealing the obscenity of what is going on’ (p. 55).

Given that the critique of capitalism has taken centre stage in critical theory ever since Marx, Pais’s approach can be regarded as a valid criticism: CME should problematise the power structures inherent in capitalist societies. Pais’s argument also resonates with critical theory’s argument that capitalist modes of production and the relations between these modes are obstacles to a thick version of democracy and emancipation.

However, at no point does CME put forward the restriction that ‘citizens should pursue democratic ideals staying within the capitalist system’. What is more, a population of citizens with democratic competencies and critical (mathematical) literacy might challenge and transcend capitalist society. However, the perspective, implicit in Pais’s critique, that emancipatory transformation in education is impossible without a revolution at the macro level can be contested: Small changes and openings at the micro level may generate hope that another world, a more just and free one, is possible, and may perhaps lead to bigger transformations (Skovsmose &
Greer, 2012). In other words, CME does not set barriers to radical social transformations of any kind.

2.12 Conclusion

The CME literature argues that mathematics education is a political domain. How mathematics is taught therefore has social, political, and cultural consequences. In short, mathematics education is not neutral. Skovsmose (1994, 2011) suggested a general framework for CME in which three concepts are central: critical distance, critical competence, and critical engagement. In his proposal, CME should distance itself from the official curriculum, promote active participation in the classroom, and address socially significant issues relevant to students’ lives.

The existing literature in CME proposes that mathematics education be reoriented to promote critical citizenship. A dialogic pedagogy may help students develop democratic values and attitudes in order to become active and critical citizens. CME argues that a critical approach must help students understand the formatting power of mathematics (Skovsmose, 2011) in order to challenge sources of dominance, oppression, and exploitation, whereas an authoritarian way of teaching mathematics that focuses on functional literacy may produce passive functionaries and obedient citizens. Mathematics education has the potential to promote critical citizenship and a thick version of democracy.

Theoretical studies in CME connect mathematics education and democracy by emphasising the importance of dialogic pedagogy and inquiry-based collaborative learning to help students to develop democratic values (Aguilar & Zavaleta, 2012; Almeida, 2010; Hannaford, 1998; Vithal, 1999). Dialogic pedagogy (Freire) and communicative rationality and the ideal speech situation (Habermas) provide the theoretical foundation on which the practice of CME can be built. However, as a new
field, CME to date offers very limited classroom-based research to confirm these theoretical studies. Moreover, no classroom-based research in the CME literature focuses on impacts of market-driven changes in relation to citizenship and democracy.

Neoliberal educational changes in the U.S. aim to produce consumer-based, passive citizens, and thus promote a thin version of democracy. On the contrary, CME aims to promote active and critical citizenship—a thick version. Neoliberal educational changes have been imposed top-down by the government and thus represent the dominant discourse in U.S. public schools. As bottom-up initiatives, individual teachers can practise CME. However, teachers’ power and autonomy has been diminished as a consequence of the control and management strategies in the market-driven approach. Thus, whether CME can survive under the neoliberal regime is a central question to which the existing literature has no answer.

This gap motivates my research, as I aim to investigate the scope and limitations of CME in a neoliberal era. The literature acknowledges that more classroom-based research is needed to establish a dialectical relationship between theory and practice. The next chapter explains the methods I used to address these questions.
Chapter 3: Methodology

3.1 Introduction

In this chapter, I describe the details of the research setting, data collection, and data analysis. For the reasons outlined in the previous chapter, this study explores how mathematics education in high school can promote critical citizenship. The main theoretical questions addressed in this study are reiterated first. The central research question is: What are the potential and limitations of critical mathematics education (CME) in the neoliberal era? Subordinate questions include the following:

1. While facing top-down restrictions imposed by neoliberal educational policies and pedagogies on a daily basis in classrooms in the U.S., how is it still possible for teachers to create ‘small openings’\(^{17}\) and a communicative sphere for humanising education through CME?
2. How can collaborative and dialogical mathematics education be facilitated to help students to become critical citizens?
3. How can mathematics lessons be developed to practise CME without disrupting the process of preparing students for standards-based assessment?

The chapter comprises four main sections. First, the qualitative research paradigm will be introduced from a critical research perspective to provide a rationale for the methods used here. The second section elaborates on case study methodology in action research. This section reviews the basis of action research methodology and introduces critical participatory action research (CPAR) as the preferred approach. The third section describes the design of this study, describing the research setting, the preparation stage, and the action cycles in the form of end-of-unit projects (EUPs). Ethical issues are also addressed. The fourth section focuses on data

\(^{17}\) The term ‘small openings’ is due to J. C. Scott (2008), a political theorist and former professor at the University of Wisconsin-Madison.
collection, describing the sources of data, the dialectical method, and other techniques of qualitative data analysis. A discussion of issues of validity and generalizability is also presented.

### 3.2 Critical Qualitative Research in Education

In a broad sense, there are three epistemological approaches in educational research: positivist, interpretive, and critical (W. Carr & Kemmis, 1986; Kemmis, 2010). At the risk of oversimplification, the positivist approach, guided by technical rationality, claims that knowledge exists objectively and is independent of human subjectivity: human beings can be understood by pre-formulated scientific methods (Hyslop-Margison & Naseem, 2007; Stringer, 2013). However, many sociologists and philosophers, including Habermas (1972), assert that humans, as social beings, can neither be reduced to linearly constructed factors nor understood in isolation from the circumstances in which they exist. Rather, humans are complex and context-dependent phenomena. Therefore, for this study—which examines mathematics education as it is experienced in its authentic context—a positivist approach falls short of capturing human-related phenomena such as life in the classroom.

Interpretive or naturalistic approaches are categorised within a qualitative research paradigm. According to Denzin and Lincoln (1994), qualitative research is multimethod in focus, involving an interpretive, naturalistic approach to its subject matter. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret phenomena in terms of the meanings people bring to them. Qualitative research involves the studied use and collection of a variety of empirical materials—case study, personal experience, introspective, life story, interview, observational, historical, interactional and visual texts. (p. 2)
A qualitative approach allowed me, as the principal researcher and the teacher, to use my observation, reflective writing, and students’ journals as the main data sources. The interpretive approach permits researchers, as McLaren and Giarelli (1995) point out, to take human subjectivity into account and consider knowledge as a social construction. It does not, however, allow the researcher to elaborate on different interpretations and their implications. Interpretive approaches may help the researcher understand a set of existing situations, but limit development of understandings that would indicate possible paths in order to challenge and transform existing situations. Similarly, Habermas (1972) suggests ‘the interpretive perspective cannot be conceived to be ontologically reflexive as they imply that there is no connection between a political stance and their methodological approach’ (p. 303). He argues that interpretive research perspectives have turned into ‘the positivism of the cultural and social science’ (p. 303). Like the positivist approach, the mainstream interpretive approach fails to acknowledge that research overtly or covertly entails ideological perspectives. Any research approach that limits understandings of a set of existing situations—without any attempt to challenge and transform them—serves to reproduce the status quo. As Habermas (1972) noted, research strategies are never neutral: they either criticise the existing system or they reproduce it.

Like interpretive approaches, critical theories hold that knowledge is socially constructed, contextual, and dependent on interpretation. However, in contrast to interpretivists, ‘critical theorists see a need and a basis for forming and understanding hierarchies of contexts and types of knowledge and evaluating them for their possibilities of contributing to progressive material and symbolic emancipation’ (McLaren & Giarelli, 1995, p. 2). Furthermore, in contrast to the interpretive paradigm, as McLaren and Giarelli (1995) argue, a critical approach aims to ‘improve
human existence by viewing knowledge for its emancipatory or repressive potential. In this way, a standard of judgement and value becomes possible’ (p. 2). The critical perspective thus establishes a clear vantage point from which we can examine knowledge for its oppressive and emancipatory potential.

Life in the classroom is a complex, interactively dynamic, context-dependent phenomenon, and so understanding it requires a qualitative approach that ‘captures the complexity of being human in an interacting social world of teaching’ (Pine, 2009, p. 17). Thus, the present project requires a qualitative approach in order to understand and interpret the dynamics in the classroom. However, my study focuses not only on the neoliberal trends in mathematics education, but also on possible alternatives to market-driven changes—not only on what is but on what can be. The research therefore qualifies as a critical qualitative study (Merriam, 2009). In contrast to positivist and interpretive approaches, critical educational research asserts that education is political action (W. Carr & Kemmis, 1986; Kemmis, McTaggart, et al., 2014; Pine, 2009; Stringer, 2013). Critical qualitative research aims to unpack oppressive, discriminatory, and dehumanising aspects of a dominant education system in order to challenge it.

3.3 Case Study

The current research was conducted in my own high school mathematics classroom during the 2014–15 school year. The study can therefore be considered teacher research and my classroom a bounded system (Merriam, 2009; Pine, 2009; Stake, 1995). Cochran-Smith and Lytle (1993) point out that ‘almost by definition, teacher research is case study: The unit of analysis is typically the individual child, the classroom, or the school’ (p. 59). Stake (1995) also considers the case study as an in-depth study of a unit. However, Yin (2008) defines the case study as more of a
process that uses multiple data sources to investigate ‘a contemporary phenomenon…within its real-life context’ (p. 18). The unit of my study is the scope and limitations of CME in a high school mathematics classroom in the neoliberal era; CME is the contemporary phenomenon under investigation.

The unit of analysis within this bounded system is an educational praxis, which aims to create opportunities in the classroom to practise CME in the presence of top-down-imposed neoliberal educational policies. Such policies constitute a significant part of the research context as they directly influence every mathematics classroom in American public schools. Because my research aims at reaching conclusions broader than my classroom, this project can be seen as an instrumental case study (Stake, 1995).

According to Pine (2009), ‘Case studies are intended for natural settings and contain data collection procedures that can help teachers in their decision making’ (p. 216). Data collection procedures in case studies are ‘compatible with instructional purposes’ (p. 216). Case studies allow researchers to ‘triangulate’ data from different sources, such as interviews, field notes, and student journals. The case study also let me produce thick descriptions of the dynamics of my classroom and present them in ways that ‘can be easily followed’ (p. 216). The style of a case study is in harmony with action research: The case study is a suitable approach for teachers who ‘try out ideas [in the classroom], reflect on their implementation, and try again….The cycle of thought-action-thought finds comfortable embrace in the case study’ (Pine, 2009, p. 213).

Skovsmose and Borba (2004) point out that ‘Doing critical research [on CME] means not only to consider what is taking place but also to consider what could have taken place and could be imagined as possible alternatives to what is taking place’ (p.
211). While acknowledging what the actual case is, this study is critical research, which aims to imagine alternatives.

### 3.3.1 Critical Participatory Action Research

Action research allows researchers to intervene at different stages. As Herr and Anderson (2005) state, ‘Unlike traditional social science research that frowns on intervening in any way in the research setting, action research demands some form of intervention’ (p. 5). According to W. Carr and Kemmis (1986), action research is a form of self-reflective enquiry undertaken by participants (teachers, students or principals, for example) in social (including education) situations in order to improve the rationality and justice of (a) their own social or educational practice, (b) their understanding of these practices, and (c) the situations (and institutions) in which their practices are carried out. (p. 162)

This definition of action research resonates with the objectives of my research.

Habermas (1972) argues that knowledge and human interests are strictly correlated, and that the desire for knowledge is divided into three categories: technical, practical, and emancipatory. Drawing on Habermas’s theory of communicative action, Kemmis, McTaggart, et al. (2014) relate each interest with a unique epistemological stance and consequently its own research methodology. Accordingly, the types of research can be categorised as technical, practical, and emancipatory action research. As Kemmis (2009) points out, these three types of action research ‘involve very different kinds of constellation of sayings, doings and relating’ (p. 469): One of them is critical participatory action research (CPAR), through which we ‘find and enact ways of doing things that are less irrational…less unproductive…and less unjust or exclusionary’ (Kemmis, McTaggart, et al., 2014, p. 68).

Considering the aims of CME as a subset of critical pedagogy, which guided my classroom teaching, CPAR conceptually resonates with my research project for
four reasons. First, CPAR itself is a learning process that performs inquiry with participants (students), not on them, and it does not objectify participants. On the contrary, it embraces each participant as an equal contributor. Similarly, a critical stance to mathematics education oriented towards critical citizenship envisions students as equal partners in the knowledge creation process and embraces a horizontal student-teacher relationship (Freire, 2000).

Second, CPAR is emancipatory and critical. It is a process in which people explore root causes of oppression in order to break free from these constraints. Similarly, CME is critical of dominant (neoliberal) education systems, and it envisions a mathematics education that empowers students. Cycles of plan-act-observe-reflect in CPAR provide opportunities to unpack oppressive dynamics and eliminate them.

Third, CPAR puts emphasis on engaging ‘in communicative action [based on dialogue]’ with research participants to reach ‘intersubjective agreement’ (Kemmis, McTaggart, et al., 2014, p. 68). CPAR opens ‘communicative spaces that permit and foster’ collective learning and doing (Kemmis, 2008, p. 126). Similarly, a dialogic teacher-student relationship is a focal point of CME; it therefore takes a central place in my research project. Freire (2000) argues that humanising education necessitates the dialogical learning-teaching process.

Fourth, Kemmis, McTaggart, et al. (2014) draw on Habermas’s theory to conceptualise CPAR. The theoretical background of this study is the complementary ground of Habermas and Freire, as reviewed in Chapter 2. This connection enhances the internal coherence of the research in terms of epistemology, ontology, and methodology.
Along with theoretical perspectives, Kemmis, McTaggart, et al. (2014) provide examples of CPAR in educational contexts from all over the world. One of these examples, from Braxton High School in Canada, exemplifies how students, teachers, and parents create flexible material conditions for students to be academically successful based on students’ socioeconomic necessities. The principal worked with teachers ‘to shift away from an instructional leadership style of mandating “best” teaching practices18 to co-inquiring with staff into gaps between visions of optimal teaching and learning and realities in classroom’ (p. 23). Students’ feedback indicated that they ‘were struggling to balance their academic and personal life’ (p. 24) as they had to work to support their family financially but wanted to be successful in school. The principal and teachers developed a non-traditional English course to meet the needs of students. This self-paced course became a CPAR project involving teachers, students, and parents.

The CPAR at Braxton High School is similar to my own study in terms of students’ participation and in terms of being critical: ‘critical’, because it aimed to develop a response, from students’ life-world, to top-down-imposed neoliberal approaches—so-called best practices. It aimed to turn the classroom into a communicative public sphere where students had opportunities to negotiate different ideas, develop their own voice, and contribute to decisions that affected their lives in and out of classroom.

Similarly, the end-of-unit projects (EUPs) in my research were aimed at shifting away from nondialogic and rote-memorised education to create small openings in which students, based on their needs as human beings and young citizens, could develop critical and creative thinking skills as well as communicative

18 ‘Best practice’ is a commonly invoked phrase in neoliberal discourse in the U.S. and many other countries.
competency to become critical citizens. In the next section, I elaborate on the stages of my action plan for classroom teaching.

3.4 Study Design

This section describes the background of the research, the reconnaissance stage, and cycles of plan-act-observe-reflect in the form of EUPs.

3.4.1 Research Setting

The research was conducted in a high school in the state of Washington, U.S.A., where I have been a mathematics teacher since 2007. The high school was built in 2003 and has two stories. In 2014–15, the school had 1,942 students. Distribution of students’ ethnicity was as follows: White 45%, Asian 17%, Hispanic 13.5%, African American 12.6%, Native American 1.4%, multi-racial 10.6%. Students came mostly from poor families: In 2014–15, 45% of students received free or reduced-cost meals as they met income eligibility requirements set by the federal government.

Students have to pass a number of standardised tests as a graduation requirement, some of which have changed several times. Under the federal NCLB Act, the Washington Assessment of Student Learning was administered between 1997 and 2009, replaced in 2010 by the High School Proficiency Exam (HSPE). High school mathematics end-of-course (EOC) exams replaced the HSPE in 2011. Since then, the EOC has been the state-level standardised test that high school students must pass. From 1997 to 2014, the success rate of students’ in my high school fluctuated between 40% and 60%. As common core curriculum standards were adopted in Washington in 2014, from the 2015–16 school year on, the Common Core
Assessment Test (CCAT), a fully computerised test, replaced EOC exams. In addition to these, as part of college admission requirements, some students take exams such as Scholastic Assessment Test (SAT) and course-based Advanced Placement (AP) exams administered by a private company, the College Board. In December 2015, the Obama government revised the NCLB Act that limits involvement of the federal government in educational decisions.

The mathematics course (Advanced Algebra) from which I collected data September 5, 2014, to March 25, 2015, was scheduled from 9:00 a.m. until 10:30 a.m. during second period every day throughout the 2014–15 school year. There were 28 students. In the classroom, I had instant access to the Internet; an electronic projector was connected to my tablet PC and document camera. The school has several laptop-carts, each with 30 laptops for students; however, none was stationed in my class. I purposefully ordered tables for my classroom. In a traditional classroom, students sit facing the board or the teacher’s table, as the teacher is regarded as the ultimate source of knowledge and therefore an authority figure. Instead, inspired by critical pedagogy, I had six groups: each group included four tables with chairs in order to create a classroom ambiance conducive to non-authoritarian and dialogic teacher-student and peer relations.

Raising test scores is the main point of the standardisation movement, and it has resulted in similar teaching practices and classroom routines in mathematics classrooms nationwide. From a market-driven perspective, this approach is called

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19 This new test is called Smart Balance Assessment Consortium (SBAC), which is a private company: their official website is: http://www.smarterbalanced.org/2013/06/practice-tests-now-available/. Even within a neoliberal perspective, the quality and cost of the SBAC is highly controversial: http://www.livingindialogue.com/strange-history-common-core-sbac-test-monster-adopted-washington-state/
20 Detailed information about the College Board and their testing business can be obtained at https://www.collegeboard.org/about
21 Further information can be obtained at http://www.nytimes.com/2015/12/11/us/politics/president-obama-signs-into-law-a-rewrite-of-no-child-left-behind.html? _r=0
‘best practice’: teachers are instructed to follow the same routine and focus on similar test-preparation activities. A typical lesson begins by reviewing the previous day’s homework or presenting warm-up problems on the board (to make the transition to a new topic or to review topics that have already been covered). The teacher then introduces a new subject based on the standardised sequence; hands out a worksheet that includes repetitive mechanical exercises on a related topic, and has students practice them; before the end of class, the teacher assigns homework for the next day.

As I would like to consider my professional practice as praxis, within this typical daily classroom routine I strove to apply CME in my class as much as possible. This point precisely frames my research question: Given the restrictions imposed by standardised education, how much space can be cleared to practise CME?

3.4.2 Reconnaissance Stage

Teaching involves communicative activities. Students’ self-disclosure and active participation were crucial to conduct the research. In order to overcome obstacles that may have kept students from opening up, it was essential to identify the obstacles. Groundwater-Smith, Brennan, McFadden, and Mitchell (2003) point out that student-teacher interactions involve emotions and power relations that may enhance or diminish students’ participation in classroom activities, [in this sense] ‘the initial lessons with a class and actions that can be taken to start up relationships in a productive way’ (p. 91). Therefore, prior to action research, it is crucial to initiate lessons as the preparation stage of action research. Accordingly, I spent two weeks teaching initial lessons to create a classroom ambiance where students can confidently and comfortably communicate with their peers and me.

More specifically, during this stage, I developed lessons to establish a teacher-student relationship based on mutual respect and trust. I initiated discussions of what
learning mathematics through collaborative inquiry in dialogical student-teacher relations may imply for our classroom, as well as how and why we should be part of this process. I ensured that my students understood well that this was participatory action research: Research with them, not on them, and therefore their voluntary participation was key to the validity and success of the entire venture. For instance, my students and I democratically negotiated two classroom rules: (a) no cell phones in the classroom, and (b) the only drink allowed is water. The students pointed out that if these rules were democratic, I as a teacher should follow them as well. I found the students’ argument fair and reasonable, and agreed to them. These form what Kemmis, Wilkinson, et al. (2014) describe as social-political arrangements that shape power agency and solidarity in socially constructed human endeavours.

3.4.3 End-of-Unit Projects

As shown in Table 3.1, end-of-unit projects (EUPs) are lessons taught over two consecutive days in 90-minute block periods, one period each day. Each EUP was intended to be an inquiry-based collaborative lesson to promote dialogic teaching and learning. During each project, data were collected from students’ journals, samples of students’ work, whole-class discussions, field notes, and my reflective journal. In terms of data collection and analysis, each EUP constituted a segment of data to answer a specific research question. Each EUP is considered a plan-act-observe-reflect cycle.
Table 3.1

*Content and Themes of End-of-Unit Projects (EUPs)*

<table>
<thead>
<tr>
<th>EUP</th>
<th>Mathematics content</th>
<th>Theme</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Linear equations and functions</td>
<td>Standardised assessment</td>
</tr>
<tr>
<td>2</td>
<td>Multipart functions: analysis of domain and range</td>
<td>Critical mathematical literacy</td>
</tr>
<tr>
<td>3</td>
<td>History of mathematics</td>
<td>Universal values of humanity</td>
</tr>
<tr>
<td>4</td>
<td>Systems of inequality</td>
<td>Community service</td>
</tr>
<tr>
<td>5</td>
<td>Exponential functions</td>
<td>Student loan debt crisis</td>
</tr>
</tbody>
</table>

As Groundwater-Smith et al. (2003) explained, ‘Learning to participate in a practice and learning how to use the tools associated with a practice does not happen overnight; one builds up competence and expertise over time’ (p. 115). I embraced an approach to teaching-learning mathematics ‘through dialogue as a collaborative, inquiry-based activity conceived of as multi-dimensional thinking, which is comprised of critical, creative and caring thinking’ (Chesters, 2012, p. 73). The purpose was to support students in developing values, skills, and attitudes oriented towards being active and critical citizens. Dialogue, collaborative and inquiry-based learning, and related skills, values, and attitudes were the focal points of EUPs. Groundwater-Smith et al. (2003) point out that ‘making explicit the tools associated with doing a particular subject is a crucial step in devising activities to help students not only learn the subject matter but also participate in the practices associated with that subject’ (p. 115). Each EUP was designed in light of this suggestion to ensure that all students in the class were actively involved. Each EUP had its own content objective within the practice of CME.
3.4.4 Ethics

Any research that involves human participants must address ethical issues. Kemmis, Wilkinson, et al. (2014) point out that the data gathering process in CPAR brings up certain ethical issues: ‘The main ethical obligations of those doing research (or practising a profession) are to respect the persons involved and affected, and do no harm’ (emphasis in original); this means that participants’ rights, and ‘physical, psychological and cultural integrity must be protected, and not to be damaged’ (p. 159). The participants of my research were students (minors) in my own classroom. Therefore, it raises ethical issues that specifically concern research with children. Leeson (2007) explains that ‘research with children is generally perceived, rightly or wrongly, as requiring great sensitivity and robust ethical consideration’ (p. 129).

Specific questions, posed by Zeni (1998), must be addressed:

- ‘Who might be harmed (personally, professionally, and financially)? What precautions have you taken to protect the participants?’ (p. 14)
- ‘Does your inquiry focus on people with less power than you? Children in classrooms are always vulnerable—especially if their families have little money or education….How does your project demonstrate mutual respect and justice?’ (p. 14)

To address these questions, I devised an ethical process that fully complies with the Charles Sturt University Ethics Guidelines. The Human Research Ethics Committee approved my research. I also obtained ethical clearance in my school district. The district has a procedure for ethical clearance, which required me to complete a written application that clearly indicated the purpose of research, potential risks and benefits for students, and confidentiality. When the school board approved my application, ethical clearance at the district level was completed.
3.4.4.1 Research with Children

Students’ self-disclosure was crucial in this study. However, it might be difficult for children to expose themselves about topics that may be sensitive for them. Leeson (2007) points out that a meaningful relationship between students and teacher based on ‘genuineness, trust and empathy’ (p. 139) is essential for children to feel safe to express their ideas and feelings, and actively participate in collaborative inquiry. Therefore, I continuously worked on creating a classroom ambiance in which students felt free to voice their ideas, feelings, and experiences. Also, I informed my students that I would keep all data gathered, including my observation notes, confidential, and save them in a locked box. Students’ names that appear in Chapters 4 and 5 are pseudonyms.

3.4.4.2 Power Relations

Because asymmetrical power relations exist between the teacher and students, it is possible for students to feel pressured to participate in research. As Nolen and Vander Putten (2007) indicate, students may not ‘possess the maturity or independence necessary to decline participation in studies conducted by researchers on whom they are dependent for their grades, access to resources, and enriching experiences while in school’ (p. 402). During the reconnaissance stage, I held whole-class discussions to ensure that my students comprehended the purpose of the research and understood that they should participate only if they would like to: There was no gain or loss in terms of course grade. At any stage of the research, they could withdraw without facing any negative consequences.

I also ensured that these points were clearly stated in the consent form for both parents and students. I held two open-house sessions for parents: One of them happened after school on a weekday, and the other was held during the weekend to
accommodate parents with different work schedules. All the students and their parents signed and turned in consent forms. Two students left the class before I began data collection: one moved to another state and the other moved to another school district for family reasons. The rest of the students participated from beginning to end.

3.4.4.3 A Humanising Approach to Research

Merely obtaining informed consent may not be good enough in my research context. For example, students from a poor socioeconomic background, whose parents are not conventionally educated, make up the most vulnerable student population (Zeni, 1998); simply signing a consent form does not fully erase their vulnerability. Therefore, I applied a humanising approach throughout the research. According to Paris (2011), humanising research should not end as soon as the data collection is completed:

Working with students in contexts of oppression and marginalization…the researcher’s effort must coincide with students’ [efforts] to engage in critical thinking about the problems and issues of interest as both the researcher and participants seek mutual humanisation through understanding. (p. 137)

In humanising research, the researcher should seek not only to take, but also to give. Paris (2011) considers non-humanising research approaches as ‘colonising’ inquiry.22 Therefore, once the data collection was completed, as Wells (2009) suggests, I shared the results and the initial analysis, and negotiated with my students ‘how they might act on the basis of the findings’ (p. 50).

22 ‘Ending colonizing inquiry is a movement and I am one small member. For now, I must reckon with how I can continue to humanize the cultural communities of my work as I myself was humanized by the young people in these communities’ (Paris, 2011, p. 147).
3.5 Collection and Analysis of Qualitative Data

Merriam (2009) considers data analysis to be ‘the process used to answer your research question(s)’ (p. 176). I therefore organised the data collection process so that the research questions could be addressed.

Qualitative research allows researchers to collect data in the form of field notes and journals (Denzin & Lincoln, 2008; Merriam, 2009; Miles & Huberman, 1994). The students, as the participants in this study, reflected on their learning experiences for each EUP. As I aimed to conduct the research with (not on) students, it was crucial to have students’ reflections. The students participated in collaborative learning activities, whole-class discussions, and made entries in their journals. In addition to students’ journals, as the researcher and teacher, I systematically wrote field notes for each EUP to record significant happenings. I also made entries in my reflective journal after each project. Finally, I took daily notes that included my observations while teaching standards-based curriculum between EUPs.

3.5.1 Student Journals

The students completed one entry in their journal for each EUP. They had 45–60 minutes for each entry to be completed. After each entry, I collected the journals and kept them in a locked cupboard in the classroom. Each student’s journal provided me with rich evidence to evaluate whether or not my classroom was a communicative sphere where inquiry-based education was practised through dialogic student-teacher relations. However, writing journal entries might exclude some students who are not so good at expressing themselves in writing. To accommodate this situation, I told the students they could articulate their thoughts and feelings through non-linguistic modes, such as drawings. But they all preferred writing.
At the end of each EUP, I provided students with prompt questions to address in their journals. These questions focused on certain skills, values, and attitudes that are necessary for critical citizenship, as discussed in Chapter 2. However, the students were not bounded by these questions: They were invited to raise any other point that they might consider worthwhile with respect to their education. The students’ reflections provided rich data for evaluating their progress towards CME. In addition, I encouraged students to review their previous entries, without changing the original, and reflect on them as well. In this way, students were engaged with self-reflection, and were able to monitor the development of their learning and how their perspectives were changing over time.

I initially thought that some students would not feel comfortable expressing their feelings, ideas, and concerns in whole-class discussions—they could expose themselves more comprehensively through their journals. For each EUP, this prediction was proven correct. Most students provided a much more compressive written reflection than they contributed in class discussions. After each EUP, I read all journals and transcribed quotations to prepare for the data analysis stage.

3.5.2 My Reflective Journal

In my journal, I wrote my observations, thoughts, ideas, and reflections throughout the study. My reflective journal included three main sections. Each was a separate file (Table 3.2). These domains produced a rich description and helped me develop a tacit understanding of the dynamics in my classroom and my professional practice as a teacher. In order to carry out effective observation and reflective writing, I applied the framework developed by Rodgers (2002), which consists of four phases of learning: learning to see, to describe, to think critically, and to take intelligent action.
Table 3.2

Sections and Content of My Reflective Journal

<table>
<thead>
<tr>
<th>Section</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations during EUPs</td>
<td>I took field notes in the classroom while facilitating EUPs. The notes includes observations on • peer interactions • group presentations • whole-class discussions • interactions with students while working collaboratively I took the notes by hand, and typed them later into the word processor.</td>
</tr>
<tr>
<td>Reflective writing</td>
<td>I kept a reflective journal and made one entry for each EUP. I made each entry after school the same day as the EUP.</td>
</tr>
<tr>
<td>Daily notes</td>
<td>I took daily notes to record my classroom observation while teaching between EUPs. These notes include my reflections during an expanded literature review and my interactions with administrators and teachers in my school.</td>
</tr>
</tbody>
</table>

As this method is primarily based on observation, it would have been difficult to fully remember what happened in the classroom yesterday, let alone an earlier period. For this reason, I went through the notes I took while teaching, and rewrote them right after each school day, because ‘immediate documentation of observation is helpful in capturing the rich context of an interaction or event’ (Pine, 2009, p. 191). However, this immediate capture brings about a challenge: I, as an observer, may be biased, and immediately contextualise what I see by filtering ‘data according to [my] personal assumptions, beliefs, and understandings’ (Pine, 2009, p. 192). I made a conscious and continuous effort to overcome this challenge.

3.5.3 Qualitative Data Analysis

Drawing on critical theory and CME (see Chapter 2), the dialectical method was the overarching approach taken to analyse the data. Winter (1989) indicates that dialectics ‘puts forward a coherent general theory both of the nature and structure of
reality and also the process of analysing and understanding reality’ (p. 51). The dialectical approach has three basic premises: First, a phenomenon—although a unified whole—is structured in relation to other phenomena. Second, a phenomenon exists within a given context that contains opposite (‘uncoupling’) forces. Third, phenomena are in constant change, which emerges out of the struggle between uncoupling forces and the unity of the phenomenon.

Theoretical assumptions play a significant role in qualitative data analysis. A phenomenon or a set of data can be interpreted according to different grand theories, with each analysis resulting in different conclusions. Miles and Huberman (1994) therefore suggest that theoretical assumptions should be made explicit to ensure that ‘the connections between theory and conclusions’ (p. 145) are well-defined. Miles and Huberman further indicate that making theoretical assumptions explicit contributes to the coherence of data analysis and ‘the strength of explanations’. This is the reason that I outlined, in Chapter 2, the assumptions of critical theory in general, and the complementary ground of Habermas and Freire in particular, in terms of epistemology, ontology, and research methods. Data analyses in Chapters 4 and 5 will be made through the lens of these assumptions.

There are various techniques of qualitative data analysis—they take different forms depending on the research context and philosophical background (Denzin & Lincoln, 2008; LeCompte & Schensul, 1999; Miles & Huberman, 1994). Nevertheless, as Merriam (2009) notes, the overall goal of analysis is to answer the research questions. According to LeCompte and Schensul (1999), qualitative data analysis is a process of turning data into stories to be able to interpret them.

Miles and Huberman (1994) identify three approaches to qualitative data analysis: interpretivism, social anthropology, and collaborative social research. Each
approach entails common ‘flows of activity’: reducing data, displaying data, and drawing conclusions (p. 10). Miles and Huberman note that *triangulation* is a type of data analysis in which at least three different segments of data are used.

Concerning action research, Stringer (2013) notes that

The end result of analysis is a set of concepts and ideas that enable stakeholding participants to understand more clearly the nature of the problematic experiences affecting their lives. These concepts and ideas may then be used to construct reports providing accounts of *what* is happening and *how* it is happening. (p. 95)

Stringer divides data analysis into two stages. First is ‘distilling’ the data, which includes categorising, coding, and analysing key experiences. The second stage is ‘enriching’ the initial analysis. LeCompte and Schensul (1999) recommend that qualitative data be analysed immediately after being collected. Miles and Huberman (1994) also suggest comparing data collected earlier with new data to be able to advance to the next step.

Coding data and framing categories are next steps. According to Merriam (2009), categories ‘must be responsive to [answer] the research question’ (p. 186). Similarly, LeCompte and Schensul (1999) suggest that researchers view data in different ways to address research questions. Saldaña (2013) considers coding as ‘one way to analyse qualitative data, not the way’ (p. 2). For example, some researchers argue that data collected from interviews cannot be coded (Miles & Huberman, 1994).

Depending on the research context, coding may take different forms. Coding may be done according to categories derived from the conceptual framework; that is, they are in place before the data are collected. Alternatively, categories may emerge from the initial analysis of the data. Saldaña (2013) defines the ‘versus’ coding approach as ‘identify[ing] in dichotomous or binary terms the individuals, groups,
social systems, organizations, phenomena, process, concepts, etc., in direct conflict with each other’ (p. 115). Saldaña writes that ‘versus coding is appropriate for policy studies, evaluation research, critical discourse analysis, and qualitative data sets that suggest strong conflict or competing goals within, among, and between participants’ (p. 115). Versus coding was used ‘for a qualitative study of teachers responding to state-developed fine arts standards for educational achievement’ (p. 194). As the standards were determined without any teacher input, teachers’ responses indicated strong conflicts. Some emerging codes included ‘grassroots committee vs. teacher input’, ‘teach to test vs. fun’, and ‘rubrics vs. personal judgement’. Because the goals of CME conflict with market-driven educational policies and implementations, versus coding is well suited to the present study, and I used it to analyse data in Chapters 4 and 5.

As noted previously, the data generated by this study were filtered through a particular theoretical lens that owes much to Habermas and Freire. Such an approach is recommended by a number of authorities (LeCompte & Schensul, 1999; Miles & Huberman, 1994). More specifically, LeCompte and Schensul (1999) recommend various techniques of qualitative data analysis, including writing significant occurrences chronologically to obtain a picture of the entire story, and selecting quotations to illustrate key points. Based on these suggestions, I selected quotations from the student journals and my reflective journals and generated initial analyses of each EUP through the lens of critical theory.

3.5.4 Validity and Generalizability

Norms to judge the quality of research—including validity, trustworthiness, and generalizability—apply to all types of research. While positivists employ the term ‘validity’, interpretive researchers prefer ‘trustworthiness’ (Stringer, 2007). A
positivist approach considers knowledge as independent from human subjectivity. Therefore, it fails to understand the full complexity of social reality and human behaviour, because ‘it is impossible to control human behaviour with the rigour and precision demanded’ (Stringer, 2007, p. 195). But this does not mean that we cannot understand human behaviour and social reality at all. Behaviourists study human behaviour at the individual level, investigating, for instance, perception, learning, cognition, and motivation. Sociologists and other social scientists study human behaviour at the group level, investigating phenomena such as class, race, gender, and culture (Stringer, 2013). Drawing on the work of Taylor (1971), Hyslop-Margison and Naseem (2007) argue that the positivist approach is doomed to miss the essence of human action, because positivists try to understand behaviour in the absence of its history and wider context. Human action

requires interpretation or analysis and explanation at the individual level in order to be properly understood rather than imposing pseudo-categories that neatly but erroneously compartmentalise human behaviour….The meaning of the act cannot be captured or understood in the absence of the specific history and context, both personal and social, in which the particular event in question occurred. (p. 21)

Lincoln and Guba (1985) recommend naturalistic inquiry as an appropriate (valid) way to do social science research. However, action research differs from both positivist and naturalist approaches, because action researchers intervene in the research process; they are ‘interested in outcomes that go beyond knowledge generation’ (Herr & Anderson, 2005, p. 49). Pine (2009) points out that ‘validity is complex, dynamic, context sensitive, and not a unitary concept’ (p. 84).

It follows that the validity of action research needs to be understood differently. For example, validity can be understood as a ‘dialogic’ enterprise (Pine,
Depending on the objectives and context of action research, the action researcher may apply different criteria of validity. This aligns with my research. I carefully established dialogic student-teacher relations with my students and facilitated activities that both my students and I continuously learned from and reflected on. My inquiry was done in collaboration with my students to produce outcomes relevant to them and to facilitate their understanding of life in the classroom in order to transform it. Therefore, my research meets *democratic* and *dialogic* validity criteria in concrete terms (Herr & Anderson, 2005).

As explained earlier, the research was conducted in a natural setting—a mathematics classroom in a contemporary American high school. Therefore I draw on Stake’s (1995) concept of *naturalistic generalizability*, according to which the lessons learned from my action research may be used by other mathematics teachers who have to practise their profession under similar conditions formed by present-day neoliberal policies.

The next chapter presents the results and analysis of the collected data. Each EUP is presented as it occurred in the classroom, along with my initial analysis.
Chapter 4: Action Research

This chapter describes the empirical practices as they occurred in the classroom, including the reconnaissance stage and the action research cycles. Each cycle includes a detailed presentation of the empirical material in the form of ‘end-of-unit projects’ (EUPs); the five EUPs are presented in Sections 4.2.1–4.2.5. Each EUP featured inquiry-based collaborative learning, dialogic teaching, whole-class discussions, and journal writing. The chapter concludes with my overall reflection on EUPs.

Each EUP began with a brief introduction to the project and distribution of the learning materials. I assigned students to groups for some EUPs but let students organise their own groups for others. Each EUP took two 90 minutes of block period time over two consecutive days. The students had an entire block period to work in groups to complete the assigned project; the next day, the groups shared their work with the rest of the class. After the presentations, I facilitated a reflective classroom discussion focusing on what went well, what could have been different, the quality of collaborative learning, peer dialogue, and how our experiences should inform the next unit.

Whole-class discussions were chosen in preference to individual interviews in the process of transition between EUPs for two reasons: First, one of the goals of CME is to have students develop communicative competency in a public sphere. Second, in the ordinary classroom setting, teachers may ask the whole class for feedback, but interviewing individual students to elicit feedback is considerably more time-consuming, and, therefore, not attainable within the restricted time frame of daily teaching.
These classroom discussions allowed the students and me to reflect on the last unit and contribute to the development of the next EUP. Once the discussion was completed, students made entries in their journals, addressing the specific questions posted on the board. Throughout EUPs, I took field notes. At the conclusion of each school day, I typed up these notes and made entries in my own reflective journal. Before designing each subsequent EUP, I typed up and read students’ journal entries. The iterative nature of this process formed the basis of my engagement in cycles of initial analysis of each EUP.

Following the ‘versus’ coding approach described by Saldaña (2013), the analysis in this chapter and in Chapter 5 is based on the categories shown in Table 4.1 below. The initial categories were derived from the research questions and conceptual framework. Thus, the a priori codes (numbers 1, 2, 3, 6, 8, 9, 10, 12, and 13 in the table) are a result of identifying various dimensions of the research questions. In the process of passing from one EUP to the next, I was engaged in cycles of analysis. Some categories (numbers 4, 5, 7, 11, 14) emerged from this preliminary analysis of the data. The a priori and emergent codes are shown in Table 4.1.
Table 4.1

*Categories Used in the Versus Coding*

<table>
<thead>
<tr>
<th>Critical Mathematics Education</th>
<th>Market-Driven, Standard-Based Mathematics Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Dialogic teaching</td>
<td>Anti-dialogic and distorted communication</td>
</tr>
<tr>
<td></td>
<td>embodied in initiate-respond-feedback and initiate-</td>
</tr>
<tr>
<td></td>
<td>respond-evaluate. Authoritative and/or transmission</td>
</tr>
<tr>
<td></td>
<td>style of teaching: banking concept of education</td>
</tr>
<tr>
<td>2 Collaborative learning</td>
<td>Competitive learning</td>
</tr>
<tr>
<td>3 Teaching-learning material:</td>
<td>Exercise paradigm: skill-drill type of repetitive</td>
</tr>
<tr>
<td>open-ended word (real-world)</td>
<td>practices</td>
</tr>
<tr>
<td>problems</td>
<td></td>
</tr>
<tr>
<td>4 Finding joy in learning</td>
<td>Teach to the test</td>
</tr>
<tr>
<td>5 Applying mathematics to</td>
<td>Applications that picture students as passive</td>
</tr>
<tr>
<td>broader social, political</td>
<td>consumers and obedient citizens</td>
</tr>
<tr>
<td>reality to form critical</td>
<td></td>
</tr>
<tr>
<td>consciousness and competencies</td>
<td></td>
</tr>
<tr>
<td>6 Inquiry-based math education:</td>
<td>Exercises and questions that have a single correct</td>
</tr>
<tr>
<td>answer may take different</td>
<td>answer</td>
</tr>
<tr>
<td>shapes based on related</td>
<td></td>
</tr>
<tr>
<td>standpoint</td>
<td></td>
</tr>
<tr>
<td>7 Critical and creative thinking</td>
<td>Exercises and questions that have a single correct</td>
</tr>
<tr>
<td></td>
<td>answer</td>
</tr>
<tr>
<td>8 Self-reflection as authentic</td>
<td>Procedural knowledge and rote memorisation</td>
</tr>
<tr>
<td>assessment</td>
<td></td>
</tr>
<tr>
<td>9 Critical-reflective learning</td>
<td>Non-critical learning aimed at adaptation and</td>
</tr>
<tr>
<td>oriented towards change and</td>
<td>domestication</td>
</tr>
<tr>
<td>transformation</td>
<td></td>
</tr>
<tr>
<td>10 Inside out and bottom-up</td>
<td>Top-down impositions of educational changes</td>
</tr>
<tr>
<td>responses (from life-world)</td>
<td></td>
</tr>
<tr>
<td>11 Dialogic peer relationships</td>
<td>Hierarchical-dominating: peer tutoring</td>
</tr>
<tr>
<td>12 Teacher as equal partner in</td>
<td>Teacher as authority figure and ultimate source of</td>
</tr>
<tr>
<td>learning process</td>
<td>knowledge</td>
</tr>
<tr>
<td>13 Small openings in life-world</td>
<td>Life-world (classroom) is colonised by the system</td>
</tr>
<tr>
<td>(classroom)</td>
<td></td>
</tr>
<tr>
<td>14 Mathematics education to</td>
<td>Mathematics education to meet the needs of the</td>
</tr>
<tr>
<td>meet the needs of students</td>
<td>system and reinforce status quo</td>
</tr>
<tr>
<td>(young citizens) as human</td>
<td></td>
</tr>
<tr>
<td>beings</td>
<td></td>
</tr>
</tbody>
</table>

These 14 categories also guided the analysis and discussion presented in Chapter 5.
4.1 Reconnaissance Stage

This stage formed the pilot to the development of the EUPs. By the very nature of critical participatory action research, students’ participation was crucial to the success of this project. Therefore, during the first five weeks of the school year, within the mandated curriculum, I integrated discussions and learning activities to articulate to the students the purpose of the study and the scope and importance of their voluntary participation. As indicated in Section 3.4.4, I informed students that the activities and assignments would not be graded in the usual fashion; instead, the purpose of the lessons was to experience the enjoyment of learning mathematics in a new way. For instance, in this preliminary stage, I developed and facilitated two block lessons, conducted over two days, on the concept of slope and tangent of a given angle using real-life environments; the school is a two-story building with stairs in multiple locations. The students spent most of the first 90-minute period out of the classroom in small groups, with cameras and measurement tools. They calculated the slopes of the stairs and took pictures. The next day, groups presented their work along with calculations and pictures. One group concluded that the slope of the stairs next to the cafeteria was too steep and should be fractioned into two or three pieces to lower its slope. I wrote in my reflective journal:

My initial intention was to use these experiences to shape the classroom into a public sphere where students could easily exchange different ideas and where a community of collective learning could be established. In this sense, this two-day lesson was a greater success than I had anticipated. (October 10, 2014)

I initially anticipated that inviting students to participate in learning activities that were not conventionally assessed (such as awarding a grade of A or B) would be a major challenge to be addressed in the reconnaissance stage. During the two-day
lesson, I asked my students whether they would have been more interested if the activity was graded and recorded in the grade book. In their feedback, students confidently stated that they enjoyed the activity, and being graded would not have made any difference to their participation level. Therefore, I concluded that my initial anticipation was not supported. Subsequently, I designed each EUP as a non-graded activity. Therefore, reflecting on the students’ positive reaction to ungraded learning activities was crucial to developing my professional understanding about participation and engagement. I inferred from this pilot lesson that if a learning activity is joyful for students, they actively and enthusiastically participate, whether graded or not. Conversely, I concluded that if a learning activity or lesson was considered boring by students, they would usually participate—because it was graded. At that point, I resolved that for this project all EUPs should be related to students’ life-world and interests in order to maintain their active participation and motivation.

As part of this pilot stage, I facilitated several classroom discussion sessions about the purpose and requirements of the study and held two open-house sessions for parents to have one-on-one conversations about the project. I answered questions raised by students and parents about the project. There were 28 students enrolled in the class at the beginning of the school year, but prior to data collection, one moved out of the state and another dropped out of school for a family-related issue. I completed the study with 26 students; all students and parents gave their formal informed consent.

4.2 End-of-Unit Projects (EUP)

The following projects were developed as part of unit plans within the district-mandated curriculum. By the time the project began, I had been teaching in the same classroom for eight years. The demographics of the student body, the neighbourhood
culture, and school district expectations were well known to me. As I was considering the contextualisation and themes for each EUP, my experiences in the school provided me with a tacit understanding that helped me frame each project. I spent considerable time developing each project to ensure that it resonated with the goals of CME. The EUPs began at the beginning of October 2014 and ended at the end of March 2015.

Each EUP was conducted in a similar format to provide initial answers to the research questions (Merriam, 2009). The EUPs are presented chronologically as they occurred in the classroom, as suggested by (LeCompte & Schensul, 1999). Data from each EUP overview contains relevant quotations from the whole-class discussion, as well as excerpts from students’ journals and my reflective writing. Initial analysis of data collected is presented through the lens of critical theory (LeCompte & Schensul, 1999; Miles & Huberman, 1994).

4.2.1 EUP 1: Student-Produced Multiple-Choice Questions

4.2.1.1 Planning

This was the first project aimed at creating a communicative space as a concrete step to moulding the class into a democratic public sphere where shared concerns and problems about our education can be negotiated. The purpose of creating such a space, as outlined by Kemmis, McTaggart, et al. (2014), is that students ‘need both space and permission to bounce ideas around’ (p. 91). If students felt uncomfortable expressing their thoughts and ideas, this study would not have qualified as CPAR.

EUP 1 focused on multiple-choice assessment in mathematics for several interconnected reasons. Multiple-choice tests take up considerable time in the students’ life-world. They are directly relevant to students’ lives; therefore, every
student in the class could easily participate in group work, classroom discussions, and peer dialogues. In other words, standardised assessment can be considered a shared concern of students.

In this project, I first scaffolded developing a multiple-choice (MC) question in mathematics on the board; students asked questions and provided feedback in this process. Students created their own groups, and each group was provided with a (non-MC) question. Then each group produced the options for their own MC question. Once each group completed designing their choices, they presented their questions and explained their process. Afterwards, each group conducted a brief search on the web to collate a range of ideas for and against using MC tests. Three primary objectives were set for this EUP:

- Improving students’ conceptual knowledge and practical skills with respect to mechanical aspects of related content (linear equations/functions and graphs): common misconceptions, procedural mistakes, and other elements that often lead to wrong answers on MC questions.
- Providing students with an opportunity to build a collaborative learning environment where dialogue is one of the forms of learning: opening a communicative space.
- Helping students gain insight into the social, political, and economic implications of MC tests to connect life in the classroom to the larger sociopolitical system and critically reflect on it: countering neoliberal hegemony.

4.2.1.2 Day 1 (October 21, 2014)

I had planned to have a whole-class discussion on the topics of collaborative learning, inquiry, and dialogue; however, I changed my mind after realising that letting students experience the project first and having the discussion at the end would be more productive, as they would have a concrete experience to reflect on. Having
briefly explained the purpose of the project, I scaffolded the process of turning a
problem of writing a linear equation into a MC question.

After this scaffolding stage, I let my students form self-selected groups of
three or four students. I then handed out questions that I had written earlier—one
question per group (see Appendix A for a sample of the students’ work on the
project). While groups were working on their own MC questions, I circulated among
them and paid close attention to the following three main points; this was necessary
since we aimed to find ways in which dialogic and collaborative learning of
mathematics can be experienced:

- Peer communication
- Active participation and exclusion of members
- Power relations

As students requested assistance, I answered their questions and provided
suggestions. For example, Nick raised his hand and said, ‘How come we can’t put
wrong random answers, how come we have to calculate?’ My response was, ‘If you
put random answers, common mistakes would probably never lead to one of the
choices…in that case, you would probably check your solution process one more
time; you would be less likely to fall into the trap and end up with wrong answer.’
Nick answered, ‘It makes sense, I never thought of it that way.’ Nick could have
addressed the same question to his peers first. But he preferred to ask the teacher. In a
traditional classroom, the teacher is regarded as the ultimate source of knowledge, as
prompted me to note in my reflective journal that it would take time to shift from a
traditional teacher-centred classroom to a facilitative classroom.

The group work progressed quite smoothly overall. In my observation, peer
discussions mostly focused on the most common mistakes for each stage in the
solution manual. As I circulated among the groups, I noticed that mechanical questions seemed easy for some students. However, other students were improving their understanding of the mathematical concepts. For example, at one point, Cincere asked his group, ‘You put it in slope-intercept form…why is that?’ Jennifer responded, ‘It’s easier to graph slope-intercept form.’ I closely observed another group as they were debating the order of operations—whether \(-1^2\) is same as \((-1)^2\). In this instance, a group member explained to his peers why these two expressions were not the same. Some groups completed their project earlier than expected and had to wait until other groups completed the task. Upon reflection, I realised that I should have planned an extended or additional activity for those who completed their work earlier. I realised that I had to plan for this type of outcome in the future projects and be proactive about it.

Once each group completed their MC question and were ready to share their work, two groups volunteered to present their work; there was not enough time for all eight groups to present. I urged the class to ask questions of their peers who were sharing their work, but this moment did not trigger a lively discussion of these presentations. I noted this in my reflective journal:

Students did not attempt to actively question the presentations: It might be because, as 10th and 11th grade students, they had already formed an attitude that does not include space for questioning, which resonates with transmit-receive paradigm. We may need to work on skills for questioning. (October 21, 2014)

Gutstein’s (2006) descriptions of his teaching experiences in middle school confirms my inference: ‘U.S. schools socialize students into non-questioning roles, creating and maintaining passive identities so that students do not believe in their own power’ (p. 88). In the middle of the second group’s presentation, all the 10th graders were
called to the Great Hall for an assembly via loudspeakers. I told the students that we would continue tomorrow.

4.2.1.3 Day 2 (October 22, 2014)

I briefly reminded my students what we had done yesterday and gave back each group’s paperwork. Before the whole-class discussion, each group did a brief web search on standardised MC exams using their mobile phones

23 to review different ideas about this type of test. I asked students to critically evaluate the information they found on the Internet and reflect on their experience of designing MC questions; each group wrote down what they found informative, and we had a short discussion. I then commenced a whole-class discussion on MC tests and their sociopolitical and educational implications. I posted the following discussion points on the board:

- How can we turn group work into a collaborative learning process?
- Is multiple-choice a fair assessment from your perspective; why or why not?
- Do you think multiple-choice tests can accurately measure the success of students, teachers, and schools; why or why not?
- What could be the sociopolitical and educational implications of multiple-choice tests?

In my efforts to facilitate the discussion, I consciously tried to let my students control their peer dialogues to learn from their experiences and interactions with each other. I permitted the entire process to run its course to enable me to determine potential obstacles to students establishing non-dominating and dialogic peer interactions. This process enabled norms and expectations to be established for discussions in future EUPs so that the overall quality of whole-class discussions would be improved.

23 My students and I agreed on the rule: No talking on the phone or texting in the classroom. Cell phones in this project were for educational purposes only.
I asked the class to reflect on their overall experiences of group work, both in our class and in the past. The students agreed that today’s group work was productive. Most students enthusiastically shared their experiences and made suggestions to improve the quality of group work. Almost all the students had something to say about group work since they have had some previous experiences of group learning. Based on past experience, the most common drawbacks were as follows:

- Unstructured assignments: Some teachers assign group projects without proper scaffolding and without clearly stating their objectives.
- An overly large group: If it contains more than four people, it is counterproductive.
- A member of the group attempts to dominate: the self-assigned leader issue.
- Some members take credit for the outcome without doing their fair share.

To solve this issue, teachers need to assign each member to a specific task.

Students’ motivation and participation level were quite high. This discussion validated students’ opinions and experiences as they contributed to the learning process in the class (Nystrand, 1997). They articulated negative experiences they had in the past and came up with suggestions for improving the quality of group work in our class. My students were taking ownership of the collective learning process. Most students suggested assigning a specific task to each student in the group to prevent potential problems. This feedback revealed that students had not yet experienced collaborative learning. Group work remained an instrument of learning subject matter. Given this instrumental perspective, the students’ suggestion to assign each student to a specific task is reasonable; however, such division of labour is nothing more than working individually in a group, as noted by Horn (2014) and Pietsch (2009). I realised that for the next EUPs, I needed to hold several whole-class discussions to explicitly scaffold students with elements of collaborative learning.
At the conclusion of the school day, I went over my literature review, which made me realise that I needed to review more scholarship on dynamics of collaborative learning as opposed to competitive or individual learning. I was well aware that collaboration and dialogue as conceived by the theory of critical education have certain philosophical roots and ethical implications that drastically differ from group work in neoliberal pedagogy. For example, critical theory ideologically promotes collaboration and solidarity whereas neoliberal ideology supports competition and individuality. I realised that I needed to deepen my own understanding of collaborative learning and dialogic classrooms in order to clearly distinguish group work guided by critical pedagogy from group work suggested by neoliberal pedagogy. I would have to elucidate this in such a way that my students would clearly understand the difference between these two pedagogies. Meanwhile, this train of thought led to the following question about collaborative learning: What if my students have already been conditioned by traditional education to believe that competition is better than collaboration when it comes to learning? I continued expanding my literature review as this classroom experience raised more questions.

Although my students indicated some negative aspects of their previous group work experience, they also agreed that learning through collaboration is more enjoyable than studying individually. For instance, Thor said, ‘My friends may know something, and I don’t, and I can learn from them…you know, they may get something from me as well.’ Students’ responses were in accord with Vygotsky’s zone of proximal development (ZPD), which specifies that collaborative learning allows students to learn from and with each other. However, from my observation, I inferred that working in each other’s ZPD might resemble transmission-style
education if students passively accept their classmates’ ideas as opposed to learning with and from each other. My journal entry documents this potential problem:

The students’ proposal for equal power for all is a good starting point. But we need to create a classroom ambiance where the students have equal powers and they feel comfortable to exercise their power. If some students hesitate or do not know how to question their peers’ ideas, regardless of the norm of equal power for all in the class, other students may possibly dominate in the group or impose their ideas on each other. ZPD outlines a good structure for students learn from and with each other, but it may potentially create power relations among the students. I need to reconsider these points for next EUPs. (October 23, 2014)

Furthermore, to be able to implement ZPD effectively, I needed to identify possible obstacles to the students’ collaboration. One goal of CME is to turn a classroom into a community of collaborative learning, which requires students to socialise, respect, and work with each other regardless of any cultural, religious, and ethnic differences they might have. However, eight years of teaching in U.S. have made me realise how difficult this might be, as students socialise with peers from their own culture, in and out of the classroom. For example, based on my lunchtime observations, Asian students sit at the same table while African American students hang out together. Based on my experience in my school, ethnic pride is mistaken for multiculturalism. This point could be one of many obstacles to establishing a culture of collaboration and dialogue in a diverse setting. I did not want to raise this point for the students yet. Instead, I tried to ascertain if students would bring up this point. I asked students whether it would be better if they set their own groups. The students responded:

Hannah: Hmm….It is not always productive because we would pick our close friends and be easily off-task and end up chitchatting.
Me: We should be able to work with each other in this class regardless of our ethnicity, gender, race, religion, or any other differences we might have. Our differences would only enrich our life in the class. What is more…what unites us as human beings is much stronger than what seems to separate us….What do you think?

I observed the students’ eyes to capture their immediate reaction to what I said. There was a momentary silence; it seemed obvious that what I just said was new to them. However, they did not appear to disagree. I then asked the class to share their opinion about dialogue and conversation; their responses were quite interesting. The whole class agreed with comments made by some that ‘dialogue’ is something that takes place onstage or in a movie; they viewed it as ‘scripted’, as something that does not leave any space for ‘free talk’. However, a few students added that conversation allows one to talk freely; this observation provoked collective agreement among the students. I did not address their comprehension of the term ‘dialogue’, but this incident made me realise that I should continuously emphasise the concept of dialogue, collaboration, and inquiry in each EUP. As Groundwater-Smith et al. (2003) pointed out, creating a dialogic classroom cannot be achieved overnight.

The second topic in the whole-class discussion was whether MC assessment is fair and objective or not. Students individually shared what they thought, after which a broader classroom discussion began. Nick spoke first:

I just read an article talking about education in Denmark….It says they rarely give multiple-choice tests in school….They focus more on conceptual understanding…and they have the best score on international exams….Maybe we don’t need to take so many tests….The article says that American high schools spend about two months of the school year testing.
A moment of silence followed Nick’s talk. Having knowingly attempted to avoid IRE and IRF moves, I reluctantly added one comment to Nick’s words:

A comparison between different countries and education systems may open our horizon to understand educational matters better….As you said, there may be no correlation between a successful education and frequent multiple-choice testing….Anyone else wants to contribute?

Ceylan articulated her opinion:

I don’t think it [MC] is a fair assessment because one may know nothing about a question and the other student may have answered almost correctly and made a small mistake just before getting the final answer….Just an example, in our group, we picked a mistake at the last stage that takes third power of negative 3 and writes 8 instead of negative 8….A multiple-choice test degrades these two students….It is not fair at all.

Ceylan formed her conclusion based on her group work experience. The point she raised aligns with similar arguments made by Kohn (2000) and Sacks (2009), according to whom MC assessment applies a binary logic that does not distinguish the answer of a student who has no clue from the response of one who only slightly missed the right answer. At this point, Eric joined in the discussion:

Let’s see…I’ve never been to a math class like this…you know, like…in math classes, teachers give you equations, and you solve it….We have taken so many tests, but I never thought it was objective or fair, things like that….Ok, we say it was easy or difficult, but that is all….I have just read an article and learned that multiple-choice is cost-effective and practical; that is why we take it so often.

It was interesting that I had already planned to bring up Eric’s point, but it came up within the flow of student discussion. As I was preparing a reasoned response in my

24 Initiate-respond-evaluate (IRE) and initiate-respond-feedback (IRF) are regarded as nondialogic, transmission-style instruction.
head, Jennifer objected to Eric’s point: ‘Yeah… but… it may be cost-effective, but that doesn’t make multiple-choice good for us…. You know, fast food is also cheap and cost-effective, but they are unhealthy.’ Multiple students’ voices were then heard agreeing: ‘Yay… sounds right’. Tibu added another dimension to the discussion:

Ok, I agree with you [Jennifer]…. You may not go to McDonalds, but how would you avoid these [standardised] tests?… Last semester, some of us opted out of EOC [end-of-course tests], but you know the curriculum coordinator lady got us together and told us that if we don’t take the test, our school would get less federal funding.

This was an exciting moment for me as a teacher! Eric noticed that this was a non-traditional lesson. He shared an idea from his web search as an explanation for the rationale for MC tests. Jennifer came up with a perfect analogy that contested Eric’s point and unpacked the tension between the needs of systems and needs of students. Tibu’s comment exemplified how mechanisms of colonising life operate in the classroom. The discussion seemed to have encouraged my students to build on each other’s ideas, aligning with Freire’s (2000) suggestion that a dialogic approach and a horizontal student-teacher relationship encourage students to express their ideas freely. This creates the possibility for a dialogic discourse that allows humanising education to take place in the class. In this sense, my deliberate move to create a non-authoritative and an egalitarian classroom ambiance succeeded: It led to more active and joyful student involvement. This classroom exchange was different from those I had had in the past and led me to conclude at this point that a communicative space was gradually emerging in the class. I addressed Tibu’s comment and carried on:

In my opinion, we may not be able to change things overnight; it may take a long time. And we have to take and pass these tests in short run. However, this doesn’t mean that we should embrace these standardised tests uncritically and
unreflectively….As students and teachers—citizens—we have the right to express our dissatisfaction and concerns about standardised assessment….And to be able to change something, we have to have a good understanding of the matter….Who else wants to add something to this topic?

Another moment of silence followed my comment. I sensed that this silence was a sign of the processing that occurs when students are introduced to new or unfamiliar ideas, concepts, or approaches. It is often suggested that moments of entering a new frontier of knowledge trigger a process of inner debate at the individual level (Rogers & Freiberg, 1994; Tsankova & Dobrynina, 2005). Tibu added another comment, returning the class to the central group discussion:

Mr Bulent, you are trying to make us think critically….Would not you get in trouble for this? You know, it is like these tests are made by the government, and if you criticise multiple-choice, just like you criticise government, you know.

His tone seemed to be half-serious and half-joking. I did not know how to respond to it at that moment. Upon reflection, however, I was surprised that a student could clearly sense that there is a hidden curriculum. His comment and my reflection on it implied that engaging in an educational practice that fosters critical literacy is not welcomed by the official thought, which resonates with the point made by Giroux (2014):

The collapse of education into training, the loss of autonomy by teachers, the removal of the conditions that enable students to be critical and engaged citizens all speak to the character of a society in which independent thought is debased; creativity, stifled; and dissent, squelched….At its best, education is dangerous because it offers young people and other actors the promise of racial and economic justice, a future in which democracy becomes inclusive and a dream in which all lives matter. (para. 25)
Tibu’s comment and my reflection on it relate to the notion that market-driven changes have turned schools into places where free thought and critical thinking are not welcome: This is more obvious when students recognise the limitations of non-critical approaches to teaching and learning. Once I gathered my thoughts a moment later, I responded to that question: ‘We all, as citizens still have a right to express our opinion by the Constitution of the U.S.A. and if making you think critically will put me in trouble, I will take that risk.’ Directly following my response, Molly addressed Tibu’s comment: ‘He is supposed to make us critically think, why would he get in trouble for that?’ This was a significant moment. Both students are referring to critical thinking: While Molly spoke of critical thinking in the sense of functional literacy, Tibu’s use of critical thinking generated an element of critical literacy that enables one to think beyond official thought and to talk back to authority.

Daryl raised his hand to join the discussion. He faced his peers when he was talking. At that moment, I was standing behind his table in the left-hand corner and writing down my observations. I realised that this kind of peer communication and whole-class discussion could only occur in a dialogic classroom. I initially decided to address the scope of critical thinking within both functional and critical literacy, but I momentarily changed my mind. Instead, I took note to save this point of discussion for projects to come. I did not want to digress too much to preserve the integrity of the ‘free dialogue’ in which these students were genuinely participating. Daryl stated:

I was thinking…like….These tests like SAT or, you know, EOC, government hires some companies to do these tests, and government pays them for this….It doesn’t make any sense to me. Our teachers can make and grade these tests as part of their job….Why involve these companies? And if our teachers graded these tests, government would not have to pay them extra….They already get paid anyway. This seems to me better cost efficiency than paying some big testing companies.
Until Daryl’s statement, the students seemed to collectively agree on standardised tests being cost-efficient, but inadequate for measuring students’ knowledge. Daryl directly challenged this point, arguing that letting teachers create and score the tests would completely eliminate the cost itself. Daryl’s comment emerged as segue to the wrap-up of the discussion. Many students nodded after he spoke. I deemed it a good time to move on to students’ making their journal entry.

Having handed out journals, I posted the following prompt questions for reflective journal entries:

- Provide one negative aspect of your group work experiences in this project and other classes in the past. How do you think we can improve the quality of our group work for projects to come?
- Based on your experience in the project, is multiple-choice (MC) a fair and reliable assessment tool? Why or why not?
- If it is not, why do you think that MC is widely used to measure students’ academic success?
- Anything else you may want to address with regards to the project.\(^{25}\)

4.2.1.4 Individual Student’s Reflections

Students’ journal entries mostly mirrored the whole-class discussion, but individual students took advantage of the exercise to articulate their thoughts in greater depth. For example, Darryl’s comments summed up the whole class’s reflection about the project: ‘Everything in our group work yesterday was great except the problem was a little easy.’ The simplicity of the task was a point I had noticed in my observation. The immediate lesson I learned as the teacher was to

\(^{25}\) The intent of this question is to allow individual students to propose agendas and discussion points they might have.
consider more overtly the level of mathematical challenge; a task that was too easy or too difficult would most likely be counter-effective in terms of students’ motivation.

However, the approach taken in the project helped some students clarify their misconceptions about some of the mathematical concepts. For example, Cincere wrote, ‘When we were talking about mistakes and wrong answers, I learned how to figure out shortest distance between a point and a line….I mean really this time with understanding.’ Mason wrote, ‘I always thought that negative sign in front of a fraction would affect top [numerator] and bottom [denominator] numbers….In our group discussion, I got it that negative sign effects top or bottom, not both.’ These reflections illustrate that in addition to connecting mathematics learning to a larger sociopolitical system, the project evidently stimulated students to improve their mathematical skills and knowledge. The comments also show that the lesson successfully achieved one goal of CME—to ensure that students are engaged in learning mathematics and successful in a traditional sense (Ernest, 2002c; Gutstein, 2006).

Students’ reflections were significant, enabling me develop an understanding of the students’ overall perception of the role of group work as compared to more traditional pedagogies that they had been accustomed to. Reflections provided me with a starting point towards jointly working on turning our classroom into a community of collaborative learners. Based on their comments, almost all students agreed that domination was a major problem in small group work. For example, Tibu wrote:

Group works usually sucks because, there is a person or two that assign themselves as group leaders...you know, big shots….and they often control entire process and that sometimes intimidate other people….I believe a
solution to this problem would be giving everybody a role based on their strengths and weaknesses, so they can complete their task.

Many students approached the same issue from different perspectives. For instance, Serano voiced another problem: ‘When we do group work, usually one person works and the others mess around. If each person’s role is assigned by the teacher, it may be solved.’ Bin echoed the same approach: ‘[If] each person is assigned to a specific task, [it] will solve it.’ Their comments were informative for me to understand students’ current perception suggesting the teacher’s domination, since our goal was to create non-dominating and dialogic peer relationships oriented towards establishing a community of collaborative learning.

Many students suggested the teacher’s direct intervention as the solution for the problem of domination in group work. These comments seemed to indicate that they prefer to be told what to do, implying that students do not consider themselves proficient in handling potential problems arising in group work. Within traditional pedagogy—which operates as a more anti-dialogic and transmissive teaching (Freire, 2000)—this demand is not unusual. As a result, as part of practising CME, during the next project, I needed to create more opportunities to challenge this expectation and work on developing students’ skills and values oriented towards egalitarian and collaborative learning, where students can self-organise, coordinate their projects, and solve potential conflicts that may arise in this process.

Although Lexus brought up the same problem, the solution she suggested was slightly different from those of her classmates: ‘Domination is always a problem in group projects. I think if everyone in the group is given the same power, this may eliminate that negative aspect.’ Lexus implies an egalitarian peer relationship is necessary for a truly collaborative learning process. In that relationship, equal power
distribution should probably come from the teacher. Bill also acknowledged the
domination problem; however, he implied that peers could resolve conflicts resulting
from domination without a teacher’s direct intervention:

Sometimes a group member, a know-it-all dominates the rest and people don’t
like this. But I believe that this could be solved through a democratic process
in the group without outside interference, because usually most people will
agree with the most logical and pleasing idea.

Cincere suggested that potential problems in group work should not be reason for
choosing to work individually: ‘When the group is working by themselves and not
together and you have two stubborn students in disagreement about who’s right. We
can eliminate this by settling the disagreement instead of working in singles.’ The
whole-class discussion and students’ journals indicate that students challenged the
myth that ‘standardised assessment is good’. As my students developed their
arguments, they referred to tests they had taken in the past, their group work
experiences, and online searches as they articulated their arguments and conclusions.

Ceylan argued that MC tests may fail at accurately assessing students’
knowledge; however, other manners of assessment may also be problematic. She
wrote:

I don’t think MC is fair because there is a possibility that a student could get a
correct answer purely through guessing and another student could study hard
and make a small mistake and end up with wrong answer….But I think
sometimes teachers are biased as well….One of my teachers in middle school,
she always favoured some students….MC eliminates this.

Ceylan’s point is that a student may achieve a correct answer in a standardised test by
guessing while a wrong answer may be caused by a minor mistake. However, her
comment also reveals her scepticism about the impartiality of some teachers.
Bin reflected on the test he took recently to question the reliability of standardised assessment:

MC is not objective or fair; for example, I just took an SAT test, which is quite important for college application; there was a third-degree equation to solve, I did not know how to solve it? So I just plugged all choices and got the right answer. I got the credit without knowing the subject matter. So how could SAT or other MCs be a reliable assessment tool to define one’s future? There should be more options to measure our knowledge.

He articulated that test results should not be the only tool to define students’ academic success and future. On the other hand, Bill approached the test fairness issue from a different perspective: ‘Based on my experience, MC is fair but not reliable. It is fair because everybody gets the same test, but there is no way to know that the correct answer is chosen by chance or knowingly.’ Tibu’s argument was more of an individual account: ‘I don’t like MC because it is made to expose your mistakes.’ Cincere framed his argument with a question by writing, 'It is so obvious that MC is not fair, but how come people keep saying it works?’ He took an important step towards linking life in classroom to a larger sociopolitical system, indicating the contradiction between his experience and the neoliberal argument that claims the standardised assessment is the best way to improve quality of public education.

While most students challenged the idea of standardised assessment without linking it to a broader socioeconomic picture, some students clearly mentioned the political and economic aspect of standardised assessment. For example, Jennifer wrote:

I think our work should be assessed by free-response problems when we can show what we know and our process….MC is used because it is cheap and quick for the government, but I think our government should do whatever is best for our learning, and they should not use MC just because it is cheap.
This quotation reveals the tension between the needs of students as human beings and citizens and the needs of the system (money and power). As explained in Chapter 2, in an education system guided by market imperatives, the needs of students are not prioritised (Apple, 2000a; Giroux, 2012; Hyslop-Margison & Thayer, 2009). Jennifer stated that a free-response exam would be a better way to assess students’ knowledge. However, grading a free-response exam is much more expensive than an MC test, as free-response exams usually require certified individuals for grading, whereas MC exams can easily be graded by a scanner machine. From Jennifer’s point of view, a preference for assessment type should be based on whether it allows students to demonstrate what they know rather than whether it is cost-effective or not. This comment reveals that students recognise that the system imposes its rationale on students and colonises their life-world in the classroom without their consent.

Jacob was also critical of MC testing. He said that MC does not correspond to the problems that exist in one’s life, but offered a reason for government’s preference:

MC puts you in a situation that you will never face in real life….When problems happen in real life, no one writes five choices for you, and you know one of them is correct….I think testing companies make a lot of money out of these MC tests….It is not a reliable assessment for students, but an efficient way for them [the system] to make money.

Mikael’s approach to the fairness of MC was unique. Demonstrating a nice sense of humour, he wrote:

MC doesn’t measure what we know…it focuses on what you do not know….In our group we made four wrong answers and only one correct one. It gives you four opportunities to fail:) how nice! Therefore MC tries to select golden chickens, but it is not fair since everybody wants and deserves to be the golden chicken.
Mikael’s response reveals the students’ understanding of standardised assessment as an apparatus to produce winners and losers. He thus illustrates the contradiction between the stated argument of neoliberal education to bridge the achievement gap and what students identify as consequences of standardised assessment for themselves (Apple, 2005; Hursh, 2007a).

Having reflected on his group work experience in this project and an online search, Nick made the connection between standardised assessment and its political economy. Like Jacob, he suggested that assessment methods should resonate with challenges in real life:

When we wrote the question in our group, I realised that MC is designed to trap you, rather than help you….MC does not require deep understanding. For example, sometimes you can plug all choices and get the right answer. In MC, all they care about is the right answer, but life doesn’t work that way. There isn’t one right answer in life; there are many, sometimes none….In our group, we have read an article that was saying that testing and publishing companies promote MC as they make money out of that. I have never heard of that before—very interesting. That explains why MC is still around after all.

Nick based his critique on arguments that link life in the classroom to a larger socioeconomic and sociopolitical structure. Darryl put forward a similar perspective. From his point of view, the government’s preference for MC tests is not for academic but rather for corporate reasons. He articulated the contradiction between corporate interests and the needs of students:

MC doesn’t reflect your real skills and tries to mess you up on purpose….Problems in real life and situations are not like this….Large companies make MC, and they make lots of money from this….Fair or unfair, they will do everything to keep MC tests like SAT around. But what is good for companies is not good for us [students] and this is a big mistake.
All my students showed their dissatisfaction with MC tests for various reasons. Some referred to their life in the classroom while others made connections between the classroom and a larger political and economic life outside it.

4.2.1.5 Discussion

This project was a process of reflective learning to experience the ordinary extraordinarily (Kemmis, McTaggart, et al., 2014; Shor, 1987). My students and I analysed the standardised tests as part of reflective learning practice. The students appeared aware of the fact that standardised tests take up a large segment of classroom time. In that sense, the topic for them was quite ordinary. However, as we conducted the project, we took a critical approach to understand aspects of standardised tests that are either taken for granted or that rarely become a topic of discussion in colonised public spheres. It was evident that through this project we, as a class, took a big step towards turning our classroom into a dialogic and democratic public sphere. The students’ reflections indicate that they realised that their voices were heard and valued as those of equal partners; their engagement and active participation were genuine.

During class discussion and group work, students listened to each other, shared their experiences and thoughts, and built on each other’s ideas. Throughout the project, most students actively contributed in the classroom instead of being passive receivers. As much as I favoured the horizontal student-teacher relationship as defined by Freire (2000), my students enjoyed dialogic peer interactions and a more equal structure of participation. In this project, my own reflections, student reflections, and classroom observations demonstrate that the students improved their content knowledge, developed assessment materials (MC questions), reflected on their experience, and linked their life in the classroom to a larger sociopolitical life.
However, I also noted in my reflective journal that the practice of CME cannot be developed overnight. The skills, values, and attitudes students needed to become active and critical citizens require continuous praxis. Based on this first project, some adjustments needed to be introduced for the development of the next phase. For example, my students indicated that certain individuals’ dominance in group work as a potential obstacle to a collaborative learning process. Therefore, in the next project, I needed to work on egalitarian peer interactions as a process to move the classroom interactions towards a more dialogic orientation, so that students themselves could learn about, conduct, and organise a collaborative learning process without requiring the teacher’s authoritative intervention.

Based on the reflections, this project uncovered a number of pedagogical challenges that required consideration in the development of the next EUP. I needed to spend additional time (one or two days) to expand the students’ thinking about the implications of the standardised tests as a way to deepen the collective insight into recent neoliberal changes that affect life in classroom. Simultaneously, from an educator’s perspective, I had to address district and state standards; in reality, as an action research approach shows, this neoliberal and managerial pressure significantly narrows the possibility for the flexible use of classroom time.

The following lessons were learned from this project that would inform EUP 2:

- More challenging mathematics content and a more structured lesson plan are necessary to coordinate group work and use classroom time productively. However, I realised that often I had to make initial revisions and changes to the original plan. My lesson plans should allow for this flexibility.
- Establishing norms for non-dominating peer interactions for collaborative learning within a dialogic classroom.
• Expansion of our discussion on obstacles to collaborative learning and possible actions to eliminate them.

EUP 1 was a successful entry into the research aimed at studying the potential and limitations of CME in a high school mathematics classroom, which has been reshaped by neoliberal educational implementations. While improving their content knowledge, the students related their learning to a larger society and developed a bottom-up response to neoliberal educational changes. The project helped the students engage in a structural analysis of neoliberal educational policies and implementations and develop and exercise critical mathematical literacy. At the beginning, I was nervous and excited. After I observed the highly positive attitude from my students, I became more excited and hopeful—and less nervous—for the projects to come.

4.2.2 EUP 2: Part-Time Worker

We began a new unit of district-mandated curriculum that covered the mathematical concept of function, including topics such as domain, range, graphical, numerical, and algebraic analysis of a function, piece-wise functions, and family of functions. In this project, students needed knowledge and skills on algebraic, numerical, and graphical analysis of a function as well as learning to calculate intercept point(s) of given functions. We had already covered the mechanical aspects of this content prior to the project. Therefore, my students were familiar with these topics as we began EUP 2.

4.2.2.1 Planning and Objectives

I developed EUP 2 in light of the lessons learned from EUP 1. This excerpt from my reflective journal describes the preparation stage:

The main theme of this project is the socioeconomic class division and inequalities. Within this theme, my intention was to focus on public interests
versus corporate interests. I have reviewed word problems in mainstream algebra textbooks to design a word problem. However, I noted that almost every word problem in these textbooks involve themes such as best-buying decisions, maximising profit, or minimising cost; my intention was to find a word problem that could be revised as Freitas (2008) suggested. But I could not find any word problem that could be useful for this project. I then designed an open-ended word problem of my own. (November 19, 2014)

As my journal extract indicates, finding learning materials oriented to practice CME was an ongoing challenge. I developed EUP 2 by applying these four criteria:

- The word problem should not be so open-ended that students would lose focus and motivation; however, it should not be too easy, either.
- It should contain rigorous mathematics at conceptual level in connection with our current unit as defined by the district standard to meet procedural necessities.
- Students should be able relate the problem to their life-world.
- The problem should be inquiry-driven so as to inspire students to collectively engage with critical and creative thoughts and have space for dialogue and negotiation through which they can approach the problem from different perspectives as they work in groups.

The word problem for this project is shown below (See Appendix B for a sample of the students’ work on the project):
Project 2: Part-Time Worker
Edward is a senior student in High Hill High School and has recently got a part-time job in a local restaurant to support himself and his brother Joel, who is a middle school student. He is going to work there after school and sometimes over the weekends. As he negotiated his weekly salary with his boss, his boss offered him two different calculations for his time:

\[
S(t) = 16t + 20 \\
W(t) = \frac{8}{5} t^2 + 20
\]

Edward’s total work time cannot be more than 40 hrs as he is a part-time worker. Before Edward made his final decision, he told his boss that he needs a day or two to think about it. Also he asked his boss whether or not he could combine these two offers as a multipart function of time (or piece-wise); the answer was YES….It could be a multipart function.
Edward will collaboratively work with his friends in his math class to come up with an offer that best presents his interests out of all possibilities.

The problem illustrates a negotiation between a young worker who is a senior student in a high school and his boss. The boss offers the student a weekly salary, which includes two distinct functions. The problem allowed my students to develop different solutions that favour either the young worker or his boss. The purpose here was to challenge the common perception that mathematics problems always have a single answer (Skovsmose, 1994, 2011). Further, because CME is aimed at fostering critical literacy, my intention was to contextualise mathematical discussions (functional literacy) in a discussion of social class issues to promote critical literacy.

4.2.2.2 Day 1 (November 17, 2014)
To begin EUP 2, I facilitated a whole-class discussion. Having reflected on the previous project and students’ feedback, my students and I together decided to negotiate and set some norms for working collaboratively. First, we addressed the students’ reflective journals; there were several suggestions by students to improve the quality of group work. For example, some students had proposed that if the teacher assigned each student in a group a specific task, it would solve problems of domination. I briefly explained that if I assigned each student in a group a specific
task, this would no longer be qualified as collaborative learning; this is evident in the following transcript:

Me: Our aim is to achieve learning and applying math through collaboration and dialogue....Dialogue is much more than our daily conversation. In our group work, if we try not to dominate each other...if we do not listen to each other...if we do not build on each other’s ideas and suggestions, our conversation would not evolve into dialogue....Without dialogue, our group work would not turn into a process of collaborative learning....It would be just an individual’s work in a group.

Nick: That sounds right, but what if one tries to run over everybody or not work at all?
Me: That is what we need to figure out together in our class....I have some suggestions, and you should come up with your own as well.
Jennifer: Ok....Honestly, I sometimes step back [in group work], specially when I am not confident about subject matter.
Multiple students: Yep!...That happens to me....Me, too.
Me: Can we, please, talk one at a time....[a moment of silence]. Then it is our responsibility to welcome our friends who, for some reasons, are left out....Similarly, if one tries to dominate, politely remind him/her that we are trying to achieve helping each other rather than racing each other.

Lexus: I think we should all agree that everybody in group work should have same power....I don’t like when people are bossing me around.
Me: I absolutely agree with Lexus....If anybody wants to add something more..

In this brief discussion, the students agreed to jointly coordinate group work on their own and ask me for suggestions as needed. Further, we decided that if a student was not part of collaborative learning, other group members should gently try to welcome them in. Finally, the students jointly agreed that if somebody tries to
direct the group, they would be politely reminded that our goal is non-dominating collaborative learning. The students’ comments revealed that they recognised that dominating behaviour in group work is not productive as it disrupts the inclusiveness of group work, which aligns with suggestion made by Horn (2014) that true collaborative learning necessitates egalitarian peer relationships.

The discussion turned out to be much more stimulating than I had anticipated. The students’ comments indicated that they enjoy the humanising, dialogic, and non-hierarchical learning process described by Freire (2013, 2000). As Freire argued, whereas dialogue allows the development of a humanising education, anti-dialogue is the root cause of oppression characterised by the banking concept of education.

After this brief discussion, I handed out the project paper and had five minutes of silence for students to review and process the word problem. Then I divided them into coeducational groups; also, each mixed-gender group contained students from diverse backgrounds. Circulating among the tables, I observed and took notes about peer interactions in groups. As I was in one corner of the classroom, from the other end, Tibu raised his hand and made a joke, which was loud enough to be heard by everyone in the class: ‘Mr. Bulent, Ben is trying to dominate our group, and he is very dogmatic too.’ Multiple students laughed at this comment. This joke generated a warm ambiance and signalled students’ awareness of what we were trying to achieve and their enthusiastic participation in this collective learning process. A group of three seemed to be engaged in lively conversation. I noted that Allen was making a T-chart to see the input-output relation; Jennifer was trying to graph the two given functions with a graphing calculator; and Nick was solving the system to find the intercept point. The next transcript shows the development of the group’s problem solving through peer dialogue:
Allen: I think Edward should go by the linear one…it seems he makes more money this way…

Nick: Are you sure? Quadratic one should bring him more money….You know each time you square, it curves up quickly…

Allen: I tried 2, 3, 4, 5, 6 hours….definitely linear one is a better choice for Edward…

Jennifer: Actually [showing graphing calculator’s screen to her peers], you guys are right….Linear one has a higher output first, but it changes after a point.

Nick: You mean the intercept point? I got 10, is that what the calculator shows?

Jennifer: Let me check….Yes! It is 10….Allen, can you plug 12 in your table and see what happens?

Allen: [he calculated by pencil] Wow! Now quadratic one is way higher.

The excerpt indicates that the students learned from and with each other as they worked in each other’s ZPD (Vygotsky, 1978). Peer discussions seemed to gradually evolve into dialogue; each student in this group followed a different method to approach the problem, which seemed to complement one another, and no student was left out.

In one particular group, each student applied a different method to solve the problem and they jointly combined their answers to developing a piece-wise function that favoured Edward. I noted that peer communication in group work was inquiry-based and egalitarian. It appeared that no student was trying to compete with the others in the group. It was also evident that in this incident, peer solidarity was an emerging and self-organising phenomenon.

All groups seemed to be engaged and making contributions to completing the task. Furthermore, no student was excluded during discussions. I focused my observation on one specific group for a while; the group was already discussing how
to frame a multipart function in favour of Edward. I noticed that no group was trying
to come up with a multipart function in favour of Edward’s boss. It seemed that
students had a strong empathy with Edward as they could relate his situation to their
own life-world. I made this assertion upon reflecting on a question I posed to the
students at the beginning of the school year. I asked my students if they were working
somewhere to make money. Twenty-six out of 28 students had a part-time job either
after school or over the weekend. In my reflective journal, right after that period
during my preparation time, I noted:

Since we emphasised non-dominant, non-hierarchical peer relations in group
work, my students seemed mindful and sensitive in their interactions; all
students enjoyed that their participation were equally valued. They took
ownership of learning process and generously contributed in their group work
without posing as power figures. My observation strongly confirms that
vertical relationships or hierarchies, either among peers or between the teacher
and students, eradicate possibilities of participation and consequently dialogic
(humanising) learning as Freire (2013, 200) argued. My observation also
confirmed that once egalitarian peer interactions occur, students feel more
confident and can productively socialise with each other and learn with and
from each other, as suggested by the Vygotskian concept of ZPD. However,
unlike Vygotsky’s argument, there is no need for a more competent student to
tutor others. (November 17, 2014)

After the problem-solving process was completed, two groups briefly shared
their solutions on the board. Noah, Taylor, and Michael’s group presentation seemed
highly effective in terms of the quality of collaboration. Noah explained, ‘I did
equalise two functions to calculate the edge point for piece-wise function.’ I
intervened and asked if this was their first action, since many students seemed to use a
T-chart to make numerical analysis. Taylor answered: ‘I was trying to do that [T-
chart/numerical analysis] first, but Noah said it would take a long time.’ Nick (who
was not in the group) raised his hand and said, ‘I believe this was a collaboration, like it’s an “us” project, not a “me” project.’ Jokingly, he added, ‘I think Noah attempted to impose his perspective on his group.’

This humorous perspective spontaneously created a warm and welcoming atmosphere; students laughed for a while. Taylor said, ‘It may look like Noah was dominating, but I don’t think he meant it.’ Michael added that they should have politely reminded Noah that each voice was supposed to be heard. Michael also agreed that Noah’s act was not intentional. I intervened with a question: ‘Can we generalise Michael’s comment that when authorities or government attempt to impose something, it is our duty as citizens to come forward and voice our concern that we are not Ok with the situation?’ After a short silence, Noah’s comment finalised this conversation: ‘I had no intention to dominate my group. I just said that a T-chart would take a long time and is not always reliable, and I can still show that a T-chart takes much longer than solving systems of equations.’ It was evident that Noah was trying to say that his action was driven not by power dynamics, but a mathematical argument.

Egalitarian collaborative learning and dialogic peer interactions were part of the objectives of this project. In this context, this conversation revealed that my students were learning and embracing dialogic and non-dominating peer interactions in their group work. Therefore, this outcome indicated that we as a class had made progress towards objectives set for this EUP and for the entire project. As Pine (2009) suggested, developing dialogic and collaborative learning has to be learned, and it requires a systematic approach.

Nicole’s group also shared their group work. She placed her group’s paper under the document camera and explained how they came up with a graphical
presentation of the situation. Jacob went on to explain algebraically and numerically why their multipart function favours Edward. At the end of the explanation, Daniel asked the class if they had any questions. Eric raised his hand: ‘This Edward’s scenario does not look like real….I mean, if he works 30 hours a week, he would make a lot of money….No job would pay that much….What were you guys thinking?’ As Jacob attempted to answer the question, he was interrupted by a school-wide announcement by the principal, and we had only a couple of minutes left for that period. I quickly tried to wrap up discussion, ‘Ok, let’s continue tomorrow….We’ll come back to this question as soon as the period starts. Please turn in your final work before you leave the class. One paper work for each group….Thank you, and have a nice rest of the day.’ I instantly noted that this question would be a good start-off point for a whole-class discussion next day.

4.2.2.3 Day 2 (November 18, 2014)

Next day, the lesson began with the question from the day before: Was the word problem compatible with current wages? Because on the previous day Jacob was cut off by the announcement, I invited him to start. He said, ‘If Edward worked 30 hours a week, his annual salary would be way higher than a part-time job would pay. I myself work in an ice-cream shop and get paid minimum wage.’ Allen added, ‘Even many full-time jobs don’t pay that much at all.’ My students were already linking Edward’s story to a larger economic system that seemed to frame this project as a meaningful learning experience.

Daryl raised his hand: ‘It is not only wage….I think it is not realistic that Edward’s boss offers him a choice. People usually work under restricted conditions.’ Selena joined in: ‘I don’t think his boss will accept our piece-wise function that favours Edward. I mean, why would he? [the boss]….The boss has the power and
Edward doesn’t. Why would any boss agree to pay more?’ Tibu responded, ‘If the boss wouldn’t accept the function that favours Edward, what could his motivation be? Like…is he trying to see how smart Edward is?’

These comments reflect a successful realisation of one of CME’s fundamental goals—to ensure that students are provided with learning opportunities to develop critical mathematical literacy and to be able to ask questions such as ‘whose interests are served and who benefits?’ (Gutstein, 2006, p. 5). I momentarily intended to make a transition from a relation between the boss and the worker in the context of Edward’s story to public versus private interests in the context of a corporate-driven world. However, we had time restrictions and needed to complete the project today. We still had to make entries in our student journals. I shared this concern with my students, and we then moved on to a journal session.

I posted questions shown below on the board using the projector. As a class, we agreed to spend ten minutes for a discussion of these questions in groups before making journal entries. While some students focused on a single question, others responded to multiple questions. I let my students know that in connection to this project, they could bring up any point that they considered worthwhile. Their responses should not be limited by the prompt questions. Because this action research was conducted with my students, not on them, they should also be able to come up with their own questions to address.

4.2.2.4 Prompt Questions

- Do you think a student needs basic mathematics skills in order to make informed and educated decisions and to be critical/active citizen?
- Do you think that we live in a class society?
- Do you think Edward’s interests clashes with his boss’s interest? Why and how?
• Can you give an example of private versus public interest?
• Do you think that the quality of collaboration and dialogue in your group work for this project was better than our previous project?
• In what ways was this project mathematically meaningful for you?

The journals were collected five minutes before the bell rang. I asked the students if they would like our next project to be on the history of mathematics. They enthusiastically asked me several questions. I told them that if we agreed, it would be an open-ended inquiry and each group could decide their own topic.

Within four days, I reviewed all journals. By referring to Edward’s employment story, my students clearly articulated that one needs to be literate in mathematics to be become an active and critical citizen. Nicole illustrated the developing perspectives on how critical literacy could allow individuals to prevent sociopolitical manipulation:

Basic math and even basic school education is necessary, in my opinion, so one is not manipulated by others, but is an active citizen instead….If Edward did not know graphical and functional analysis, he may have ended up with a package that favours his boss. In our group, we came up with two piece-wise functions….One was in favour of Edward, and the other was good for his boss.

While Selena agreed with others on the necessity of basic mathematics skills for becoming an active citizen, she argued that advanced math skills are not necessary for one to become a critical citizen. She stated, ‘In order to be an active citizen, [and] also to survive, you need basic math skills; however, not everybody has to know advanced complicated math.’ This point was also a challenge for CME: Where do we draw the line between basic and advanced mathematics in the context of critical citizenship?
Jennifer added another layer to the discussion: ‘I don’t think one needs only basic mathematics skills not to be manipulated or be active citizen; I think one should be able to speak for oneself.’ Her comment implies that functional literacy itself may lead to adaptation to a given socioeconomic condition. However, one needs critical literacy to demand change for the better, as well as courage to stand up for one’s rights. She questions whether there is a relation between knowing something and the courage to act upon that knowledge.

Darryl agreed on the necessity of mathematics skills; he looked at the issue from a wider perspective and provided examples to back up his point:

I do believe that one needs basic math skills in order to not be manipulated by others and to be active citizens. Math is everywhere in our lives. This includes our economic exchanges. For example, when purchasing insurance, one must know about all the different parts and fees as well as what they will cost and cover to make an informed decision that is best for you….Large companies are smart and know how to take advantage of people to gain profit….Having basic math skills helps one not to fall in these traps. One must also have these skills to make informed decisions when voting on taxes and budgets.

Darryl’s comment reveals an emerging critical mathematical literacy: He appeared to be aware that corporations have no self-imposed limitations when it comes to making a profit. One needs critical (mathematics) literacy to detect and challenge these situations, which aligns with the definitions of critical mathematical literacy made by Frankenstein (2005) and Skovsmose (2011).

Lexus’s comment focused on Edward’s story. She wrote:

People like Edward need to have basic skills. If we take this scenario like real, if Edward did not know his math, his boss could have easily manipulated him into a piece-wise function that benefits the boss. If you have basic understanding of math, you will be able to pick the one that benefits you.
My students’ journal entries indicated that they believed mathematics knowledge and skills to be necessary for survival and being active and critical citizen. They also agreed that they live in a class society by providing evidence from their life-world. However, the matter of class is a sensitive subject to discuss in public in the U.S. According to Chomsky (2003), there is a top-down imposed perception that Americans live in a classless society. Thus, I did not anticipate that my students would be able to smoothly connect Edward’s story to the larger socioeconomic system. I was expecting that at least a couple of students would repeat clichés such as ‘This is a land of freedom and equality…we are all same’, etc. However, as I evaluated the results of this project along with the previous one, the dialogic, collective, and inquiry-based learning process in the class was becoming a more self-directed process. The students’ reflections indicated that inquiry-based dialogue could create a communicative sphere where even a top-down imposed perception—such as that of the U.S. as a classless society—could be challenged.

In her response, Jennifer identified asymmetrical power relations in society to explain why we live in a class society: ‘No matter how free and classless we think we may be [in our country], some people have more power than others. There are bosses and workers; the rich and the poor; and the government and the people.’ From her point of view, a classless society would require equal power distribution. Sidney’s reflection was complementary to Jennifer’s, linking Edward’s story to the significance of critical literacy for ordinary people to make informed decisions:

Yes, we live in a class society where high-class people have lots of money and education and power to manipulate for their benefit; therefore, people like Edward should have math skills and other skills as well not to be deceived or exploited.
Sidney implied that in a class society, those who are in power position may exploit ordinary individuals’ ignorance; therefore, citizens need critical mathematical literacy to protect themselves as much as possible for these possible deceptions and exploitations. Her journal entry does not imply that being literate in mathematics would prevent deception and exploitation of all kind. She refers to Edward’s story to support her point. Molly goes beyond Edward’s story and directly refers to her everyday experience in the school to address to the question:

> Obviously we live in a class society….Even in our school, one can see that there are students who live in families where [they] don’t always get a full meal every day or warm clothes to wear. One can also see the students who always have whatever they need whenever they want.

Molly was the only one who pointed to the socioeconomic background of children at our school to exemplify the class society. Even though most students in my school come from a poor socioeconomic background, some are from higher-income families; my long-time experience in the school confirms Molly’s argument. The students’ comments revealed that critical mathematical literacy was emerging as they examined their lives and Edward’s story ‘in relationship to sociopolitical’ and socioeconomic contexts, in the same vein as Gutstein (2006, p. 5) distinguishes critical literacy from functional literacy.

During this project, we did not have any direct discussion about clashing interests in a given society. However, the students’ comments indicated that the word problem had two meaningful solutions. While one favours Edward, the other would increase his boss’ profit. They noticed that the solution that benefited the boss was not good for Edward. For example, Leonardo made several connections to indicate that
we live in a class society, including a recent campaign\textsuperscript{26} in Seattle, Washington, to increase the minimum wage:

Yes, we live in a class society. The rich people’s interests and regular people’s interests are quite opposite. For example, while workers in the Seattle area demand higher minimum wage, $15, which is currently $9, the rich oppose this, as a wage increase means their profit will decrease. The same thing for Edward….If we make a piece-wise function to take side with Edward, his boss will get less profit. On the other hand, if we favour the boss, Edward will get underpaid. However, the point is that Edward doesn’t have any power, while the employer has money and power and others.

Leonardo’s comment revealed that Edward’s story enabled him to develop different approaches to the problem. This outcome resonates with Skovsmose’s (1994) suggestion that word problems should be designed in ways that provide a landscape of investigation for students to develop multiple answers as opposed to a single correct answer. Leonardo, who also had a part-time job to meet his school expenditures, noticed the basic fact that employers have asymmetrical power over employees; he implied that Edward’s story in this project does not reflect reality as far as power relations go.

One of the prompt questions listed earlier invited students to think about public interests and corporate interests in order to make a transition from the interests of employer and employee to corporate interests. We did not have much time to discuss this point during whole-class discussion. However, some students established a coherent logic to make the transition. For example, Ceylan addressed the question in the context of Seattle’s minimum wage campaign:

\textsuperscript{26} Since 2014, there has been a civil movement for increasing minimum wage from $9 to $15 in Seattle, WA. This movement triggered a larger public debate in Seattle area and spread to other part of the U.S. Further information can be obtained at http://www.occupydemocrats.com/seattle-mayor-says-behind-living-wage/
An example of class interest in our society is the upper class arguing that minimum wage should not be increased because it will only cause inflation. Obviously, it would be in their [corporate] best interest to keep the working class’s salary as low as possible, so they don’t lose revenue in paying employees more….Public interest is in favour of the overall good of society as whole. Private interest is in favour of one individual’s personal gain like owners of big companies….I am not sure how salaries exactly are determined, but it is probably determined by the bosses who calculate the minimum amount of money they can give to their workers that will pay them enough to sustain themselves while maximising the boss’s personal profit.

Looking at socioeconomic structures through the lens of the working class, Ceylan’s reflection revealed her understanding of critical mathematical literacy as she identified oppressive and exploitative aspects of labour relationships (Freire, 2013; Giroux, 1983; Gutstein, 2006).

The students linked this EUP to an ongoing public discussion over wage increase; it was one of the tangible successes of practising CME here. In fact, I was going to do one EUP on the wage-increase campaign in Seattle, but the school principal (head teacher) did not approve it: I was told that this was a controversial issue and may cause problems. Therefore, I did not raise this topic in class. Nevertheless, some students made the connection in a different context by linking Edward’s story to a larger socioeconomic and political structure.

Creating a dialogic classroom culture through inquiry-based and collective learning process was the ongoing goal. We attempted to make group work a more democratic and collaborative learning process. My students reflected on the quality of group work as it was experienced in the project. Nicole acknowledged that we had improved: ‘The group work in this project worked a little better than the previous one.
We worked very well together and did same amount of work without overpowering one another.’ Nick provided a more detailed account:

All members of my group were actively participating, asking questions to each other, listening to each other’s questions, pulling their own weight, putting out good work and ideas….As working on the project proved that we count on each other to do their parts, but also we respect everyone and try not to be dogmatic and suppressive. Everyone wants to be able to work in an equal, non-dominant environment, and I think we achieved just that in this project….First time I felt that I got why do we need piece-wise functions and how intercept of two graphs could be useful.

His reflection indicates that in this project, dialogic peer relations and collaborative learning of mathematics have improved; this notion seems to be consistent with that of Pietsch (2009), who argues that establishing dialogic collaboration requires systematic effort and time.

Kohn (1992) indicated that collaborative learning creates a sense of belonging and community; Jennifer’s reflection resonates with this notion: ‘I felt like a part of my group in this project was way better than last time….I spoke my ideas on how to find edge point for the piece-wise function without thinking, What if I was wrong and they would make fun of me?’ Jennifer felt that she would not be judged or embarrassed if she made mistakes. It is evident that her group achieved respect and tolerance that are vital for dialogue and collaborative learning. Darryl also indicated that their group work was a success:

Hannah, Nick, and I communicated very well to make all calculations and algebra to figure out the edge point [intercept point] to graph the system and write a multipart function for Edward….We greatly contributed to each other’s thoughts as well as the final work itself.
In my observation, I noticed that Edward’s story triggered inquiry-driven peer collaborations. As I had anticipated, the problem called for multiple approaches to possible solutions; the transitions from mathematical argumentation to students’ life-world to a larger socioeconomic system were dynamic.

4.2.2.5 Discussion

In EUP 2, all the students genuinely worked on establishing dialogic peer interactions as they worked in groups. Based on the students’ reflections, inclusiveness in group work in this project was another emerging point. They had previously said that when someone acts as a self-assigned leader, others might be intimidated or discouraged. Once non-dominating peer interactions have matured, students seemed to become more careful not to exclude anyone.

The students appeared to develop a democratic and egalitarian sensitivity—as witnessed in their reflections—and at the same time improved their critical mathematics literacy as they connected their life-world in the context of Edward’s story to the larger socioeconomic system. Although I provided scaffolding for the process, I aimed to be an equal research partner with my students. The dialogic classroom environment gradually created a self-regulating framework, which led to small openings within which my students had an opportunity reflectively question concepts such as the class society and the difference between private, public, and corporate interests. This project met the four objectives listed earlier and thus was another step towards creating a dialogic mathematics classroom and successfully practising CME.

4.2.3 EUP 3: History of Mathematics

The day before the third project began, a brief lesson was conducted on operations with zero and infinity. I wrote mathematical expressions on the board (as
shown below) and then facilitated a whole-class discussion to encourage students to develop some insight into these topics. My intention was to evoke their curiosity about the history of mathematics by briefly introducing the historical background of these concepts.

| a) \( \frac{3}{0} = ? \) | b) \( \frac{0}{3} = ? \) | c) \( 2\infty + \infty = ? \) | d) \( \frac{3}{\infty} = ? \) | e) \( \frac{\infty}{3} = ? \) | f) \( \infty \times \infty = ? \) |

This brief lesson incorporated an instrumental aspect. The next unit contained the concept of asymptotes, which required working with the notions of zero and infinity. Students usually have difficulties comprehending topics that require a deeper conceptual understanding than the mere procedural knowledge needed for solving skill-drill-type repetitive examples. I facilitated a brief class discussion to negotiate each key question. For example, students’ first reaction to expression (d) indicated that they assumed infinity was a known number. The discussion helped students clarify some confusions and misconceptions. However, from a CME stance, to help students comprehend such concepts in more depth, it would be necessary to explain the historical context in which each concept emerged.

### 4.2.3.1 Preparation and Objectives

My reflective journal contains a summary of the key objective of this lesson:

This project is the continuation of EUP1 and EUP2, which aimed at creating small openings in our mathematics classroom for students to develop values, attitudes, skills, and knowledge to become critical citizens. Similar to the previous two projects, we intended to achieve these goals through an inquiry-based collaborative learning and dialogic classroom interactions. D’Ambrosio’s (2010) approach to mathematics education was my main inspiration for this project. The objective of this unit was to provide my students with the opportunity to realise that mathematics is a collective
production of humankind and recognise that collaboration historically is more beneficial and less destructive than competition. (December 13, 2014)

The reflection refers to D’Ambrosio’s (2010) conceptualisation of critical mathematics education as *ethnomathematics*. He argued that mathematics education could contribute to world peace and provide students with the ability to survive with dignity:

As a mathematician and mathematics educator myself, I accept as a priority, the pursuit of civilization with dignity for all, in which inequity, arrogance and bigotry have no place. This means, to achieve a world in peace and to reject violence. (p. 53)

Inspired by D’Ambrosio’s (2010) point, I considered the history of mathematics the main topic of this project. I asked the students if they would like to inquire into the history of mathematics. As students requested some examples, I told them that they could research, for instance, the contribution of non-European cultures or the historical emergence of certain mathematical concepts such as pi, zero, or infinity. The students were excited to be able to select a research topic for their group.

4.3.3.2. **Day 1 (December 11, 2014)**

I posted the details for this project via electronic projector and then explained the scope of EUP 3. I handed out the paperwork as a hard copy and let the students review it for five minutes. The project included a problem-posing aspect; it did not directly involve any mathematical problem in a conventional sense. Students were supposed to collectively determine the topic of their study and its extent; they were able to use Internet links provided in the package to do the research. Therefore, in comparison to the previous projects, this one required higher coordination skills,
collaborative decision making, and dialogic peer interaction in order to achieve inquiry-based learning.

In the previous two projects, I set the groups. This time, responding to the students’ emerging collective initiative, I allowed them to select their own groups. Each group was provided with two laptops with Internet access and a printer. As I circulated among groups, I observed that students were becoming collaborative, deciding what to do without excluding anybody. Trying to dominate one’s peers and expecting the teacher, as an external authority, to assign each student to a specific task were no longer notable issues—they all participated and contributed equally to the work. However, as I reviewed the lesson package at the end of EUP 3, I noticed that the project was too open-ended. I did not specify where to end research on pi, for example. Each group had to decide where to start and how to conclude, which could be frustrating and confusing for students.

Nevertheless, the students were more self-organised, excited about collaborative learning, and enjoyed taking ownership of their learning; I observed that they handled the entire process very well. In my reflective journal, I noted the progress towards collaborative learning and dialogic classroom interactions:

My students have been becoming active participants and agents more and more in the class. All groups had a brief discussion and agreed on a topic from the packet. They coordinated the whole process much better than in previous projects. They prepared posters out of their research by including everyone in the group. During this time, the quality of questions they asked each other and ways in which they responded made me think that there is a dialectical relation between dialogic peer interactions and collaborative learning: one would not exist without the other. My students improved their skills of active participation and taking initiative. Observing this while facilitating made me feel great and gave hope for a better future as a public school teacher. My
students have been changing their ways of learning and doing mathematics, and interacting with each other. (December 11, 2014)

As noted in this reflection, it was evident that students’ ability for collaborative learning and peer dialogue had improved since EUP 1. I realised that inquiry-based egalitarian collaboration and non-dominating peer interactions had been transforming the class into a community. The reciprocal was also true: As the class became more of a community, egalitarian collaboration and dialogic relationships among students were more apparent.

An incident occurred during this project that was significant in terms of highlighting limitations of CME within a neoliberal educational system. While students were working on the project, the school principal visited our class as a part of my teacher evaluation. He talked with the students in each group and took notes on his laptop. At our school, teachers receive their evaluations through an Internet-based system. Four days later, I received an email with a link to review my evaluation. In the comments section, the principal left the following note:

I talked with most students in the class. They were extremely engaged with their projects. They answered my questions with excitement. I reviewed the lesson plan, but I could not see any standards. I wonder what common-core state standards were addressed in this lesson.

Even though an instrumental part of the project focuses on the content, I was not able to relate this lesson to any standard in the paperwork for the project. In fact, the new national standards, common core state standards (CCSS) do not include any information about history of mathematics. There was also a section in the evaluation form for me to respond to the principal’s comment. I recorded the same comment in my journal:
The standard (CCSS) for high school mathematics, unfortunately, does not cover history of math. Therefore, I was unable to directly refer to a specific standard. However, the lesson indirectly relates to learning objectives in the standards. For example, students need conceptual understanding for usage of zero and infinity to fully comprehend concepts such as undetermined situations, undefined fractions, and asymptotes. In this sense, the lesson covered multiple standards. Also the National Council of Teaching Mathematics (NCTM) strongly advises integrating history of math into math classes to empower students. Moreover, in all developed countries, the math curriculum includes history of math. In my professional opinion, our curriculum should cover the historical dimension of math. I believe that it is our responsibility to provide our students with the opportunity of experiencing cultural and social side of math beyond repetitive equations, isolated facts, and theorems even though current standards do not suggest doing so. (December 15, 2014)

This incident forcefully reminded me of the intricate daily dialogue between neoliberalism, standardisation, and innovative teaching and learning; it confirmed the critical analysis of neoliberal education by numerous scholars (Apple, 2000a; Hursh, 2007b; Kohn, 2000; McNeil, 2009). Commenting on the restrictive aspect of neoliberal educational changes, Apple (2000a) noted that some states in the U.S. ‘not only have specified the content that teachers are to teach but also have regulated the only appropriate methods of teaching. Not following these specified “appropriate” methods puts the teacher at risk of administrative sanction’ (p. 70). This episode reminded me that I needed to be more cautious in balancing the needs of the standardised curriculum as I practised CME.

I kept circulating around the class and observed groups learning about different symbols for numbers, having discussions about pi, zero, infinity, and the connection between mathematics and technology. Towards the end of the class, Malik asked: ‘My brother is taking calculus, and I looked at his textbook. Every chapter
cites all these individuals who developed calculus. But our algebra textbook doesn’t cite anybody. Does algebra come from an unknown history? What do you think, Mr. Bulent?’ I did not expect such a question, and I was not sure if Malik had his own answer and was just checking on me. I responded, ‘Why don’t we research this question tonight when you go home? We may be able to see different opinions and approaches to Malik’s question. And we can add this question to our class discussion tomorrow.’

At the end of the class, students suggested posting their work on the wall instead of doing presentations. Their rationale was that when we did presentations, only one or two groups could present their work due to time constraints. On the other hand, if they hung their work on the wall, it could remain there for much longer, and other students in different periods could read them as well. It was a strong idea that involved students in the decision-making process, which resonates with the spirit of participatory action research. All the groups hung their posters around the classroom walls and talked about each other’s projects.

**4.3.3.3 Day 2 (December 12, 2014)**

Students were ten minutes late because of a fire drill from the previous period. Having briefly reviewed the material from the previous day’s class, I facilitated a whole-class discussion. Because students selected different research topics, I opened the discussion with a broad question:

Me: As the quote from NCTM\(^\text{27}\) says, math is a joint production, and humankind and we should appreciate it; however, math has not always been used in the past to make our life better. Perhaps we need to look at math beyond our classroom walls and see how it functioned in past.

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\(^{27}\) The quotation is cited on the second page of project’s paperwork.
If you need to say something about your research yesterday, what would like to share with your classmates?

Nick: It was interesting that math people in ancient times visited other cultures to exchange ideas. They did math collaboratively just like we do in this class.

Multiple students laughed for a while. There was a sense of community spirit spontaneously generating a warm and inviting ambiance as they began thinking and talking about the history of mathematics:

Lexus: Yes! Greek mathematicians went to Egypt to study math, for example.

Jacob: It was amazing to see how different cultures used different symbols for numbers and did addition and multiplication with two-digit numbers...way different than we do today.

At this point, I tried to redirect discussion onto specific questions.

Me: Do you think mathematics has been more of a help or harm for people?

Bin: We would not have any technology without math, and you know technology is something good for humanity. But I am not sure it is always beneficial for people.

Ceylan: We read an article talking about nuclear wars, and atomic bombs would be impossible without math. I guess math can be used to harm people, but it can be used for good causes. For example, math was used to make weapons and bombs to kill people. But the energy of a bomb could be used to heat up households during wintertime.

Me: I agree with Ceylan. It depends on who uses math with what purposes. A corporation may use math to increase their profit while communities can use math to optimise their needs based on their resources for public good....We learned that math is an outcome of collaboration, but we all know that people may not always work together. What do you think keeps people from collaborating instead of making them fight or compete?
J-Paz: If people have different [conflicting] interests, they would probably not collaborate. That makes sense, but I never understand why people fight with each other when their interests are the same. If they help each other, it would be in every one’s favour.

These comments revealed that the students considered collaborative learning as more productive and less distractive than competitive learning, a discovery that resonates with a comprehensive analysis by (Kohn, 1992). Kohn argued that whatever can be achieved through competition can also be achieved by collaboration. Unlike competition, collaboration is not destructive. The discussion continued:

Me: What about our classroom; do we have any rational reasons for not collaborating?

Darryl: It is different….You know…our collaboration is for learning, and, as Jen said, we all want, to learn and we learn better when we study with each other.

Sidney: I personally feel good about myself, I mean, whenever I learn something new from my friends or they learn from me. It makes me feel I am part of this class.

Me: We learned in this project that mathematics is a joint production of the human family. What would you say if a friend of yours came up to you and told you that he/she was proud to belong to the human family?

Tom: I would tell him to go to the nurse.

[Multiple students laughing]

Ceylan: It sounds like a cheesy word to say, but we actually belong to a human family. My friend may be a Filipino, she may be a Korean or Mexican, but we are all human beings.

Andrew: Yes…but, I feel different, like, you know…I belong to my own culture. I am German-Irish. If you say human family….You know, you also include Chinese or Russian or something else, and I am not part of their culture.
Ethnicity and race are delicate topics in the U.S. I carefully attempted to come up with an analogy to convey my point:

Me: Well...if you think of your ethnic background as a flower in a big flower garden, your flower looks nice as it is situated next to others; it enriches the entire garden. Without the big garden, your flower would not be the same. As you are proud of the big garden, you are also proud of your own flower while appreciating existence of others.

These discussion questions took up almost the whole period. However, I felt as if we were jumping from one topic to other. I had planned to address the question that Malik had asked yesterday, but instead I included his question in the prompt for journal entries as we ran out of time. On that day, we had a 15-minute-shorter period as the office scheduled a student assembly at the end of the day. I posted the prompt questions on the board and gave students 10 minutes to exchange ideas with each other. I then handed out their journals. They quietly made their entries for the rest of the period.

4.3.3.4 Prompt Questions

- Would you consider mathematics as a joint achievement of all cultures of the world or specific nations/cultures? Explain your rationale.
- In your opinion, can differences be barriers for people preventing people from collaborating to produce common good? Provide examples to back up your answer.
- Do you think that it would be a good idea to make history of mathematics an integral part of our mathematics curriculum? Why or why not?
- While all calculus textbooks cite their contributors, algebra textbooks do not mention the historic background of algebra. Can you think of any reason behind this?
I anticipated that some students would think mathematics a Western production as opposed to a product of many nations and cultures. But all students stated that mathematics emerged from a diverse human history. For example, Taylor perceived mathematics as a universal value:

Before this project I had always thought that math was something into itself; through this project, I realised that it is a collective achievement of humanity. Math is something that brings people together like music….I also learned the concepts of zero, infinity, and pi and their philosophical stuff.

Lam drew on an argument that opposes the idea of a cultural superiority. Given that all the world’s cultures contributed to mathematics, this could not have been done by one single nation or culture. He also asserted that dialogue is needed to work with others productively, and people’s differences do not have to be barriers for dialogue:

This project made think that no culture is superior to another culture….Throughout history, different cultures contributed to the development of math. We did research [in our group] and learned that some mathematicians in ancient times visited different places, civilisations, and exchanged ideas to improve math….One thing I got from this is that differences should not keep us from having dialogue with others.

Some students made clear connections between mathematics, culture, and politics. For example, Ceylan shared her ideas about the underlying reasons for cultural superiority of Western countries. She linked advanced technology to mathematics and their emergence in the Western world. However, Ceylan also indicated that without other cultures’ contributions, advanced mathematics or technology would be impossible:

Yes, all cultures contributed to math, but advanced math resulted in more advanced technology, which generally correlates to feeling of cultural
superiority like developed Western countries. But these Western countries
developed advanced math on top of what other cultures had already done, like
algebra came out in Babylon. Western countries are politically strong and
have huge military power, but I don’t think that makes them culturally
superior to other nations.

Ceylan’s comment was consistent with that of D’Ambrosio (1999), who
suggests that integrating the history of mathematics into the curriculum may motivate
students who come from historically marginalised and underserved communities.

Jacob used the history of mathematics as an example to support his conclusion that
collaboration is beneficial: ‘It made me think that without collaboration, math would
not be a math as it is today….We agreed in our group that, by any measure, I think,
collaboration produces more common goods than competition.’ It appeared that more
students began to see value in collaboration, and the classroom became a better
community of collaborative learning.

Reflecting on the previous EUPs, I facilitated whole-class discussions in EUP
1 and EUP 2 to probe common obstacles to collaboration. Students provided valuable
feedback and suggestions to overcome those obstacles. Collaboration had always
been voluntary. I gave students the option to work individually if they wished to do
so. Interestingly, however, all students preferred to work with their peers. The class as
a whole considers collaborative practices to be useful for their learning. In their
journal entries, students reflected on the dynamics of competition and collaboration.

Darryl argued that our common roots are stronger than our differences:

Some people think that differences are obstacles for collaboration to achieve
common values. I don’t agree with that. We are all human beings [and] should
understand that we come from the same beginning. We should respect each
other to work together to change the world for the better for everyone.
In Darryl’s view, there is no reason to consider people’s differences as obstacles to collaboration; such differences can be overcome. Nicole stressed the irrationality of the notion of cultural superiority. Feeling superior to others eradicates possibilities for collaboration and dialogue:

Cultural, religious, and other kinds of differences do not have to be obstacles to work collaboratively, but sometimes it can be as well. When people from a certain cultural/ethnic background see themselves superior to others, differences become barriers for collaboration. On the other hand, interacting with different people can actually teach you something. Learning to appreciate others’ ideas and values can also make it a value of your own.

From these comments and others like them, I concluded that having a discussion on collaboration versus competition helped the students and me gain a deeper insight into potential obstacles to collaborative work. It also helped make their collaborative activity more conscious and deliberate during small group work. Nicole implied that the notion of cultural superiority feeds arrogance, whereas one needs to be humble to appreciate other’s ideas and values.

Jennifer-Paz put forward a perspective similar. She seemed to consider our collaborative learning efforts as an outcome of a conscious joint decision. She implied that we have to overcome being dogmatic and dominant:

I would like to think cultural, religious, or ethnic differences should not get in the way of working collaboratively, but the reality is that it becomes an obstacle sometimes. The sad but inevitable truth is as long as people remain opinionated, differences may keep people from working collaboratively. We do collaborative learning in this class, because we agreed on non-dominating communication, and we are trying not to be dogmatic.

Some students, on the contrary, appeared to be sceptical about humankind’s capacity to achieve peace. For instance, Cindy wrote, ‘I am not so sure if differences are
barriers for people to work collaboratively to produce common good. Yes, we have universal values that unite us like love, art, music and math, but how can we explain all these wars and bad things that took place in history? She seemed to realise that collaboration is positive and beneficial for everyone. At the same time, she recognised the violence and injustices of the past, thereby avoiding a romantic optimism. Her line of thought can be considered to contain a healthy level of scepticism, rather than plain pessimism.

On the first day of this project, one of my students had asked why algebra textbooks do not cite their founders or historic background, while every calculus textbook credits mathematicians who contributed to the development of calculus. Students’ responses to this question were beyond my anticipation. When I reviewed their journals, I realised this question would be a great point of departure in order to discuss cultural superiority, world peace, collaboration, and dialogue. Malik, who had raised the question earlier, felt strongly about it:

Algebra textbook is not credited because it is not from the Western hemisphere….Anything that comes from non-Western countries is not important in our [U.S.] culture….Even when we use things like yoga, we degenerate, commercialise, and strip it of its authenticity and roots.

Hannah agreed with Malik. She wrote, ‘In our country, things are valued if they come from Western cultures/countries. Otherwise, it is either ignored or belittled. That is why founder of algebra is not cited in our algebra textbooks.’

Reviewing the students’ journals, I also noticed that students whose cultural roots are non-Western strongly believe that their cultural existence is marginalised and not valued at all. Most of them were born in the U.S., but that does not change their feelings. This was a very delicate situation about which I exercised caution, as my reflective journal indicates:
A sense of being marginalised in a classroom (or in a society) may be the result of chauvinism or other discrimination. A society in which people have no dialogue would soon become atomised individual consumers. Therefore, through critical mathematics education and critical pedagogy, I strive to help my students develop universal values to deconstruct and reconstruct their thoughts, skills and values to become critical citizens (December 13, 2014).

As a mathematics teacher, I can confirm that algebra textbooks in the U.S. do not cite any historical figures or backgrounds. For example, it would be extremely unusual for an algebra textbook to cite the historical background of the quadratic equation. However, calculus textbooks credit the founders of calculus, whenever appropriate. Calculus was invented in seventeenth century in Europe, whereas algebra was created by Islamic scholars in the near East in the tenth and eleventh centuries. Founders of algebra are known figures, and they could be credited if authors and publishers chose to do so.

Students explained the choice to not credit those scholars by referring to the ongoing tension between two worlds: the East and the West. Nicole said, ‘I think Western dominance is the reason why the founder of algebra is not credited in our algebra textbooks.’ I realised that integrating ethnomathematics into daily lessons, as D’Ambrosio (2010) proposed, could effectively challenge the Eurocentric vision of mathematics. Helping students recognise that mathematics is a joint production of the human family aligns with the ideas of Frankenstein (1990) and Powell and Frankenstein (1997), who argued that integrating the history of mathematics into the curriculum would empower students.

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28 Further information about this topic can be found at http://www.storyofmathematics.com/islamic.html, as well as many other websites that deal with the history of mathematics.
Despite the fact that EUP 3 was only a two-day project, the students’ reflections showed that it seemed to expand the their horizons and make a positive difference in their life-world. Molly shared her overall impression of the project:

Learning HOM [history of mathematics] will promote conceptual understanding. I had never understood the concept of zero and infinity until this project; as we had discussions, I now better understand why a number cannot be divided by a zero. Also I had never thought of math as being potentially used for bad things such as nuclear warfare. This was an interesting point of our discussion; how do we keep people from using math for bad?

Molly posed an important question: What can we do to keep mathematics from being used to harm people? Similarly, Ceylan implied that the project gave her a new perspective on the formatting power of mathematics in the modern world. She wrote, ‘I never thought about the ways mathematics could be used for positive and harmful purposes.’ These comments resonated with Skovsmose (2011) point that mathematics may lead to both ‘wonders and horrors’. He provided examples of both: ‘It is very difficult to think of any medical research without mathematics playing an integral part….Military enterprises can not be carried out without mathematics’ (p. 68). He suggested that ways in which mathematics affects our world should be subject to critical reflection and ethical evaluation.

4.3.3.5 Discussion

As demonstrated by the students’ comments and journals, and my reflective journal, the students embraced the imperative to be active and reflective participants in their learning process. My role was no more than a facilitator; this is in line with Rogers’s (1995) suggestion that the teacher’s role is crucial to establish a dialogic classroom. As the students’ participation was encouraged and valued, they took
ownership of the learning process. For example, instead to having presentations—with some groups not having time to present—they proposed hanging posters on the wall to extend the discussion and include every group. Their highly reasonable suggestion was a turning point, showing that our classroom was becoming both a community of collaborative learners and a dialogic classroom, as described by Alexander (2006).

The whole-class discussions and journals indicated that, in the students’ view, collaboration produces more common good for humanity than competition does, as evidenced by the historical development of mathematics. However, students also developed this view through experiences in the projects, gradually noticing that they learned and felt much better doing peer collaboration. Their reflective understanding of dialogue, respect, and solidarity helped them construct this view, as action and reflection are mutually reinforcing.

This project helped students consider mathematics from a historical-political perspective, as opposed to simply being an educational subject, and helped them revise some of their ideas. In the case of the history of algebra, for instance, they problematised the notion of cultural superiority. I did not know that some of my students felt strongly about this dominance. However, their reflections did not reveal any sign of aggressive ethnic pride. I was expecting some students to make chauvinistic arguments; fortunately, that did not happen. Instead, they talked about universal values, indicated world peace as a common interest of all people, and opposed the idea of cultural superiority and arrogance. In that sense, we partly achieved our objectives. Nevertheless, from a larger perspective, as I noted in my journal,
I realised that our objectives in EUP 3 being too broad resulted in superficial learning. For example, presenting the idea of peace and history of mathematics in the context of citizenship was a very broad topic. Two days were absolutely not enough to cover all of the objectives. But I had no chance to spend any additional time on this project, as we had to cover a certain number of topics within each month as mandated by standardised district curriculum. (December 13, 2014)

Despite good intentions, looking at EUP 3 from this perspective showed this project to be, to some degree, a fast-food style of education. I realised that the history of mathematics should be incorporated into every unit throughout the year.

The principal’s classroom visit reminded me of the ever-present tension between one’s life-world and the system, as described by Kemmis, McTaggart, et al. (2014). While my students and I were trying to decolonise our life-world, I neglected fulfilling the system’s requirement by not including codes for learning standards. The principal’s observation of our classroom (and the project) could lead to a very lively discussion between the classroom teacher and the principal beyond technical formalities such as referring to the standards. However, as part of its control and management effort, the system was more concerned with the codes for learning standards—regardless of whether the lesson was actually educationally beneficial for students.

In order to create small openings in our life-world (our classroom), I need to keep my job. Teaching a lesson that was not linked to district standards did not cause any serious consequences for me on that occasion—I received a verbal warning that I should not teach anything not covered by the curriculum. However, under different circumstances, it could have led to serious consequences, resulting in a poor teacher evaluation. I realised that I needed to be more careful and creative to meet system requirements while reorganising our life-world.
4.2.4 EUP 4: Community Volunteer Service

4.2.4.1 Planning and Objectives

We began this project after winter break in January 2015. Prior to the project, we had probed mathematical inequalities and practised mechanical aspects of the subject matter, so that students were able to solve given systems of inequality by graphing or algebraic approaches. My reflective journal frames EUP 4 as follows:

My intention was to develop a project relating systems of inequalities in mathematics to inequality as a socioeconomic concept. I was going to use a word problem, which I had developed and used in my classes many times. In that word problem, students initiate a campaign for raising money with certain restrictions to serve their community. But I was not completely satisfied with the context and its linkage with critical pedagogy. As I was simultaneously working on my literature review, I reviewed Freitas’s (2008) article again. One of her word problem examples was a system of inequality, which inspired me to revise my word problem in a way that would incorporate social issues such as homelessness, poverty, community service, solidarity, and social responsibility, in order to counter neoliberal pedagogy. (January 10, 2015)

I framed this project as another story of Edward, whose name was featured in EUP 2. To begin the lesson, I handed out the following word problem (see Appendix D for samples of the student work).

**Project 4: Spirit of community and solidarity**: Edward, along with his friends David and Jane, volunteered for a community organisation called Solidarity Here and Now (SHN), which is a non-profit organisation, run by the local community and aims to support those who are in need. SHN owns a building, which is used as temporary housing in winter sessions. Edward, along with his friends David and Jane, are in charge of this housing project. This is a two-story building with foldable (accordion) walls: It is 6,000 square feet that could be used for different purposes. In summertime, it is used for community activities, and during the winter, they host people, who are victims of great financial crisis, for six months only. They have two kinds of units: (1) Individual units and (2) family units. SHN building can be divided into two types of units: A family unit is 400 square feet and an individual unit is 200 square feet. The building can host 20 units in total. Based on experiences from previous years, they have figured out that families can afford $3,500; however, individuals can contribute to SHN only $2,000. Edward, David, and Jane hope to serve people in need and also maximise the amount of money that SHN receives from this project to support people in need in other areas. How many individuals and families should they host? What is the maximum amount of money Edward can raise from this project?
My reflective journal provides a further account about the development of this project:

Poverty in the U.S. is a delicate issue; blaming the victim is the prevailing ideology. Whenever a poverty or social inequality issue comes up, average people (including some students) tend to explain the issue through the dominant discourse that points at victims rather than system and structure. This point is one of the premises of neoliberal ideology that considers concepts such as success, failure, poverty, and unemployment are individual matters and have nothing to do with sociopolitical and socioeconomic system. Therefore, when it comes to poverty-related issues, students from poor families feel very strongly. They usually feel ashamed. I anticipated that a discussion on this topic might make some students reluctant to join in conversations and feel comfortable exposing themselves. For this reason, I came to that conclusion that I had to frame the project in such a way that students were able to consider poverty and its related issues as a sociopolitical and socioeconomic problem, instead of merely an individual matter. (January 10, 2015)

Victim blaming is a way to individualise injustice and deflect from a discussion of public good and public sphere. This ideology blocks open and transparent discussions, enabling the top-down imposition and the manufactured consent of masses (Chomsky & Herman, 2008). This notion aligns with Gramsci’s (1971) theorising of hegemony. What is new here is that neoliberalism contracted the public sphere resulting, according to Habermas (1975), in a legitimation crisis. From that point on, if people are provided with opportunities, they may develop alternative perspectives to look at social and political issues. Therefore, to achieve this end, in this project I aimed to make small openings in the lives of students where they can connect mathematics to larger societal problems through dialogic pedagogy and collaborative learning.
However, in my teaching I ensured that this process not become a propagandist approach, about which Freire (2013) cautioned us:

Propaganda, slogans, myths are the instruments employed by the invader to achieve his objectives….True humanism [and humanising education], which serves human beings, cannot accept manipulation under any name whatsoever, for humanism there is no path other than dialogue. To engage in dialogue is to be genuine. (p. 101)

Freire (2013) said that liberating teaching can be achieved by genuine dialogue, whereas propaganda and manipulation result in domestication. While improving their mathematical content knowledge, the students in my class were provided with an opportunity to discuss and elaborate on the issues of homelessness, poverty, and inequality from different perspectives. They shared their own initial thoughts, experiences, and ideas while listening to their peers deconstruct and reconstruct their knowledge and values.

It may seem unusual to align inequality as a mathematical concept with larger sociopolitical problems. Within a traditional education view, it would be very unusual, if not impossible, to have such a discussion in a mathematics classroom. In fact, direct teaching and content-based monologues are the preferred instructional strategies in mathematics classes (Alexander, 2006). However, this study was aimed at praxis: As W. Carr (1995) explained, teachers who only cover content in their classroom are not engaged in educational praxis.

4.2.4.2 Day 1 (January 8, 2015)

We began the lesson with a brief discussion about the quality of our group work by reviewing the regulative norms of the collaborative learning process. As Pine (2009) pointed out, ‘collaboration is not achieved naturally’ (p. 155); it takes time and practice to be learned. Reflecting on our previous projects, as a whole class we
reviewed important factors in collaborative learning and dialogic interactions, raising such features as mutual respect, trust, listening to others, not excluding anybody, welcoming every member to contribute, and non-dominating communication. We continued these objectives in the next projects in line with suggestions made by Groundwater-Smith et al. (2003), who indicated that inquiry-based collaboration cannot be achieved overnight—it takes time and continuous effort.

Immediately after the discussion, I handed out the paperwork for the project and had students quietly read the problem. The students then configured their own groups. As groups began working, I circulated around the class to observe peer interactions and assist students as needed. As I was circulating and taking notes, two students in a group raised their hands:

Jennifer: We are kind of lost…With getting variables.
Tibu: I am confused….We could not figure out the number of unknowns, would that be two or four?

I took a seat in the group and attempted to lead them to use their abilities instead of telling them the answer directly. I was thinking out loud to collaboratively determine the variables, as the following dialogue illustrates:

Me: Ok, let’s see….Assuming that Edward accepts one family and one individual, can we write an equation for the total money?
Me: Ok, good…What about if he accepted two hosts for each?
Multiple students: Ok, it would be…7000 + 4000.
Me: That is correct….We then could also rewrite as 2 × 3500 + 2 × 2000 [I wrote them down] and reiterate this more like 3 × 3500 + 3 × 2000….Would you agree with that?
Multiple students: Yeah! It makes sense.
Me: We can do the same for the first equation…[I wrote] 1 × 3500 + 1 × 2000….Then what would we think that that numbers that increase 1, 2, 3 represents in the problem?
Multiple students: How many rooms are…family and individual units.

Me: Then these are the variables…Let’s say x = number of family units, and y = individual units….Great, can you continue from this point?

Tibu: Yes, I guess we can, thanks, Mr. B [other students nod their heads].

From the excerpts above, I immediately noticed two elements about the quality of group work: First, the students’ questioning techniques were much better than before. Second, they seemed to be very careful not to dominate the group. The students seemed to negotiate the question in their group, which was specific about variables as they indicated their confusion if it should be two or four, rather than posing a generic statement like ‘I don’t get this’, which is a very common expression used by students when they do not understand subject matter.

With respect to the scaffolding I provided, as shown in the above excerpt, Vygotsky’s (1978) concept of ZPD focuses not only on peer interaction, but also on the teacher’s role. Similar to Freire’s (2000) suggestion of a horizontal teacher-student interaction, ZPD requires the teacher to be an equal partner in the collaborative learning process, transforming the teacher’s role from knowledge transmitter to facilitator (Wells, 1999). When the group asked for assistance, I provided scaffolding and let them continue the process on their own instead of telling them the answer. My communication with the group was based on a validity claim instead of a power claim. Dialogic peer interactions and horizontal student-teacher relationships have been gradually transforming our classroom into a community of inquiry and a dialogic classroom. This aligns with Kennedy’s (2009) and Skovsmose and Alrø’s (2004) arguments for a community of inquiry model in the classroom.

As the group continued working on the project, I followed peer communication in another group. This group did not ask me for help; they managed the problem-solving process on their own, as shown in the following dialogue:
Nadia: If we maximise something, don’t we need a quadratic function?

Nicole: I think so…but some linear inequalities can be optimised. You know, just like we did in exercises last week from the textbook….

Nadia: We need to maximise a function, but what function?

Nicole: Yeah…that is what we need to figure out….We need to write that equation down first….Here we need to add family and individual unit prices [she wrote inequalities down].

Tom: Why did we set inequalities in standard form and put into slope-intercept later?

Nicole: It is easier to graph it in slope-intercept form.

Tom: Then we should have set it in slope form in first place.

Nicole: I don’t know how to set it. It comes easy to set it in standard form when I read the problem…. [They wrote all inequalities and the objective function down].

Tom: I wonder if Edward gets money for this work.

Nadia: Maybe some pocket money….Why not?

Nicole: He is doing a community service here; he has a part-time job…you know, in the other project [EUP 2] he got a part-time job.

Once students agreed that the system of inequality corresponds to Edward’s story and the objective function, they moved on to calculations and graphing:

Nadia: Looks like our solution area has three edge points…x-intercept is 20 and y is 15.

Nicole: I got the same, but let’s plug them in and see how they work….I also got the intercept of two lines.

Tom: Yes, x and y intercepts are correct.

Nadia: Let’s evaluate objective function….Tom, can you do it by calculator?

Tom: Yes, P(x,y) is the objective function right? [Pointing to their objective function]

Nicole: Oh yeah, that’s the equation [she pointed to the equation]….This is going to calculate the total money they could get.

This quotation reveals that each member of the group contributed to the process of collective thinking and working within each other’s ZPD. Even though
their skills and knowledge vary, they learned from and with each other. Their learning processes align with Cesar (1998) argument, according to which collaborative learning does not necessitate one peer to be more competent than the others.

Based on my field notes, I noticed that the students embraced egalitarian peer relationships; they seemed to be sensitive about not dominating each other. The students’ reflective journals in EUP 1 indicated their antipathy towards competitive learning in their past experiences in other classes. Egalitarian collaboration in group work has gradually matured. A quotation from my reflective journal supports this conclusion:

My observation notes in this project [EUP 4] confirm Kohn’s (1992) argument against competitive learning. Competitive learning is not necessary to achieve any educational success. Excellence in education can be achieved without putting students against each other; without producing losers and winners; without applying a paradigm of reward and punishment; without having students racing with each other. Knowing their work was not to be graded, they were still engaged in the collaborative learning process and enjoyed the whole process. In fact, they consciously put in effort to create and improve egalitarian group work. As Alfie Kohn argued, cooperative learning helped my students to create healthy peer relations and develop self-confidence. I also came to the conclusion that Habermas’s notion of the ‘ideal speech situation’ in education cannot be achieved without a collaborative learning process in place. One requires the other. (January 13, 2015)

The quotation shows that the students worked on the project enthusiastically. They did not compete with each other; they appeared not to consider their peers as barriers to their success. This lesson structure did not use the reward-punishment paradigm: It did not produce winners and losers. Instead, the process created an egalitarian learning environment where peers stood in solidarity with each other—Flecha’s (2000) description of collaborative learning promotes precisely this type of
process. This project—in Kohn’s (1992) words, without ‘the use and salience of extrinsic motivators’ (p. 221)—promoted a sense of community, and encouraged students to take ownership of their learning. My reflective journal also indicated that students appeared to learn better when their learning tasks were designed as collaborative rather than competitive.

4.2.4.3 Day 2 (January 9, 2015)

We began the second day by briefly reviewing our activities the day before. We then spent an hour on whole-class discussion and half an hour on journal entries. Taking Edward’s story in the context of mathematics in action, the discussion aimed to foster values such as social responsibility, caring about each other, and cooperation in order to counter elements of neoliberal pedagogy such as individualism and competition. I posted topics of the discussion on the board through a PowerPoint.

• What kind of knowledge and skills do you think Edward (and his friends) need to carry out their community voluntary service?

In response to this question, students exchanged ideas and made comments about skills and knowledge oriented towards citizenship responsibility. An excerpt illustrates:

Tom: Edward should know how to label unknowns [in a word problem] first.

Jennifer: He needs to know how to turn sentences into inequalities correctly.

Akil: He’s got to know his algebra.

Multiple students: Yeah, obviously….

Selena: He should know how to set the objective function….We struggled with that but figured it out in the end.

Nick: Edward must know stuff like how to graph [linear] inequalities.
Jacob: Edward’s got to know his math, but it is not enough. He also needs collaboration skills to work on this problem with his friends just like we do in this class.

Michael: I think, before all, he’s got to be someone who believes in social responsibility and helping and caring about each other.

Nick: He volunteered for SHN; I guess we can assume that he is a responsible and helpful guy.

This dialogue reflects the main claim of CME that students need certain values, skills, and attitudes to become critical citizens (Aguilar & Zavaleta, 2012). The students agreed that Edward needed not only mathematics knowledge and skills, but also critical thinking and critical literacy. Their intuitive understanding parallels Gutstein’s (2006) distinction between critical literacy and functional literacy:

A literacy is functional when it serves the reproduction purpose….In contrast, critical literacy means to approach knowledge critically and sceptically….Being critically literate also means to examine one’s own and other’s lives in relationship to sociopolitical and cultural-historical contexts. (p. 5)

When we apply this distinction to Edward, algebra skills and knowledge (functional literacy) would help him adapt himself to existing conditions. However, he needed critical mathematical literacy to take the initiative to challenge the existing conditions, for example, helping those in need in his community.

Helping people in need frames the sense of social responsibility of participatory and social justice-based citizenship (Westheimer, 2015). Continuing the discussion in this context, I asked the following questions:

• What do you understand by social responsibility? Do we have any social responsibility?

Tom raised his hand:
Yes, we all have social responsibilities. For example, the other day, I was walking down the street and saw a couple of banana peels. Well, I picked up the banana peels from the ground and put it in the garbage. What if some dude runs over his car, slips, and crashes his car and dies….That did not happen, because I did my social responsibility….I was a good boy!

Some students responded to Tom’s speech with smiley faces. Tom himself had a smile as he said this. I momentarily thought that he was making up this story. But I could not find any conceivable reason for him to make up this story. Regardless of my momentary scepticism, he explained what social responsibility might mean with a very simple and effective example. It is about caring about each other in a community.

From a neoliberal perspective, however, responsibility can be defined only at the individual and family levels (Ventura, 2012). Acknowledging the notion of social responsibility, Tom’s comment challenged the neoliberal standpoint. Therefore, I transitioned from the question of responsibility to the question of choice between a competitive society and a collaborative society. I posed a question to invite students to share their ideas and reflect on their experiences. As I asked it, I was aware that the idea of competition is continuously imposed in the U.S., as Kohn (1992) indicated:

> [In the U.S.] we are systematically socialized to compete—and want to compete—and then the results are cited as evidence of competition’s inevitability….The message that competition is appropriate, desirable, required, and even unavoidable is drummed into us from nursery school: it is the subtext of every lesson. (p. 25)

My next question, therefore, focused on the desirability of competition:

- What kind of social environment would you wish to live in, cooperative or competitive? Would you want our learning activities be cooperative or competitive?
Andrew: Because of the Manhattan Project’s competition, nuclear power was invented. I think competitive society is more productive.

Tori: Nuclear power could be invented by cooperation as well. I personally believe that competition overall produces a hostile environment….I mean, if the U.S. and Russia [had] cooperated, the world would have been a better place.

Bin: Frankly, I believe that collaboration is better than competition. You know….I enjoy studying together in our class, I think we all do, but there are some other things that I would like to do on my own, and I don’t think that contradicts with the first.

Jennifer: I like how we study in this class. Our chemistry class starts like, there is ‘do-now’ question on the board, like, you know….whoever gets the answer first gets bonus credit. Sometimes, I mean, even when I know the answer, I don’t want to raise my hand.

The excerpt relates to Kohn’s (1992) claim that competition is ideologically imposed from the top down and is perceived almost as a national religion. Based on my observations, student journals, and my reflective journal, the hegemony of competitive learning can be countered. EUP 4 revealed that small openings can be created for students to learn without competition.

As I was preparing to ask if they remembered a sports game they had lost, Ceylan added another dimension to the discussion:

Ceylan: I really would like to live in a world [where] people help each other and collaborate to solve their problems; they play games for just fun. But this is not the world we live in….We have to compete one way or another. Even in our school, next period we go to another class and Mrs. [X] will say, ‘There is a do-now question on the board, who gets the answer first, will receive extra credit.’ Welcome to the competition!

Nick: That is not only Mrs. [X]; that happens in many other classes too.

Bin: It is not only school. My father says that the business world is a place where dog eats dog.
Me: Unfortunately, our world is formed by the competitive perspective, and we have to survive in this world. But always remember that competitive approach is a human-made decision; it does not come from god. And, therefore, it can be changed. If we think cooperation is more humanising, you, young citizens, have a power to challenge the existing system that glorifies competition. We began this change in our classroom, and we can continue in other parts of our life.

This was a challenging moment for me in relation to critical pedagogy. It was not easy to respond to Ceylan’s comment; this aligns with McLaren and Leonard (1993) point that ‘critical pedagogy must serve as a form of critique and also a referent for hope’ (p. 69). In other words, the practice of critical pedagogy runs the risk of generating hopelessness. In that sense, I attempted to promote the idea that as much as we have to survive in a given system, we should try to transform it if we are unhappy with it.

As Edward’s story involved a homeless shelter, I invited my class to reflect on the notions of success and failure. Neoliberal ideology defines success and failure as individual matters (Ventura, 2012). I posted the following question on the board:

- Do you consider success or failure as an individual or social matter? Explain your reasoning.

Me: Let’s evaluate this question a bit more. We sometimes fail at something and succeed at another. That is how life goes on. But when it comes to explaining reasons behind our failure or success, you may have different thoughts…

Tibu: My dad was laid off from Amazon and could not find another job for six months. We had a really hard time….He got a job in a grocery store with minimum wage. Was this my dad’s mistake? He is a very hard-working man.

Andrew: Yeah, but what about lazy people who just give up?
Jeff: People lose their job because of the bad economy….It would be unfair to blame individuals in this case. You know, you get wet when it rains, they say, why don’t you have umbrella? Well, if it rains heavy, you get wet even with a nice umbrella.

Darryl: Success and failure, I think, is an individual matter. If you have a rich father and you can have everything you need and want….Let’s say, you don’t study and then fail all your classes; that is definitely an individual matter. But if you didn’t have dinner last night as your dad lost his job or you live in foster care, and you failed your classes…I would not blame you for this.

These comments went against my anticipations. I was expecting that most students would tend to think of success and failure more as individual matters, as that is the dominant discourse in the U.S. The students seemed to challenge the neoliberal assessment of failure and success with reasonable arguments. Specifically, Darryl’s example of a student’s failure demonstrated situations under which failure could be either an individual or a sociopolitical and socioeconomic matter.

As I gazed at the clock on the wall, I saw we had 28 minutes left for journal entries. I posted the questions below through the projector and handed out the journals. For the rest of the period, the students made their journal entries for the project.

**4.2.4.4 Students’ Journals**

I posted prompt questions on the board:

- Briefly describe your group’s collaboration story.

The students’ journal entries indicated that they genuinely exposed their feelings, thoughts, and ideas. For example, Taylor articulated his joy of applying mathematics to a real-life situation:
We did really well on this project. We did not get sidetracked at all. It was enjoyable; we did not agree on the objective function at first, but we talked about it for a while and figured it out. Actually, I felt like I learned it with understanding this time. Also I began looking at things from mathematical perspective that I have never done before like we did in this project, using systems of linear inequalities for community service problem….Math is used pretty much for everything.

Taylor’s comment shows that in collaborative learning, students can learn subject matter ‘with understanding’ as they learn with and from each other through dialogue. This finding is consistent with educational theories arguing that fellow students can widen your horizons much better than you can do on your own (Skovsmose & Alrø, 2004; Vygotsky, 1978; Wells, 1999). Ever since the first project, the students emphasised dialogue and collaboration as we had agreed on improving the quality of group work. Bin’s comment was unique, as he consciously considered the limitations and possibilities of collaboration:

The quality of our collaboration is improving, but I would not say it is perfect. When I first saw the problem, I knew what to do, but I let my friends speak their ideas, before I shared mine….Although everyone made a contribution, they have remained silent time to time….I pushed more and more to hear what they think, but I also tried not to be dominant as well. I would have liked to exchange ideas more enthusiastically and ask questions to each other more.

Bin’s entry indicates that he consciously avoided being dominant in the group. Although he immediately knew the mathematical aspect of the word problem, he was considerate enough to include his classmates in the process by thinking that his peers’ thoughts would enrich the overall quality of their work. Bin’s entry reveals that empathy plays an important role in egalitarian peer interactions. His approach to his classmates was not only dialogic, but also empathetic. Considering Bin’s comment in
light of other students’ journal entries, I concluded that Habermas’s ideal speech situation could be attained. I will return to this point.

The next question invited the students to share their experiences and ideas about whether there need to be institutions to assist people in need:

- Do you think that societies should have a safety net?  

This question relates to whether success or failure is an individual or a socioeconomic and sociopolitical matter. If it is not an individual matter, then how do we approach it? Jeff wrote:

Societies definitely should have a safety net, due to the fact that some people get placed in a situation of disadvantage just from sheer bad luck….Safety net should exist to aid people in growing back into a state of independence since even hard-working and good people may get down on their luck and need help.

Jeff implied that within the existing system, everyone, including hard-working people, can be a victim; therefore, a society should have institutions to assist citizens in need. Jennifer added another layer:

I think we should have a safety net, but I don’t think that it is a permanent solution….As long as there is an unfair system, there will be people in need. The only people, who care about these in need, are people who volunteer for non-profit organisations….The government is not genuinely interested in helping people who struggle to make basic needs.

She argues that one needs to focus on the root causes of socioeconomic problems if one really wants to solve these problems. She also points out that a safety net is the responsibility of government; citizens should not be at the mercy of non-profit organisations.

29 The safety net refers to the welfare state and institutions that are in place to provide citizens in need with help.
Michael proposed a similar approach:

This safety net thing is meaningless; it does not solve the problem. If people work for six days a week and twelve hours a day, and still can’t survive and need food stamps….Well, we need to look elsewhere to get the real problem here. These smart gentlemen better stop blaming lazy people and explain why hard-working people fail to make their bread. We should work for a more equal society.

From Michael’s point of view, our efforts need to focus on making our society more equal and just. If that were achieved, social programs such as the social safety net would not be needed.

Tori agreed on the necessity of a safety net by pointing to unfair conditions resulting from a market-driven society: ‘There is so much competition that some people like elderly, veterans, disabled are left on the side and have little or no option. That is why we need safety net.’ Jacob represented the necessity of a safety net as an indicator of caring about each other. From his point of view, a group of people turns into a community when they care about each other: ‘Yes, we need a safety net, we need to care about each other and help those who are in need. This is what being a community means.’

The students’ comments showed that they envisioned a society where people care about each other. Without exception, all students disapproved of the neoliberal standpoint, according to which individuals are responsible for themselves and have no right to expect help from society (Ventura, 2012). The next questions concerned Edward’s possible motivation:

- Why do you think that Edward volunteered for SHN organisation? What could be his motivations? How and why?

I framed the rationale for this question in my reflective journal as follows:
With this question, I aimed to help students to unpack two main philosophies...behind community volunteer services and further elaborate on them. First is that community service is equated to being nice and helpful; it aims to help people in need, but never questions why people are in need in the first place. The second one politicises the process: While providing help for people in need, it questions the root causes that put people in need. (January 12, 2015)

To respond to this question, students reflected on their life experiences to come up with educated guesses. For example, Derek wrote, ‘Perhaps Edward himself was once was homeless or struggled financially and decided to volunteer for SHN.’ Daryl responded, ‘Edward wanted to help people in his community who needed it. He probably felt socially obligated to stand up for those who are not as lucky as himself.’ Nicole wrote, ‘Perhaps Edward volunteered for SHN to improve his resume as universities require community involvement….Or maybe he was thinking that it was a social responsibility…or he was a very caring individual.’ Ryan stated, ‘As we know he was looking for a job earlier, he may have realised that people are better off when they help each other in hard times.’

In their answers, the students actually articulated why they would volunteer for SHN from their individual point of view. The students indicated that their sense of social responsibility, empathy, and caring about each other and their community were emergent. These qualities are what Westheimer (2015) considered essential for justice-based and participatory citizenship. The next question invited students to reflect on their experiences with notions of generosity and selfishness:

- Do you think that people are caring or selfish by their nature? Or people’s actions should be evaluated within given social-economic system? Give examples from your life experiences.

My journal indicates the rationale for the question:
Peer collaboration is central to our projects. However, if students tend to think that people are selfish by nature, why would they genuinely participate in collaborative learning process that requires caring about each other and negates selfishness? As Skovsmose and Alrø (2004) indicated, peer collaboration cannot be imposed: This implies that collaboration can be achieved if students embrace values and attitudes such as collaboration, solidarity, and caring about each other. (January 12, 2015)

The students provided thoughtful responses to the question by reflecting on their life experiences both in and out of the classroom. Christian wrote, ‘People are caring or selfish not by nature but by experiences…How others treat them in a community forces them to be one way or another. For example, when my buddies help me with my math in our group, I would like to help them any way that I can.’ Christian’s comment reveals that socioeconomic and sociopolitical structure urge people to act in certain ways. A collaborative and dialogic learning environment motivated Christian to help his classmates. In education, if a learning task is designed to be competitive, students might seem to be competitive. However, this does not mean that competition is inherent in human beings.

Jennifer specified that people become selfish as they try to survive in a competitive society: ‘I don’t think people are selfish by nature, but a winner-takes-all society makes people selfish; in order to be the best, you need to defeat rivals. Sometimes these rivals are your neighbours and sometimes your friends.’ She implies that survival in a market-driven society requires a selfish attitude. Otherwise, in agreement with Kohn (1992), Jennifer does not consider people inherently selfish. The students’ comments clearly indicated their understanding that people’s actions cannot be separated from the social, economic, and political system in which they live.
The next question invited students to identify certain issues either as individual or social problems:

- Do you think homelessness is an individual problem or a socioeconomic and political problem. Why?

Nadia wrote, ‘Homelessness in the end is a social problem. Nobody enjoys being homeless or [being] in a difficult situation.’ Her conclusion is based on the premise that if homelessness is not a choice of individual, then it has something to do with the socioeconomic and sociopolitical system. Cindy supported her conclusion with concrete examples:

It doesn’t make sense to me when people say that homeless people are homeless because they are lazy. What if you lose your job or get fired, or you are disabled or sick? It is same for people with drug addiction. They need guidance and medical help to get better, but they are thrown into jail instead in our country….There are homeless orphans on the street. If society or the government helps them, they may be able to stand on their feet; otherwise they will always be on the street.

In Cindy’s view, homelessness is a social problem and so is its solution; it might result from being laid off or fired, sick, or addicted to drugs. If we as a society stand in solidarity, these problems can be effectively solved. Darryl seemed to agree: ‘If you are lazy, [homelessness] is a personal problem, but most people are not so. They may have been laid off or have had bad luck and circumstances that have prevented their success.’ Nick argued that homelessness cannot simply be reduced to an individual matter:

I think that homelessness is a political and socioeconomic problem because no one would like to be homeless. Something had to happen to them, such as being denied a job or failing to graduate, or any other reason that caused a setback for them. And we can’t blame all of that on them.
Neoliberal ideology rejects thinking of problems at a social level. Instead, it considers one’s failure, success, misery, or happiness as individual matters rather than social, economic, and political problems (Ventura, 2012). However, even they were not able to use the word ‘neoliberalism’, the students’ comments show that they challenge basic tenets of neoliberal ideology.

The next question encouraged the students to provide an overall reflection on this project:

- This is our fourth project; overall do you feel empowered in a sense that you can apply your math knowledge and skills to real-world problems?

Please be as specific as possible.

Students’ responses indicated that they took ownership of the collaborative and dialogic learning process; they were empowered, as they could use their mathematics skills and knowledge outside of class. They also critically evaluated the joint objectives oriented towards creating a community of collaboratively learning mathematics. For example, Jeff noticed improvements in peer dialogue:

I feel that this project, I felt for the first time that I learned when my friends answered my questions as well as listening to my classmates asking or explaining each other’s questions and confirmations….Also I enjoyed our classroom discussion that connected Edward’s CVS to social problems.

It is evident in Jeff’s statement that students experienced CME by dialogic learning and by connecting mathematics to larger social, political, and economic problems. Nicole articulated achievements up to this point:

As we do more projects, our collaboration and dialogue got better; no one dominates, everybody contributes, and we listen to each other to solve the problem, and we all take pride [in our] final work….I also realised that skills I developed in our class for collaboration helps me for other classes as well.
Nick shared how our projects gradually changed his perspective:

As we did projects, I feel I am much more empowered on looking at things in life from a mathematical perspective….Until I got in this class, math was for me to study for the test subject. But these projects opened my eyes….Edward used his math knowledge to figure out his best salary and now he uses math to raise maximum money to serve people in his community. This is a very unusual story for me.

The comments suggest that EUPs were successful in fulfilling one of the fundamental goals of CME: to transform students’ ‘foreground’ (Skovsmose (2011). Students clearly recognised their changed and changing ways of thinking about and with mathematics, doing mathematics, and reflecting on it. Aligning with Kemmis, McTaggart, et al.’s (2014) description of critical participatory action research, my students and I critically examined learning practices in the class and jointly attempted to transform it into a community of learners. Darryl outlined how he benefited from collaborative and dialogic learning:

I do feel empowered. This class has been very fun for me because solving real-world problems helps me to understand concepts and how they apply to real-world situations. Working with my classmates made everything easier and has allowed me to make connections to more complex mathematical concepts and theories.

These comments show that the whole class made tangible progress towards being a community of collaborative learners. Having reflected on the previous EUPs, I realised that there is a dialectical connection between practising CME and the classroom being a community of learners.
4.2.4.4 Discussion

Inquiry-based collaborative learning has been central to this project as well as the previous projects. We negotiated potential obstacles to collaboration and agreed on eliminating these obstacles collectively. For example, in EUP 1 students expressed concern about ‘self-assigned leaders’ in group work, and proposed a more egalitarian peer relationship. We set certain goals to make our communication more dialogic and collaboration more egalitarian. For each EUP, the students continued reflecting on these overarching goals as well as specific objectives within each EUP. Having triangulated my observations of whole-class discussions and peer interactions, students’ journals, and my reflective journal, this project has led me to certain conclusions.

First, when the students worked on Edward’s story in groups, they learned from and with each other to construct knowledge collectively. Based on Vygotsky’s (1978) conception of peer interaction as a ZPD, we know that through collaboration, students can achieve much more than they could do on their own. My observations also confirm that for a ZPD to be realised, a more competent peer within a group is unnecessary. Once dialogic peer interaction is achieved, students can work in each other’s ZPD, as suggested by Cesar (1998). Vygotsky’s notion of ZPD intersects with Freire’s horizontal student-teacher relationship (Freire, 2000; Wells, 1999). As the students asked me for help, I provided the scaffolding as another member of the group, instead of posing as the ultimate source of knowledge and an authority figure. This is not to deny that I had authority, as the classroom teacher. However, in light of CME and critical pedagogy, I used my educative authority to establish dialogic (horizontal) student-teacher relationships and a facilitative learning environment.
Second, the students stated their preference for collaborative learning over competitive learning; they said they learned better if they studied in groups in which peer communication was dialogic and peer interactions were egalitarian. This confirms Flecha’s (2000) argument that dialogic learning requires egalitarian peer relationships, non-authoritarian teacher-student relationships, and reflexive dialogue. From a holistic perspective, our classroom became a dialogic classroom (Alexander, 2015). Inquiry-based learning and dialogic interactions transformed the classroom into a community of inquiry, as described by Kennedy (2009). Being a community of collective inquiry creates the material conditions necessary for Habermas’s (1987, 1990) theory of the ideal speech situation. When applied to educational settings, these material conditions are as follows: (1) the students included their peers in a collaborative learning process; (2) each student was free to question any argument that came either from the teacher or from peers; (3) students were free to express their arguments; (4) no external force was imposed on the process—all decisions were reached through non-coercive arguments. As a result, knowledge was constructed based on validity claims, not power claims.

Third, this process could not be fully constructed by a mechanical exchange of ideas and thoughts within regulative norms. With respect to this question, Freire (2000) introduced another dimension of educational process: love and hope. From Freire’s point of view, dialogue requires love, humility, and faith:

Founding itself upon love, humility, and faith, dialogue is a horizontal relationship of which mutual trust between the participants is the logical consequence….Nor yet can dialogue exist without hope. Hope is rooted in men’s incompletion….The dehumanization resulting from an unjust order is not a cause for despair but for hope. (p. 91)

30 Love in this context should not be confused with romantic love; hooks (2003) described this kind ‘as a combination of care, commitment, knowledge, responsibility, respect and trust’ (p. 131).
My observations and students’ reflections guided me to conclude that the ideal speech situation in an educational setting could not fully be realised in the absence of love and hope. Egalitarian peer interactions and horizontal student-teacher relations require more than an exchange of cold arguments. As indicated earlier in connection to Bin’s journal entry, I realised that empathy plays a significant role in peer and teacher-student interactions. At times, I would teach the students skills, knowledge, and values over and over again until I succeeded. This was not a mechanical process. Freire (2000) points out that a critical pedagogy cannot be materialised without love and hope, and it was during EUP 4 that I realised that without hope and love, it would be easy to become discouraged.

More importantly, I observed the students genuinely striving to explain their arguments for their classmates and respectfully listening to their peers’ views without trying to dominate each other. As suggested by Freire (2000) and hooks (2003), the students interacted through empathy and love. It can be concluded that creating an ideal speech situation in the classroom can be achieved not only through better arguments, but also through empathy, as evident in Bin’s journal entry. This finding confirms the philosophical foundation of this study—the complementary ideas of Freire and Habermas.

Fourth, as I answered the students’ mathematics questions, I drew on the validity of mathematical axioms and algebraic properties without posing as an external authority to impose some arbitrary rules or laws. While answering their questions, I did not move to the next step unless the students were convinced by the justification I provided. In other words, my teaching mathematics was not based on a power claim but on a validity claim. This approach to teaching mathematics resembles that of Almeida (2010), who compared teaching mathematics through
dialogic pedagogy with the dynamics of democracy as a political system. In his view, democracy refers to the Constitution as mathematics refers to theorems, properties, and axioms to verify validity of an argument.

Fifth, in the context of Edward’s story, the students negotiated, formed, and reformed values that are necessary to become critical citizens. The students articulated that they preferred collaboration to competition; they spoke in favour of a society where people care about each other in solidarity; they supported egalitarian peer interactions; they established a dialectic connection between the individual person and society (life-world and system). In other words, the students embraced values that challenged the hegemonic values of neoliberalism. As we engaged mathematics through inquiry-based collaborative learning and dialogic pedagogy, students had opportunities to experience democracy in the form of dialogue and internalise democratic culture (Ellis & Malloy, 2007; Hannaford, 1998; Shor, 1987).

4.2.5 EUP 5: Student Loan Debt Crisis

4.2.5.1 Planning and Objectives

This project contextualises the student loan debt (SLD) crisis, which has long been a major issue for students who come from working class families in U.S. Resulting from the neoliberal policies over the last thirty years, the SLD crisis has been getting worse. As I examined in Chapter 2, since 1980, the average income of working class people in the U.S. has decreased significantly while the cost of living has increased (Hayase, 2013; Torbat, 2008; West & Smiley, 2012). This increase

includes college tuition and student loan interest rates (Brown (2015). Employment opportunities for college graduates have diminished, which makes paying back debts more difficult than ever for new graduates (Blacker, 2013). I describe the preparatory stage of this project in my reflective journal:

While the SLD issue directly affects students in high schools, they usually have no opportunity to understand and critically reflect on this matter. Although there is a career centre in our school that helps students with college applications, student loans, and scholarship problems, this project invites students to adapt to the existing conditions without any criticism. That brings us back to the hegemony of neoliberal ideology. In neoliberal view, issues such as SLD are individual, not social matters (Brown, 2015; Ventura, 2012).

I developed another story of Edward, a senior student in Liberty High School. Previously, Edward in our projects was a part-time worker and a volunteer in a community service. Now he is getting ready for college in the U.S. (February 12, 2015)

CME should provide students with opportunities to connect their life in the classroom to the larger social, political, and economic systems in which they live (Ernest, 2002a; Gutstein, 2006; Skovsmose, 2011). In this vein, inspired by Freire, Shor (1993) said, ‘In the liberating classroom, teachers pose problems derived from students’ life, social issues, and academic subjects, in a mutually created dialogue’ (p. 25). In light of these critical educational theories, the SLD issue exemplifies a problem derived from the lives of my students.

I designed the paperwork so that each group would engage in an inquiry-based collaboration and dialogue to envision Edward’s life after college and reflect on it. The first page of the project paperwork was as follows (see Appendix E for a sample of the students’ work).
**Project 5**
Edward is a senior student at Liberty High School. He is planning to study engineering (this is your group's decision). He is going to need a loan for his college tuition (4 years) and for some of his expenses while he is in college. Assuming that his loan comes with 6% annual interest, soon after graduation, Edward will start paying back the loan as a monthly-fixed amounts and he is supposed to pay it back in 10 years. Edward is aware of the student loan debt crisis in the U.S.; therefore, he wants to take educated and conscious steps towards his life in college and after college. From this point, through his mathematics knowledge and skills, he wants to model different scenarios and be ready for some of the risks and possibilities he may encounter. He is going to do mathematics for:
1. The amount of the loan he is to apply for and the total money he is supposed to pay back in ten years.
2. The amount of his monthly payment in connection with his other expenditures such as rent, food, and other monthly bills.
3. Ideally, he wants his monthly payment to be between 10% and 20% of his monthly income when he gets a job. Based on his loan payment, what kind of job should he hope to get?
4. He wants to make some predictions about how his financial situation and social life will look like while he will be paying back his loan.
5. Edward also wants to reflect on the student loan crisis not only as an individual, but also as a responsible and critical citizen.

The project was designed as an open-ended word problem that aimed to provide students with a landscape of investigation, as suggested by Skovsmose (2011), through which the students could develop multiple scenarios for the life-world of a senior student who has to deal with the SLD issue.

The students were to consider Edward’s life-world from a sociopolitical, socioeconomic, and cultural perspective and critically reflect on his life. Each group was required to reach a consensus on multiple points such as the initial amount of the loan, Edward’s possible expenses, lifestyle, employment situation, and payment plan. Furthermore, through mathematical calculations, each group was expected to come up with several different scenarios, such as Edward has a decent paying job, a minimum wage job, or no job at all. In comparison to the previous projects, this one necessitated a high level of collaboration and dialogue in order to reach consensus on each point.

**4.2.5.2 Day 1 (February 10, 2015)**

Prior to this lesson, we spent four periods on processing exponential properties, functions, and equations. But we had not covered compound interest
calculation, which this project required. Therefore, we began the project by learning
the compound interest calculation formula. I aimed to construct the formula with my
students to ensure they learned it through active participation and with understanding.
I had a lesson plan to follow, but I did all the calculations with my students as our
dialogue progressed. I projected an example on the board to begin the lesson:

Me: Alright….If you guys are ready, before working on our student loan
project, we have to develop a mathematical model to calculate compound
interest….Let’s say, we invest $1,000 in a bank with 3% annual interest
for five years. We will figure the total amount of money we are supposed
to get back at the end of the fifth year….How much money will be added
at the end of the first year? [The question was directed to the whole the
class.]
Multiple students: Just multiply $1,000 with 0.03…it is $30.
Me: That will calculate how much is to be added to the starting amount. If we
invested for a year, how much would we get back?
[An office assistant stepped into the class to give some letters to a student. I
asked her to come back five minutes before class ends—I try to avoid
disruptions.]
Nicole: You know, just add 30 to 1000….[Multiple voices] It is $1030.
Me: Correct….Ok, then, let’s move on to calculate the amount for the
second year. Should we start with $1030 or the original amount,
which was $1000? [The question was directed to whole class.]
Multiple students: It is $1030.

Another student from the left back-corner table, however, Nadia, said ‘Original
amount.’ As I was getting ready to begin a dialogue with her, Tom responded: ‘If we
take the original amount each year, it would just be $30 extra per year after five years.
It doesn’t make sense.’ Ceylan joined this discussion: ‘I think you would be right if it
was 3% rate without accumulation, but it says 3% annually for five years.’ Nick
joined in: ‘If I earned $30 for the first year, then I have $1030. It is my original money, not $1000.’ At this point, a discussion between the students and me triggered peer-to-peer communication and transformed the classroom into a dialogic sphere where collaborative learning was taking place. Nadia seemed to be convinced by Ceylan’s and Nick’s explanation.

The peer communication revealed that the students seemed to understand each other’s point of view. They felt comfortable expressing their ideas as they had already developed a sense of belonging to a community of collaborative learners. This finding is consistent with that of Kohn (1992), Rogers (1995), and Kennedy (2009), who suggest that dialogic and collaborative peer interactions in the classroom generate a sense of community. At the end of EUP 5, I wrote the following in my reflective journal:

It would be very difficult, if not impossible, to create this learning environment in a classroom where communication takes the form of a monologue between the teacher and an individual student. Even though I was teaching whole class, the students worked in multiple zones of each other’s ZPD as the emergent dialogic classroom allowed this to happen. Collaborative learning within EUPs created a classroom atmosphere where the students perceived their classmates as members of their community of mathematics learners; therefore, they appeared to respect and trust each other’s friendly intention (February 13, 2015).

Nadia seemed to appreciate Ceylan’s response, as learning in our class was a collaborative activity. In a classroom where learning is competitive, this egalitarian peer dialogue would be very unusual. The dialogue between Nadia and Ceylan in a competitive classroom could be seen as a race between two rivals. Perhaps Nadia would not have responded to my question in the first place, as making a mistake or giving the wrong answer is equivalent to being a loser in the competitive learning
environment (Kohn, 1992). The preceding excerpt signalled how central an egalitarian community is to sustaining collaborative and dialogic learning.

The next segment shows how a mathematical formula may be introduced in a dialogic form, as opposed to authoritative teaching:

Me: Ok….If we all agree on this, let’s calculate it for the second year.

[Writing on the front board:] $1030 \times 0.03 = 30.9$, and if we add this to the first year amount, $1030 + 30.9 = 1060.9$. Let’s do this for all five years; you help me with calculations with your calculators. [Writing on the board, several students made calculations out loud, and I wrote them on the board.]

At this point, I noticed that having participated in creating the table above helped the students to remain engaged and be an integral part of this knowledge construction process. Tom noticed that the final outcome in the third row was incorrect. Jennifer also indicated the same mistake. I corrected it. This indicates that the students were actively listening to and participating in this collective meaning-making process.

Once the table was complete, I carried on talking with the whole class:

Me: [To whole class:] Can we see any patterns here?

Lexus: Looks like each time we multiply by 0.03.

Me: Do we all agree with that?

MSR: Yep! Yes, we do…

Me: All right then….Is there any other pattern we can locate? [A couple of students raised their hands.]

Me: Yes, Nick?

Nick: I think each year we start with the last year’s amount.

Multiple students: Yep! That is right.

Me: Perhaps we can reorganise this table in a slightly different form to make these patterns a little more visible by factoring the expression.

As we multiply each year’s amount by 0.03, we can rewrite them as below:
I write them on the second board, at the back of the class. Multiple students did calculations with calculators to double check the outcomes for each year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Calculation</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000 ( (1 + 0.03) )</td>
<td>$1030</td>
</tr>
<tr>
<td>2</td>
<td>1000 ( (1 + 0.03)(1 + 0.03) )</td>
<td>$1060.9</td>
</tr>
<tr>
<td>3</td>
<td>1000 ( (1 + 0.03)(1 + 0.03)(1 + 0.03) )</td>
<td>$1092.727</td>
</tr>
<tr>
<td>4</td>
<td>1000 ( (1 + 0.03)(1 + 0.03)(1 + 0.03)(1+0.03) )</td>
<td>$1125.5088</td>
</tr>
</tbody>
</table>
on the right side of it:} Instead of multiplying them by years, we can express it exponentially. Would you please help me with calculations with your calculators?

[Several students did calculations and I wrote them on the board.]

<table>
<thead>
<tr>
<th>Year</th>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1000 + (1000 \times 0.03) = 1000 \times (1 + 0.03)^1 = $1030$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$1000 \times (1 + 0.03)^2 = $1060.9 $</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$1000 \times (1 + 0.03)^3 = $1092.727 $</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$1000 \times (1 + 0.03)^4 = $1125.50881 $</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$1000 \times (1 + 0.03)^5 = $1159.274074 $</td>
<td></td>
</tr>
</tbody>
</table>

Me: Alright….We may be able to generalise these patterns anytime soon. Do these outcomes match with the other table?

Multiple students: [Silent for a while.] Yep….Right column is exactly the same.

Me: Then we can generalise these patterns and set a formula. We all agreed that the year (time) is a variable; let’s label the year as $t$, as it stands for time.

Multiple students: Ok….Let’s go for it.

Me: In our example, the interest rate was 3%, but it could be different. Should we take the interest rate as a variable as well?

Nicole: But it doesn’t change in the table.

[Another student in the class responded before me:]

Noah: But we may have another example with a different rate, you know, just like credit card rates….Each one is different. We can’t put 3% for every example.

Nicole: Got it... It doesn’t change in this example, but it might change for others.

The excerpt shows that the students were actively following and contributing to construct a formula collectively. As I later wrote in reflective journal: ‘This way of teaching made me think of myself as more of a facilitator in a dialogic classroom,
rather than an authoritarian teacher….I felt empowered’ (February 10, 2015). We as a whole class continued constructing the formula:

Me: Let’s call the interest rate \( r \)…by the way, different textbooks mark them differently. It doesn’t have to be \( r \) everywhere. Does inside of parenthesis change for each year?

Multiple students: No…Looks the same.

Me: Ok, then we can simplify it as 1.03, and the starting amount of money also remains the same in the table. Then we can generalise the entire table through variables in one expression [I wrote the formula under the table on the board.]

\[
\text{Final amount} = \text{Original amount} \times (1 + r)^t, \text{ where } t \text{ stands for year, } r \text{ stands for interest rate}
\]

Me: Can we verify the numbers in the table with this formula to make sure that it matches with our long calculations?

Taylor: I did the fourth year, it seems correct.

Jacob: I just did the third…that is true as well.

Me: [To whole class:] Let’s verify all the other years with this formula.

Multiple students: Yep…it works.

Me: Then, from now on, whenever we need to calculate compounded yearly interest, we can use the formula we just developed.

As shown in the excerpt, I attempted to introduce the new mathematics topic in a dialogic form. I constructed the formula together with my students. I justified my instruction through mathematical concepts and encouraged students to participate in the process of developing the exponential model. The students actively contributed by verifying the equations’ numerical accuracy and algebraic coherence. In other words, I facilitated the process and did not continue my teaching without consent of the students.

Science and mathematics teachers usually take an authoritarian (nondialogical) approach when they introduce a new theorem, concept, or topic
(Alexander, 2005; Mortimer & Scott, 2003). Traditionally, mathematics teachers are not supposed to spend this length of time to teach a formula; it is considered time-consuming and not ‘effective’. The authoritarian approach would be to directly provide the formula on the board, followed by several repetitive, skill-drill examples, so that students acquire procedural knowledge of this calculation.

Skovsmose (1994) called this approach to mathematics education the ‘exercise paradigm’; Freire (2000) called it the ‘banking’ concept. According to Skovsmose (1994, 2011), this approach creates a comfort zone for students in mathematics classrooms and thereby makes students bored; they lose motivation for learning. Emphasising its sociopolitical implications, Freire (2000; 2013) argued that banking concept is anti-dialogical: it produces authority-dependent personalities. Establishing and sustaining a democracy, on the other hand, requires active and critical citizens who have communicative competencies and are able to hold authorities accountable (Giroux, 2012). In my reflective journal, I elaborated on dialogic teaching in this situation:

I could have followed the authoritarian path to introduce this formula, which would have taken less time and effort on my side. Instead, I preferred the dialogic approach as I aimed to practice CME, which emphasises dialogic and collaborative teaching and learning of mathematics to promote critical mathematical literacy. The extract from my whole-class teaching reveals that the dialogue was not simply a means to teach the formula: Dialogue in that context was also an end in itself as Wells (1999) elucidates. (February 13, 2015)

I co-constructed the knowledge—the formula—with my students. Even though we did not invent the formula, through dialogue and collaboration, we brought it to life. With my facilitative instruction and students’ active participation, the exponential model of interest calculation became meaningful for us. Drawing on
Freire’s ideas, Shor (1993) pointed out that liberating teaching invites students to active participation; it rejects rote memorisation and the transmitting style of education that results in the alienation of students and teachers:

Teaching and learning are human experiences with profound consequences. Education is not reducible to a mechanical method of instruction. Learning is not a quantity of information to be memorized or a package of skills to be transferred to students. Classrooms die as intellectual centres when they become delivery systems for lifeless bodies of knowledge. (p. 25)

In contrast to Mortimer and Scott (2003), who considered the authoritarian approach appropriate and to some extent necessary, I favoured a dialogic approach. Dialogic pedagogy leads to humanising teaching whereas authoritarian, dehumanising teaching leads to authority-dependent and passive citizens (Frankenstein, 1983; Freire, 2013; Gutstein, 2006; McLaren & Kincheloe, 2007). As evident in the excerpt above, the students responded to my dialogic teaching by verifying the numeric accuracy of the calculations, identifying numeric patterns in the formula, and exchanging questions and answers. In short, the students actively participated in knowledge construction processes; their contributions and ideas were welcomed and valued, just as Nicholas and Bertram (2001) recommend.

4.2.5.3 Introducing the Project

I posted the paperwork for the project and demonstrated an example scenario for Edward’s life after college. Each group had to agree on a study field for Edward. They then determined a possible amount of loans for tuition and made a payment plan by calculating the interest rate for five years (see Appendix E). I handed out the paperwork for the task and allowed the students set their own groups depending on their choice of study field. Once the students began working on the project, their peer
interactions intensified. Each group began by deciding on a study field and researching tuition in that field. They also factored in Edward’s expenses over four years, such as textbooks, housing, food, and other personal items.

Thus each group determined the total amount of money Edward would need for the loan. Then they calculated the compound interest to determine an instalment plan for paying it back in five years after graduation. Depending on Edward’s job situation, they pictured different scenarios for social, economic, and cultural aspects of his future life. By the end of the period, three groups had not completed their project. They asked if they could have lunch in my classroom while completing their project, which I agreed to. An extract from my reflective journal for that day reads:

I noticed that student loan debt is a very burning issue for my students. The students’ motivation was much higher than I anticipated. I observed intense peer discussions, exchanges of arguments, and collaboration on the calculations. As the students had to reach a consensus on multiple subjects, I had expected some conflicts or antagonistic arguments, but I was wrong. All the groups worked in harmony. The effort we, as a whole class, made throughout the past four projects gave its fruits. We have made a qualitative progress oriented towards critical mathematics education. Now our class displays qualities of a dialogic classroom and a community of collective learners. (February 10, 2015)

As my reflection indicates, I realised that turning a classroom into a community of learners is a continuous process. Since EUP 1, the whole class has attempted to achieve egalitarian collaboration, dialogic peer interactions, and horizontal student-teacher relationships. Our learning snowballed as we cumulatively applied what we learned from one project to the next.
4.2.5.4 Day 2 (February 11, 2015)

The second day began with online research. The students reviewed articles, comments, and statistical information about the student loan debt (SLD) issue in U.S. In this project, a different format for the whole-class discussion was trialled. First, each group shared their findings. Then, if the whole class agreed with their conclusion, a student volunteer typed them in a PowerPoint file on the laptop on my table while it was projected on the board. On this first occasion, Nicole volunteered. Once the file was completed, we ran through each slide from beginning to end; this led to whole-class discussion. Some of the information compiled about SLD is presented below:

- Over the last 20 years, the amount of SLD doubled.
- In 1994, less than 50% of the graduates had SLD; that number in 2015 is 70%.
- SLD has increased by 84% just since 2008, and in 2015 hit a grand total of approximately $1.2 trillion in the U.S.
- In 2014, four out of every 10 family men or women younger than 40 years old were paying off SLD.
- SLD is one of the root causes for new college graduates in U.S. to delay marriage.
- From year 1970 to 2013, the average cost of college tuition increased by 275%.
- Since 2005, SLD has drastically increased, but salaries for young college graduates have decreased. It became difficult for young college graduates to get a decent paying job. Many college graduates end up having a minimum wage job.
- There are some countries in the world where public universities are tuition-free.
This activity seemed to expand the students’ horizons and helped them situate SLD as their problem in the larger socioeconomic structure. While they were working on their presentations, I observed in my reflective journal that none of the information or data about SLD surprised my students. They indicated that they knew someone either in their immediate family or relatives, who was or is having trouble with SLD. However, the students did not want to believe that there are places in the world (about 40 countries) where college education is tuition-free or tuition costs a symbolic amount of money. I was surprised that my students were not aware of that information. (February 11, 2015)

I observed that as much as the students appeared to be empowered by the discussion, they seemed worried and upset with the current SLD situation. While developing the PowerPoint slides, a spontaneous discussion in the class emerged. I wanted to bring up the consequences of market-driven education policies and discuss SLD in that context.

I did not, however, want to impose my viewpoint. Instead, I aimed to make some small openings in the colonised life-world of the students where they could be engaged in an open dialogue with their peers. When I asked whether education should be free for all or only for those who cannot afford it, without exception, all of them said that college should be free for all. For about 10 minutes, I observed the students posing questions, exchanging opinions, and answering each other’s questions, all without interrupting each other. Unfortunately, I had to bring this discussion to an end and begin the journal entry session, as we were running out of time.

My ongoing reflections about this experience prompted the following question. In the context of critical pedagogy and CME, where should teachers draw the line between expressing their standpoint and making small openings for egalitarian dialogue? This question was raised by Shor (1993), one of leading figures
in critical pedagogy. Based on his teaching experiences in a community college in the U.S., Shor elaborated:

Dialogic teachers do not separate themselves from the dialogue. The teacher who relates economic power in society to the knowledge under inquiry in the classroom cannot impose her or his views on students but must present them inside a thematic discussion in language accessible to students, who have the freedom to question and disagree with the teacher’s analysis. (p. 30)

Although I agreed with my students about the SLD issue, what I inferred from this dialogic experience aligns with Shor’s (1993) point. As long as a discussion does not turn into a process of propaganda or manipulation of any kind, and students can comfortably question the teacher’s point of view, the teacher can participate in dialogue as an equal partner.

4.2.5.5 Students’ Journal Entries

This is the third time that Edward figured in our word problems. The students seemed to have much stronger empathy with him in this project than in previous ones. They imagined the next 10 years of Edward’s life after college with debt. To provoke discussion, I posted a number of prompt questions on the board:

- How do you think paying back his student loan for 10 years will affect Edward’s life?

Jacob thought that Edward’s life would be restricted by his debt: ‘I picture Edward, struggling financially and living in an apartment with a roommate to save money after graduation as salaries are decreasing.’ As the students determined the field of Edward’s study and an estimated amount of tuition, each group’s emphasis was slightly different. Selena mentioned her group’s calculation and linked Edward’s debt issue to her family’s debt problem in the past:
Edward’s life doesn’t seem so bright….According to our plan, he has to pay off $100,000. Whatever he does in his life, he has to think about his debt….Being in debt is a horrible thing. When my father had an accident, he had to stay in bed for five months; we did not have any income and got into debt.

Darryl emphasised the connection between one’s debt and getting married and starting a family: ‘It would be very difficult, if not impossible, to start a family in his strict financial situation.’ During the online research, the students compiled some statistical data, which confirmed Darryl’s claim: SLD forces many college graduates to postpone their marriage plans.

Tibu voiced another common concern of students who are in debt: ‘Edward’s life depends on many factors; his debt will definitely limit his choices, but if he gets a good paying job, he may pay off comfortably, but what if he doesn’t get a job, that would be a nightmare.’ Jennifer had another term for what Tibu called a nightmare. She wrote, ‘Edward will most likely face the dark side of the American dream….And I hope he will not.’ Cindy provided a brief but vivid description of Edward’s future life:

Life after college for Edward may look like working long hours, maybe a second job, and restricted social life. He may not want to get married and have a child; he would not want his child to go through same problems….According to our project, Edward has to pay back approximately $20,000; he could buy a nice house with that money, but he will pay to loaners instead….It is not fair for anybody.

The students’ entries consider Edward’s next 10 years as stolen years. With the next question, I invited my students to share their experiences and opinions about socioeconomic consequences of SLD:
• How do you think debt in general affects people’s life? Examples from your experiences?

In Akil’s view, debt enslaves people: ‘Debt ruins people’s lives….If you are in debt, you are kind of slave.’ Cindy shared her father’s situation, which resulted from SLD: ‘My father works overtime to pay off my sister’s loan debt and two of my cousins, whose parents are not fit to support.’ The students pointed out that SLD influences not only individual students, but also their families. As Akil articulated, debt can become the apparatus for social control and oppression.

With the following question, I invited the students to share their views about neoliberal perspective on education:

• Do you consider education as a human right or a commodity?

From the neoliberal perspective, education, like any other item in market, can be bought and sold. In this view, education is considered a commodity rather than a human right (Chubb & Moe, 1990). However, the students seemed to disagree with this neoliberal tenet in their journals. I was expecting that a couple of students would disagree with an idea of tuition-free education. However, my expectations were wrong. All the students in my class argued that education should be a human right. Therefore, from students’ point of view, it should be free of tuition. If a citizen wants to study, there should be no financial barriers whatsoever to their educational journey.

For example, Nicole disagreed with the neoliberal ideology, arguing that education should be a social investment. She wrote, ‘If a student has determination and will, she should be able to go to college regardless of her money situation….When an individual is educated, the whole society benefits. So education should be a human right and social investment, not an individual commodity.’ Darryl voiced a similar sentiment: ‘Everyone should be able to go to college regardless of
their economic status. Education’s cost should be shared by everyone because it is a necessary part of our society that affects everyone and everything.’ Cindy argued that education could not simply be reduced to an item in the market: ‘Education can’t be priced. It should be free for all….We need to change our pledge…like liberty, justice, and free education for all.’ Her comment seemed to imply that the pledge of allegiance\textsuperscript{32} is empty rhetoric: there could be no liberty and justice without education for all.

Leonardo was another who envisioned education as a basic right of citizens:

Education should be a human right to give everyone a good career. But unfortunately, college education is something you pay a lot of money for in our country even though these days you have no guarantee that you’re going to have a decent job after graduation.

Contrary to my expectations, students’ arguments for free education were emotional but nevertheless well-informed. Their comments challenged the neoliberal stance that education is an individual commodity and thus each student has to pay to receive it.

As I observed the tension between neoliberal impositions and the students’ needs as human beings, I encouraged them to share their concerns and feelings:

• Does dealing with student loan problem in this project make you feel informed and empowered or instead make you worry about your college education?

The students articulated their thoughts and feelings about the SLD issue, as it directly relates to their own life-world. For example, Nadia, like her classmates, voiced her anger about the unfairness of college tuition in the U.S.: ‘Now I have a better sense of my college education. But it is unfair and makes me angry. I am not

\textsuperscript{32} It is a ceremony held every morning in public schools in the U.S. The words are as follows: ‘I pledge allegiance to the Flag and to the Republic for which it stands: One Nation indivisible, with Liberty and Justice for all.’
sure how, but I feel like this needs to be changed.’ Jacob indicated that he felt empowered as well as informed about the problem; however, SLD made him anxious, too:

I usually worry about my future, and facts about SLD are startling. At the same time, I feel like I am aware of the problem. I don’t think we can solve this problem individually. This is the first time I am experiencing a math problem that connects to the student loan problem.

Jacob’s reflection also implies that the student loan issue is not critically addressed in his education journey, as this project is the first one for him.

Some students’ reflections clearly indicated that being able to receive college education in the U.S. is directly connected to one’s socioeconomic situation. Selena wrote, ‘The whole idea of SLD does worry me, because my family does not have that much money. I am hoping to be qualified for a scholarship maybe.’ Darryl voiced similar sentiments: ‘As much as I became informed, it made me worry about my college education and my future. My family is not that rich to pay my tuition if I go to a good college….Something needs to be done for high college tuitions.’

Jennifer agreed with Darryl that something has to be done:

I have mixed feelings; I feel like I am informed about a catastrophe of SLD and I am glad that I am aware of that. But it also scared me a lot…the way things are in this country is stifling our future….It is not fair….People in this country—I mean all of us—should do something about it.

She considers the socioeconomic and political system as the source of this unfair situation and indicates that citizens collectively need to take action to ensure that the system will no longer be able to destroy young people’s futures.

The students’ responses to this question revealed one of the challenges of critical pedagogy: How can hopelessness and pessimism be transformed into hope
and love oriented towards ideas and actions for a more just and free world? If critical pedagogy fails to make this transition, it may do more harm than good, making students feel hopeless and helpless about their college education.

Therefore, the next question invited students to think of possible solutions to the problem:

• What do you think that can be done to remedy student loan debt issue?

Finally, the students shared their thoughts about possible solutions to the problem. Along with anger and frustration, they came up with suggestions aimed at solving it. Darryl envisioned a free college education and proposed a strategic step: ‘In my opinion, college tuitions should be significantly lowered and that would be a good step towards a free public education….Some countries offer free university education. Why not in the U.S.?’ Through this project, the students learned that there are many countries in the world where university education is free: They were fascinated with this information.

The students also articulated the link between the job market and the SLD crisis. They wondered how college graduates could pay back their loans if there were not enough employment opportunities. Noah wrote, ‘Decreasing college tuitions and increasing minimum wage would be a good start to solve SLD crisis; also, you don’t have to pay your loan until you get a job’. Tibu put forward a similar perspective: ‘Creating more jobs, not charging the going interest rate for loans, and increasing the minimum wage would be a good start.’ The students argued that if more employment opportunities are created and the minimum wage is increased, it would eventually affect the jobs that require a college degree by increasing their salary.

Although most students considered SLD a crisis and came up with possible solutions, they did not attempt to explain or question the root cause of the problem.
Jeff was an exception. He pointed out that the first step should be to end corporate involvement in college education: ‘We need to find ways to remove big corporations, corruption, and greed from education….We need to speak out for better conditions and better opportunities for all.’ Jeff framed the issue as part of a larger struggle for a free and just society.

It was also surprising to me that some students pointed to excessive military spending. Liam wrote: ‘I believe that tax money that is spent on military should be shared with funding for universities. This way it would lower the cost of going to university.’ Jennifer agreed, saying that taxpayers’ money would be better used lowering SLD instead of killing people. She wrote, ‘Our army is dropping bombs overseas and says that they will bring democracy to these places, but would it not be better spending this money for universities rather than spending it on bombing people?’

Cindy pointed out that the system keeps people from publicly discussing problems such as SLD:

To be honest, I don’t know how the SLD crisis can be realistically tackled….These problems are not discussed in public; TVs are loaded with celebrity stories….American people are kept in dark. People need to be able to talk about issues like the SLD crisis, and something may come out of these discussions….But people think that they don’t matter; for example, my parents don’t vote, they think an election is completely meaningless. It doesn’t change anything in their life.

Cindy articulated the liberating power of open dialogue. She implied that the system colonised the life-world and does not leave much space for people to be engaged in a free and open dialogue oriented towards possible solutions to real

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33 My observation confirms that the military budget is one of the taboos in the U.S.; it takes intellectual capacity and civic courage to speak in public about overinflated military spending.
problems. Cindy’s comment relates to Habermas’s (1975; 1996) argument that neoliberalism maintains its hegemony through colonising the life-worlds of people, shrinking the public sphere, and then imposing the system’s imperatives top-down. This conclusion derived from students’ comments about EUP 5 signifies the importance of creating small openings in the classroom where empowerment and liberation can emerge.

4.2.5.6 Discussion

In EUP 5, I integrated the standard curriculum (exponential modelling) into the SLD issue through an inquiry-based dialogical education without compromising rigour and academic standards. Freire and Faundez (1989) reasoned that ‘democracy and freedom are not denial of high academic standards’ (pp. 33–34) and pointed out the misconception that academic rigour can be achieved only by authoritative teaching. A false connection is often made ‘between a democratic [dialogic] style and low academic standards…[and between] high academic standards and authoritarian [teaching] style’ (pp. 33–34). My experience in this project is that dialogic teaching does not prevent high academic standards from being enacted. To the contrary, dialogic pedagogy makes the learning process more meaningful and empowering. For example, in EUP 5, as well as learning the interest rate formula, the students participated enthusiastically, which improved their collaboration and communication skills. Based on the students’ comments and journals, and my reflective journal, the students improved their content knowledge and seemed to develop critical mathematical literacy—one of main goals of mathematics education (Ernest, 2002b; Gutstein, 2006).
Instead of providing students with the interest rate formula and a couple of repetitive examples, I aimed to engage them in ‘problem-posing’ in the sense of Shor (1993):

In problem-posing, in teaching subject matter dialogically, academic material is integrated into students’ life and thought. Students do not simply memorize academic information about [mathematics], but rather face problems from their lives and society through the special lens offered by an academic discipline. (p. 31)

SLD is a significant issue for students who wish to pursue university education. The students applied the interest rate formula as well as their mathematical literacy to envision social, cultural, and economic aspects of Edward’s life and reflect on the SLD crisis in this context. During this project, I made small openings by raising questions about the SLD issue and encouraging students to do the same. In this sense, I critically distanced myself from mainstream mathematics instruction and textbooks. Instead, I constructed the project around three interrelated concepts oriented towards CME (Skovsmose, 1994): critical distance, critical competence, and critical engagement.

Through my teaching of exponential modelling, I concluded that introducing a new mathematical concept, theorem, or formula can be achieved through dialogic pedagogy. This is precisely the question raised in Section 2.8: ‘To what extent would CME agree with the idea that authoritative (or nondialogical) teaching is necessary when mathematics teachers teach a new subject?’ The main point here, as Freire (2000; 2013) argued, is ethical and political. On ethical grounds, I prefer the dialogic approach because it is empowering and humanising. The students actively contributed to creating calculation tables; they responded to each other’s questions and comments and worked within multiple ZPDs. All of which is to say that the interest rate formula
already existed, but it was not yet alive. Through dialogue, we collaboratively brought it to life by unpacking its internal logic and coherence. Instead of memorising it, we developed the formula with understanding. Thus, how we learn mathematics is as important as what we learn (Gutstein, 2006).

Teaching the interest rate formula, I referred to a set of mathematical axioms, properties, and theorems. That is, I made a validity claim rather than a power claim, just as democracy itself must constantly refer to the Constitution for its validity check (Almeida, 2010). Because I want students to develop their participation skills, democratic values, and mathematical literacy en route to becoming critical citizens, I encouraged them to actively participate, check the accuracy of calculations and validity of conclusions, and take ownership of their learning. My approach is consistent with Ellis and Malloy (2007) and Hannaford (1998) who argue that mathematics education should promote critical literacy oriented towards critical citizenship.

From my observations and reflective journal, I noticed that ever since the beginning of the school year, we had been working to become a critical community of mathematics learners. We had made qualitative progress towards the goal of dialogic learning, developing egalitarian peer interactions and horizontal student-teacher relationships.

As noted earlier, the sharp increase in college tuition since 1980 in the U.S. is directly linked to the impact of neoliberal ideology on higher education. As Coco (2013) argued, neoliberal forces initiated ‘a shift away from education as public good [to] education as an individual investment’ (p. 566). These forces convinced ‘state and federal governments to disinvest in institutions of higher education’. When universities received less public funding, tuition was raised drastically. Considering
neoliberal policy the root cause of the SLD crisis, Coco (2013) argued ‘the vision of education and the resulting education policies has led to tuition hikes, reduction in scholarship and grant funding for students, and an increase in students drawing on loans to pay tuition cost’ (p. 566).

While these top-down neoliberal policies resulted in the SLD crisis, the students through this project developed bottom-up responses that radically challenged neoliberal educational tenets. The students’ comments resonate with Coco’s (2013) point that ‘many low and middle income students are either prevented from attending college, or they are required to assume an enormous debt loan, equivalent to a thirty-year home mortgage, to obtain a university education’ (p. 601). My students challenged neoliberal ideology: They proposed steps oriented towards tuition-free education. They made tangible short-term suggestions to remedy initial SLD crisis, such as creating more employment opportunities for college graduates and minimising the interest rate for tuition.

The students’ reflections indicated that they do not give their consent to neoliberal educational policies. According to Habermas (1975), this tension between the life-world and system brings into question the very legitimacy of the system. He conceptualised this problem as the legitimation crisis. Habermas (1984) concluded that this tension could only be solved by initiatives originating from life-world; emerging bottom-up responses could transform the system. Creating small openings in colonised life-worlds—such as mathematics classrooms—is crucial to counter neoliberal hegemony and generate hope for a more just and free world.

4.2.6 Conclusion

In this critical participatory action research, I facilitated five projects (EUPs) based on the objectives of CME. The students’ reflections and whole-class
discussions, together with my observations, field notes, and reflective journals, provided a broad corpus of empirical data. My analysis of the data yields five general conclusions.

First, following Skovsmose’s (1994; 2011) description of CME, the EUPs were developed to ensure that they were open-ended, critically engaging, and distanced from official-standardised curriculum. The students gradually deconstructed and reconstructed skills, attitudes, and values to initiate collaborative learning and non-dominating, dialogic peer relationships. They genuinely made efforts to establish and then improve the quality of collaboration and dialogic relationships.

Second, because students’ participation was valued, the cycles of plan, act, observe, and reflect were highly effective. Critical participatory action research resonated well with the dynamics of the classroom. Each project interconnected inquiry-based learning materials and activities, collaborative learning, and dialogic peer interactions. Interactive dynamics in the classroom created an inclusive and egalitarian atmosphere, through which the students learned to embrace the values of collaboration, inquiry, and dialogue. Students gradually identified elements of collaborative learning—respect, unselfishness, fair-mindedness, patience, tolerance, and empathy—thereby transforming our classroom into a dialogic one.

Third, as the classroom became a community, it allowed me to become a facilitator and establish dialogic, non-authoritarian relationships with students. This was also a dialectic process, as I aimed to have a more horizontal relationship with the students, while the students developed more egalitarian attitudes towards each other.

Fourth, the life-world of the classroom was gradually transformed into a communicative space where we could develop bottom-up responses to the system’s top-down attempts to colonise it. The students connected mathematics in the class to
the larger sociopolitical system. Our classroom became a micro society in which the only language spoken was democracy.

Fifth, as much as participating in EUPs was an enjoyable and meaningful professional experience, I found it to be a highly challenging process. Engaging in dialogic pedagogies that countered neoliberal hegemony proved to be the easy part. When given space and opportunity, the students themselves developed strong arguments against neoliberal ideology. For me, this forged a sense of hope for a better education and a more just and free world.

At the same time, across EUPs, I received support neither from the school administration nor from my fellow teachers. The administrators were preoccupied with implementing standardisation and increasing test results. My colleagues, perhaps concerned about their job security, hesitated to engage in critical pedagogy, which challenges the recent neoliberal educational changes. Also, I had to develop all the teaching materials from scratch, since there were no ready-to-use curriculum materials for CME. I also had to create space for CME within the standardised curriculum, and connect the projects with scripted learning standards to satisfy system’s requirements. As Kemmis, McTaggart, et al. (2014) warn, to be sustainable, changes in the life-world must take into account the imperatives of the system.
Chapter 5: Analysis and Discussion

In this chapter, I will present a conceptual analysis of the end-of-unit projects (EUPs) from a holistic perspective in order to address the research questions. Sections 5.1 to 5.3 frame dialogic pedagogy, collaborative learning, and inquiry-based learning in the context of critical mathematics education (CME). Section 5.4 considers the implications of CME for citizenship. Data for analysis, presented comprehensively in the previous chapter, were drawn from my field notes and reflective journals, from student journals, and from class discussions.

CME centres on the idea that school mathematics can be oriented to promote democracy and justice-based participatory citizenship (Skovsmose, 1994; Westheimer, 2015). The main findings from the EUPs suggest that three concepts are especially important: (a) dialogue as pedagogy, (b) collaborative learning, and (c) inquiry-based education. In the education literature, these terms typically reference mainstream educational perspectives. Therefore, it is necessary to define the distinctive elements of dialogic pedagogy, collaborative learning, and inquiry-based education in order to elucidate their scope in a CME approach to mathematics in the high school setting.

In this research, a learning community emerged out of dialogue, collaboration, and inquiry. My students and I began practising democracy in the form of dialogue in a mathematics classroom that had been previously colonised by top-down neoliberal educational policies. In this analysis, I therefore recognise neoliberalism as an ideological and pedagogical construct in order to understand the limitations of CME. Sections 5.1–5.4 present data from EUPs to answer the central question of the research:
• What are the potentials and limitations of critical mathematics education in terms of classroom teaching in the neoliberal era?

5.1 Critical Mathematics Education and Dialogic Pedagogy

Dialogic pedagogy played a defining role in the process of meaning making and knowledge creation in this study. For example, during EUP 1, the students in their reflections and evaluations identified non-egalitarian peer relationships as a barrier to establishing dialogue. As a response to this finding, the class made collaborative attempts to overcome this barrier through cycles of reflective actions. As each EUP unfolded in response to what had been previously experienced, it seemed that a more dialogic classroom was developing, as evidenced in student reflections and in my reflective journal. This transformation was especially noticeable across all five EUPs: the students became more proficient in exchanging ideas, deconstructing and reconstructing their values, attitudes, and ideas. This finding mirrors Frankenstein’s (2010) teaching experience in a U.S. community college, where she used statistical data to unpack the oppressive and discriminatory structure of society.

The concept of dialogue is often presented in the educational literature as an effective instrument for teaching content. From this perspective, dialogue is the opposite of monologue (Cesar, 1998; Horn, 2014; National Council of Teachers of Mathematics, 2000; Pietsch, 2009). Dialogue is one type of instrumental rationality; it is ‘a way of leading others to pre-performed conclusions’ (Nicholas & Bertram, 2001, p. 1102). Therefore, pedagogy based on dialogue in this sense may promote functional literacy, adaptation, and domestication.

However, the authors of this approach do not consider mathematics education a part of the political domain. They do not question who decides what mathematics is
being taught and in what ways, or how the teaching process influences students’ growth as human beings and citizens. In short, it is a very restricted definition of dialogue. In contrast, dialogic pedagogy as practised in the present research has a deeper reach and a more profound meaning, as explained below.

Through small-group discussions across cycles of actions, the students became increasingly willing to push their comfort zones and revise misconceptions they might have had. This adjustment seemed to improve their conceptual understanding of mathematics. For example, during EUP 1, many students said that they identified some misconceptions and improved their understanding of linear functions and equations. Through whole-class discussions and written reflections, the students began to relate their life in the classroom to the system—namely, to externally imposed standardised assessment as the measure of learning. In other words, as the students engaged with mathematics content learning, each EUP and the dialogue that constituted it connected students’ lives in the classroom to a larger social, political, and economic system, thereby promoting critical mathematics literacy. Dialogue inspired by an ‘ideal speech situation’ Habermas (1990, 2005) helped create the necessary freedom for students to construct knowledge on their own volition, without it being imposed by a more powerful agent.

As indicated in Section 2.7, the theory of communicative rationality as framed by Habermas (1984, 1987) is complementary to dialogic and humanising education as outlined by Freire (1998, 2000, 2013). Across EUPs, dialogue as experienced by students seemed to promote critical literacy and transformative action as well as functional literacy. Therefore, dialogue—as an approach to teaching and learning mathematics—was here based on communicative rationality, which is fundamentally different from the instrumental rationality approaches that students may have
experienced previously (Nicholas & Bertram, 2001). In this study, dialogue was not used as an instrument to transmit pre-formed knowledge and increase standardised test results.

Results showed the specific ways in which dialogue in the CME approach formed a distinctive and new form (for myself and my students) in its relation to learning, curriculum, peer interactions, students-teacher relations, and overall classroom ambiance.

5.1.1 Open-Ended Projects as Mathematics Lessons

An important element of this study’s dialogic pedagogy was open-ended word problems. In the traditional approach to teaching mathematics, the skill-drill types of repetitive exercises are favoured. Solutions to these questions require procedural knowledge only; they lead to one-dimensional, single correct answers. Thus, such approaches foster rote memorisation. Thanks to top-down imposed neoliberal educational policies, this ‘exercise’ paradigm (Skovsmose, 1994, 2011; Skovsmose & Alrø, 2004) has become the driving force of most mathematics classes (Hursh, 2007b; Gutstein, 2006). In an era of market-driven policies and curriculums, teachers are urged to focus on exercises that will most likely be assessed on standardised tests (Leistyna, 2007; Schneider, 2015). Consequently, student-teacher communication that emerges in this context draws on instrumental rationality and implies strategic action. Habermas (1984, 1987) defines such communication as ‘distorted’; Freire (2000) calls it ‘anti-dialogical’.

In contrast to the traditional approach to mathematics education, I designed the EUPs as open-ended projects. This allows multiple perspectives to interact; it creates a learning ambiance conducive to dialogue (Aulls & Shore, 2008). For example, in EUP 1, students produced their own multiple-choice questions to show
their conceptual understanding, procedural knowledge, and numerical fluency with linear functions and equations. The project did not require the students to follow a prescribed direction that would lead to a single correct answer. Instead, it was open-ended, allowing each group to collaboratively negotiate possible choices—both correct and incorrect ones—and agree on one set of choices. My choice of inquiry design, as a teacher, was thus not based on an instrumental rationality (transmitting pre-packed knowledge), but rather communicative rationality.

Although direct teaching (lecturing) in mathematics has been suggested to improve students’ learning of mechanical aspects of mathematics and enhance their functional literacy, my experience as a mathematics teacher suggested it was more appropriate to teach linear functions through an inquiry-based and dialogic lesson. The dialogue, coupled with the open-ended task, appeared to be a liberating dynamic for the process of knowledge construction. With an inquiry-based approach, I aimed to open a dialogic sphere in my classroom where the students would be able to develop their own solutions and negotiate with their peers.

Inquiry-based processes seemed to enhance the quality of learning mathematics in this class. Each EUP involved an inquiry that allowed students to apply and improve a range of algebraic concepts, as well as connect their learning to a larger sociopolitical structure. This was evident in whole-class discussions and in students’ reflective journals. In order to connect content learning to social, economic, and political issues, a shift from a problem-solving (exercise paradigm) to a problem-posing approach is required (Gutstein, 2006; Skovsmose & Alrø 2004). For example, in EUP 1, an investigative approach provided students with opportunities to unpack standards-based assessment and evaluate them in terms of the needs of individual students.
The students’ journals and classroom discussions confirmed that peer and student-teacher communication was a key to knowledge construction. For Freire, knowledge is socially constructed through ‘subject-subject dialogue’; for Habermas, it is built through ‘intersubjective consensus within discursive communities’ (cited in Morrow & Torres, 2002, p. 54). The students’ reflections on group work indicated that peer interactions enabled them to learn with and from each other. For example, in EUP 2, the process of developing the multipart function encouraged students to exchange thoughts, listen to, and build upon each other’s ideas. This leads to the conclusion that open-ended word problems create dialogic spaces for students to challenge the hegemonic dynamics that affect their life-world in and out of the classroom. In short, CME can promote critical consciousness (Freire, 2000) and communicative competency (Habermas, 1987).

5.1.2 Dialogue within Inquiry

Results revealed four fundamental elements of an inquiry-based approach to mathematics education. First, themes of inquiry must be derived from students’ life-world in order to motivate them and increase the quality of their participation. I noted that the students readily joined discussions about social and political matters embedded in EUPs when the topic related to their social circumstances. Second, the inquiry process must welcome ideas that may be unpopular within the mainstream curriculum. Third, the inquiry process must be facilitated by the teacher to enable students’ critical engagement. Fourth, the process must include a self-evaluation (reflection) as an alternative to assessment in the mainstream system. Traditional education has its own measurement system—normally a letter grade or percentage. Therefore, an inquiry-based approach within CME should also include a critical
reflection and an evaluation processes to ensure that CME becomes sustainable as a more subversive educational practice.

In the process of conducting the action research cycles, I realised that the whole-class discussion could function as a process of self-evaluation for students’ learning as part of CME practice. This finding is in line with Rogers’s (1995) notion of person-centred education. The whole-class discussions at the end of each EUP gradually became critical self-evaluations, in which plans for the next project were initiated within the continuous cycle of planning, acting, and reflecting. Just as whole-class discussions became a process of collective evaluation, students’ reflective journals facilitated an individual evaluation process. These collective and individual reflection processes dialectically informed each other throughout all five projects.

5.1.3 Power Relations and Dialogue

During EUP 1, students identified unequal peer relationships as an obstacle to establishing productive dialogue in small group work. In response, they, with my encouragement and scaffolding, worked on establishing non-dominating peer interactions. The measure of success of these endeavours is that in subsequent EUPs, students reflected on equal power relations in group work. Thus, the data show that the nature of communication among students became a distinctive element of dialogue in CME.

In the traditional approach, communication between students and teacher occurs within an asymmetrical power relationship. This power relation is considered normal: Teachers possesses authority and knowledge; they transmit it to students, who passively receive the transmitted knowledge (Rogers, 1995). Notwithstanding widespread acceptance, this approach creates a toxic, anti-dialogic atmosphere in the classroom (Carr, 1995; Freire, 2000; Rogers, 1995; Shor, 1987).
The data in my research, however, strongly confirms Freire’s (2000, 2013) and Rogers’s (1995) conclusions; namely, that teachers can turn a classroom into a supportive and liberating ambiance. Throughout the study, I aimed to be a facilitator who understands students empathetically, who is an equal partner in the collaborative learning community, and who establishes horizontal relationships with the students. As I became a better facilitator, the classroom ambiance became friendlier and more positive. In their journals, some students said that my facilitative attitude had a positive impact on peer interactions. But whatever the cause, students deliberately and enthusiastically worked on developing non-dominating peer relationships throughout the project. They felt themselves as an integral part of the classroom and related to their classmates with empathy.

As I shared my power with students and fostered a facilitative learning environment, the typical discipline problems and classroom management issues that I’ve experienced over the last seventeen years seemed to mostly dissolve. This observation led to me to conclude that how students behave is connected with whether or not a classroom is dialogic.

Resonating with Freire’s notion of humanising education, Rogers and Freiberg (1994) considered a teacher to be a facilitator who is a key to achieving a person-centred instruction. According to Rogers and Freiberg, ‘There is no resemblance between the traditional function of teaching [authoritative] and the function of the facilitator of learning [dialogic]’ (p. 170). A teacher’s facilitative stance is not based on a power claim. Rather, teachers are ‘catalyzers, facilitators, energizers; they give students freedom and life and the opportunity to learn. Most importantly, they are co-learners with students’ (p. 167). Similarly, Freire (2013) recommended the empathetic approach and horizontal student-teacher relationships to achieve liberating
communication. According to him, dialogue is the only real communication as ‘it is nourished by love, humility, hope, and mutual trust’ (p. 42). Freire (2000; 2013) expressed the centrality of trust in humanity and indicated its transformative potential.

Rogers and Freiberg (1994) elaborated the teacher’s role as facilitator and its impact on peer relationships:

When teachers are empathically understanding, their students tend to like each other better. In an understanding classroom climate, every student tends to feel liked by all the others and has more positive attitudes towards self and school. This ripple aspect of the teacher’s attitude is proactive and significant. (p. 161)

Findings from the current research support Rogers and Freiberg’s (1994) conclusion. For example, in EUP 1, the students identified dominating attitudes as a barrier to productive and enjoyable group work; as a response, they enthusiastically worked on developing egalitarian peer relations. Initially, as recorded in my reflective journal, I considered it a coincidence that the students were working well together, listening to each other respectfully, and being kind to each other. However, I gradually realised that my empathetic approach to students, instead of enacting evaluative stance, seemed to inspire them to embrace non-dominating peer interactions in subsequent EUPs. As Freire (2000) pointed out, mutual trust is key to establishing a facilitative attitude and classroom ambiance. After five cycles of action research, I concluded that the preliminary reconnaissance stage played a significant role in establishing grounds for mutual respect and trust.

In line with Gutstein’s (2006) and Frankenstein’s (1994) suggestion that CME should serve critical mathematical literacy, students connected their mathematics knowledge and life in the classroom to the social, political, and economic system to interrogate unjust and oppressive aspects of the system. Findings illustrate the ways in
which these discussions created small openings to counter neoliberal hegemony. Therefore, the discussions inevitably involved ideological and political views.

Contrary to traditional classrooms where the teachers are not supposed to express their views, I openly shared my point of view. In EUP 1, for example, I shared my ideas about standardised assessment; about socioeconomic inequality in EUP 4, and about tuition-free education in EUP 5. As a facilitator and an equal partner of the knowledge construction process, I provided them with opportunities to question and criticise my standpoint. Although it was not antagonistic, I observed that some students opposed their classmates’ views as much as mine, which contributed to the process of creating a dialogic classroom. My experience is consistent with Freire’s (2000) conclusion that critical teachers can openly share their views as long as they can establish and sustain horizontal relationships with the students through which they can freely disagree with the teacher’s view.

Neoliberal educational changes have resulted in the implementation of a scripted curricular sequence (Hyslop-Margison & Thayer, 2009; McNeil, 2009; Schneider, 2015). As a response to this framework, I incorporated the scripted curriculum into each EUP. For example, students needed to be familiar with basic properties of exponential expressions, equations, and functions in order to pass the standardised test. Therefore, EUP 5 required students to understand and use exponential expressions to complete the project. As mentioned in Section 2.8, the CME literature is silent about whether mechanical aspects of mathematics can be taught through a dialogic approach. Clearly the nondialogic way is the dominant approach in most developed countries (Alexander, 2006, 2015; Pietsch, 2009).
Prior to this study, as a high school mathematics teacher, I had often used authoritative\textsuperscript{34} approaches in my teaching practices. These approaches urge students to memorise and accept what the teacher says with or without understanding. Over time, I began to notice that the authoritarian approach generated frustration, anxiety, and boredom among my students. Furthermore, it seemed very difficult to learn mathematics through memorisation. In my ongoing professional development and extensive reading, I came to the understanding that authoritative teaching was nurturing neither an enjoyable nor a humanising learning experience for my students. When I discovered the critical pedagogy literature and CME on my teaching journey, I began to practise dialogic teaching of mathematics.

In the present research, I applied dialogic teaching even when I taught mechanical aspects of mathematics. As there was no classroom-based CME literature to use as a guide, I had to develop my own dialogic teaching practices from the ground up. For example, in EUP 5, students needed to have a basic understanding of exponential properties to work on Edward’s college application. I facilitated a process of collective construction of the interest formula. I demonstrated numeric patterns in order to formulate a generalisation. For each step, I posed prompt questions to the whole class, which students voluntarily responded to. However, this class revealed itself to be much more than typical student-teacher talk. As discussed in Section 4.2.5, what began as a single student-teacher exchange evolved into a whole-class dialogue. Students actively listened to and responded to each other with their own arguments and ideas; they ultimately took control and ownership of their collaborative knowledge construction process.

\textsuperscript{34} This approach is also known as direct teaching or lecturing.
In this facilitative teaching, I did not continue to the next step until all students indicated that they understood the concept under discussion. While I was demonstrating numeric patterns, the students made all the calculations and verifications as active participants of this knowledge construction process. When I answered the students’ questions and confusions, I justified my conclusions by means of algebraic properties and other mathematical theorems—not by asserting my authority as classroom teacher. A quotation from my reflective journal after that lesson underscores the connection between facilitative teaching and the dialogic classroom:

This lesson exceeded my expectations. The students were engaged. They actively participated in the process of reinventing the interest formula. They were as excited as if we had developed a new formula. My dialogic (non-authoritarian) approach seemed to create a different classroom atmosphere. I noted that it would be very difficult, if not impossible, to succeed at this end if we were not a community. After four EUPs, being a community of collectively learning mathematics nurtured this authentic ambiance. (February 12, 2015)

It is evident that being a community enabled us to co-construct the formula. It is difficult to see how a traditional classroom—where students are positioned as passive receivers of knowledge, ruled by the authority (Rogers, 1995)—could have done the same.

5.1.4 The Mathematics Classroom as a Community

Taken together, the findings from the EUPs lead me to conclude that teaching a mathematical concept in dialogic form requires the classroom to be an egalitarian community of learners (Kennedy, 2009; Murphy & Fleming, 2010). Teacher reflection indicated a dialectic connection between becoming a community of learners and quality of dialogue. Five elements of this connection emerged.
First, teachers’ role as facilitator is to establish and sustain a horizontal relationship with their students. I noted that my facilitative attitude was one of the tipping points that reshaped the entire classroom ambiance, confirming Rogers’s (1995) claim:

When teachers are empathically understanding, their students tend to like each other better. In an understanding classroom climate, every student tends to feel liked by all the others and has more positive attitudes towards self and school. This ripple aspect of the teacher’s attitude is proactive and significant. (p. 161)

Acting as a co-constructor of knowledge rather than the ultimate source of knowledge and authority, I opened up space in the class, which allowed the students to become more agentic actors. As student journals indicated, this space encouraged students’ voluntary participation. It enabled them to take ownership of their learning and develop responsibility, self-discipline, and egalitarian peer interactions.

Second, structuring learning as a collaborative (non-competitive) activity created a friendly environment where students did not see their peers as rivals and, therefore, were able to empathetically understand them (Flecha, 2000; Kohn, 2006; Pine, 2009; Rogers, 1995). In this friendly environment, student-teacher conversations were transformed into whole-class dialogue. I observed that as my relationship with the students became more horizontal, peer interactions in the class became more empathetic and positive—a transformation that resonates with ideas of Rogers and Freiberg (1994) and Rogers (1995), who found that a teacher’s facilitative attitude promotes more dialogic and empathetic peer relationships.

Third, there is a close connection between becoming a dialogic classroom and establishing mutual respect and responsibility. My facilitative attitude generated a liberating ambiance through which the students developed self-discipline. Students took responsibility for and ownership of their own learning; thus, learning activities
became a self-regulated process. In traditional, authoritarian education, students are regarded as strangers who need to be controlled (Kohn, 1999, 2006; Rogers, 1995). Not so in the dialogic classroom. As my students and I established mutual respect, empathy, and trust, I could rely on the students’ responses as to whether to continue to the next step in learning activities. Students, meanwhile, exchanged ideas and comments without hesitation or fear due to this friendly environment.

Fourth, the students’ being a community, in which learning is a collaborative and open-ended process, changed their perceptions about what it means to be confused or make mistakes. At the reconnaissance stage, we as a whole class agreed that making mistakes and being confused are a normal part of learning and not subject to criticism in our class.\(^{35}\) As a result, students gradually felt free to share their ideas during group work and whole-class discussions.

Fifth, my efforts to establish horizontal student-teacher relationships acted as a catalyst for students to create egalitarian, non-dominating peer interactions of their own. In EUP 1, they agreed on equal peer relationships as a regulative norm, and continued acting upon this reflection for each subsequent EUP.

**5.1.5 Democracy and Dialogue**

Dialogic pedagogy has implications for the connection between mathematics education and democracy. Almeida (2010) argued that teaching mathematical proofs is a domain where dialogic teaching can be materialised in a form of democracy. A proof refers to mathematical axioms, theorems, and properties in order to justify each step in the process. Similarly, in democratic societies, tensions and problems between citizens and authorities are resolved based on the Constitution. Viewing the mathematics classroom as a micro society, a teacher can structure the proof process

\(^{35}\) My experiences confirmed that students are usually afraid to share their answers, thoughts, and ideas when there is a single correct answer and learning is competitive.
on the “constitutional” basis of axioms, theorems, and properties, instead of on the teacher’s authoritarian power. The National Council of Teachers of Mathematics (2000) also suggested that mathematics teachers construct teaching through the mathematical verification process—not their authoritarian power.

In EUP 5, in response to the students’ question, we constructed a proof of zero power property: $x^0 = 1$. Inspired by Almeida’s (2010) suggestion, I broke the proof process into several steps and negotiated each step with the students by justifying it through algebraic properties and logical verifications. I ensured that every student clearly understood before continuing. My reflection on EUP 5 supports Almeida’s claim. The dialogic approach enhanced the existing democratic culture in the class because the validity of the claim was based on not my authority as teacher, but on the mathematical logic, properties, and evidence of the proof itself.

According to Alexander (2006, 2015), teaching is dialogic if it is collective, reciprocal, supportive, cumulative, and purposeful. EUPs were based on collaborative learning and emphasised the value of collectivity. The students were equal partners in the decision-making process. It was evident that we established horizontal student-teacher relations and non-dominating and reciprocal peer interactions. The students expressed their opinions, ideas, and suggestions during group work and whole-class discussion without internal constraints, such as being afraid of making a wrong answer. We established a facilitative and supportive environment. Our projects were cumulative and consistent. At the end of each EUP, we reflected on our experiences and jointly planned the next project. The projects included a specific focus on mathematics content and its socioeconomic and sociopolitical context. Considering all the EUPs together, the findings confirm that the classroom was a dialogic sphere.
I conclude that Alexander’s (2006, 2015) framework for elements of a dialogic classroom resonates with CME’s vision. CME promotes critical literacy, aiming to provide students with opportunities to relate their learning experience to understanding and challenging oppressive and undemocratic elements in their life-world (Skovsmose, 2011). Taking Skovsmose as a point of departure, I derived the themes of all EUPs from the students’ life-world, which enabled them to contextualise their learning experience within a larger system.

Habermas (1984, 1987) provided a comprehensive and interdisciplinary epistemological and ontological framework (reviewed in Chapter 2) that elucidates conditions under which non-dominating communication could be materialised. He conceptualised this notion as the ‘ideal speech situation’ that enables a consensus reached by non-coercive acceptance of better argument. Kennedy (2009) argued that the realisation of an ideal speech situation requires a community of inquiry, within which inquiry proceeds through dialogue. The EUP sessions in my study confirm Kennedy’s point of view.

The reflections—specifically the students’ reflections—indicated that empathetic understanding, both between student and teacher and among peers, is a necessary element to fulfil the conditions for ‘ideal speech.’ In all EUPs, I observed that empathic understanding played as significant role in developing non-dominating peer communication. Darryl’s journal entry in EUP 2 is relevant here:

As soon as I looked at Edward’s job offer, I figured out coordinates of the intercept point….You know, it was 10 hours that adds up to the same weekly salary when you plug them in both functions….But I did not say anything at first. I waited for my buddies to take time and think it through. Otherwise, I would spoil the moment. Instead I did the kind of loud thinking, just like Mr. Bulent helps us to ask questions to figure out what we need to solve the question.
Darryl indicates that he acted with a sense of responsibility and empathy that comes from being a community. From this incident, I concluded that if a group were not a community that promotes empathetic understanding, peer interactions could easily produce power relations and hierarchies. Darryl’s empathetic approach can be applied to different situations. For example, a student whose mathematics knowledge and skills are above those of the others in a group may tend to dominate others; a student who is a native English speaker may consider non-native speakers less competent. Under these circumstances, reaching a consensus through better argument would fail to meet the conditions for an ideal speech situation.

Having facilitated five EUPs, I conclude that a dialogic approach to teaching and learning mathematics in a high school context, drawing on communicative rationality, can be achieved. It happens when a community of learners is formed, in which the teacher is a facilitator, where peer relations are egalitarian and empathetic, and where learning is collaborative and driven by inquiry. Dialogic pedagogy helped my students develop both functional literacy and critical mathematics literacy and communicative competency oriented towards critical citizenship. Dialogue as pedagogy in CME must enable students to disagree with their peers and teacher; it must consciously and systematically reject imposition and propaganda of any kind when life-worlds and the system are connected.

5.2 Critical Mathematics Education and Collaborative Learning

Collaborative learning in the current study was organised through the formation of small group work and whole-class discussion. As reported in Chapter 4, in each EUP, students reflected on their experiences of group work to improve the overall quality of collaborative learning process; they also acted upon their reflections to transform the classroom into a dialogic one. Our classroom was consistent with
Alexander’s (2015) and Wells’s (1999) description of such spaces. EUPs gave students the chance to negotiate egalitarian collaboration as opposed to competitive learning in learning mathematics. In EUP 3 and EUP 4, the whole class discussed social and political implications of collaboration and competition, which seemed to enable them to make a conscious effort to turn their group work into a more equal collaboration over time. Their reflective journals provided evidence that across all EUPs, they made genuine efforts to improve the quality of peer collaboration.

My observations and students’ reflections confirmed that the quality of collaboration in small group work did indeed improve. From this finding, I conclude that the collaborative approach to learning of mathematics is a continuous pedagogy of praxis. The quality of collaborative learning can always be improved. As Pine (2009) and Groundwater-Smith et al. (2003) indicated, it requires time and systematic effort to establish a classroom environment that fosters collaborative learning.

At the reconnaissance stage, I asked the students whether they would prefer individual, collaborative, or competitive learning. All students without exception favoured collaboration. Students’ journal entries across EUPs indicated that they collectively and consciously developed a culture of respect, inclusion, empathy, and listening to each other’s ideas. My field notes also revealed that EUPs enabled students to improve their content-based communication. As these interactions turned into dialogues oriented towards specific educational objectives, it was evident that students’ learning and empowerment were emergent throughout the process. The students wrote in their journals that EUPs enabled them to correct some mathematical misconceptions, and made them see themselves as an integral part of a community of collaborative learning (Alexander, 2015; Kennedy, 2009; Nystrand, 1997).
As Skovsmose and Alrø (2004) argued, dialogue and collaboration cannot be imposed on students—any meaningful participation in collaborative learning has to be a voluntary action. Findings revealed that it was important for students to reflect on group work experiences in the past to revise their attitudes about collaboration. For example, in EUP 1, students identified power relations as the main obstacle to productive and enjoyable collaboration. Students described ‘group’ experiences in which everyone worked on a different task, or everyone worked on their own while sitting around the same table.

Students also described graded projects in which one member did all the work but everyone received credit. When I asked students their opinions about possible solutions to unequal power relations, they suggested that ensuring equal power for each member would overcome the problem. I had intended to make this very proposal, but the students were faster at becoming agentic participants. After EUP 1, establishing egalitarian collaboration became one of our shared objectives.

With respect to barriers to the collaborative learning of mathematics, Horn (2014) concluded from her ethnographic study that students’ negative experiences with previous group work—due to unequal power relations—may be a serious obstacle. Horn also implied that neoliberal ideology is contrary to collaborative approach. If students are allowed to derive power from their social or cultural status to dominate their peers, truly collaborative learning will not succeed. In that case, ‘Over time, this system may reinforce negative ideas [students] have about themselves as mathematics learners, because they may conclude that their ideas are not valuable’ (Horn, 2014, p. 21). Findings from the current study underscore that students’ agreement on egalitarian interaction was key in establishing collaborative learning.
Findings also supported the claim that collaboration cannot be imposed (Skovsmose and Alrø (2004). Collaboration involves sharing, helping others, and non-dominating interactions—in short, learning with and from each other. But what if students do not consider these values to be virtues? To be proactive and overcome a potential barrier, I held whole-class discussions to provide the students with an opportunity to negotiate educational and sociocultural implications of egalitarian collaboration to enable them to make a conscious decision about their voluntary participation in small group work. For example, in EUP 3, we discussed the productivity of collaboration versus competition in the context of the history of mathematics. The students agreed that collaboration has been much more beneficial and created more prosperity for humanity. In EUP 4, we discussed socioeconomic inequality, the caring community, and the virtue of helping those in need. Based on students’ comments, these whole-class discussions seemed to expand their horizons about collaboration not just in the classroom but in society generally.

The existing literature shows that collaborative learning is difficult to realise within traditional approaches (Alexander, 2006; Kohn, 1992; Rogers & Freiberg, 1994; Smith & McGregor). Horn (2014) noted that students may bring existing power relations to group work; teachers should intervene if they hope to set up truly collaborative learning. Unfortunately, she did not provide any practical suggestions for such intervention.

The CME literature offers a very limited number of classroom-based studies about the dynamics of collaborative learning oriented to promote critical citizenship. Against this background, the current research makes an original contribution. In order to prevent unequal power relations in group work, a cultural and ideological confrontation with neoliberal pedagogy is necessary. If the classroom is to be
transformed into a community of learning, students need to accept, through whole-class discussions, the need for egalitarian interactions. As evidenced in my reflective journal, my intervention to establish collaboration drew on such virtues as mutual respect, sincerity, and transparency. This finding is consistent with Freire (2000), who noted that these kinds of interventions must not involve manipulation, imposition, or propaganda of any kind.

5.2.1 Collaborative versus Competitive Learning

My reflective journal includes a section on collaborative learning:

It was emergent that students embraced egalitarian collaboration as humanising and productive, which helped to transform our classroom into a community of learning mathematics. As EUPs opened a space in the class to talk about sociopolitical issues such as inequality, poverty, unemployment, and homelessness, taking Edward’s story as a point of departure, the students strongly countered neoliberal ideology. (February 12, 2015)

For each EUP, I facilitated whole-class discussions in which we negotiated the implications of collaborative (versus competitive) learning, caring for others, and developing non-dominating peer communication. In EUP 1, students agreed that egalitarian interaction is necessary to establish effective and empowering collaborative learning processes. In EUP 3, students considered mathematics as an outcome of the collaborative work of humankind throughout history.

Although we did not practise competitive learning here, the students criticised competitive learning based on their experiences in other classes. The students’ reflections revealed that they participated more actively and learned with greater understanding because our class was collaborative. Students did not consider their classmates to be obstacles to their success. On the contrary, students said that they developed empathy, transferable skills, and knowledge through collaborative
learning. This finding is consistent with those of Alexander (2015), Kohn (1992) and Rogers and Freiberg (1994), who argued that collaborative learning helps students both academically and socially.

EUPs were not evaluated: students were not rewarded with a grade in the traditional sense. Thus, students’ motivation to help each other did not arise from extrinsic rewards. The source of motivation was the learning process itself. This supports Kohn’s (1992) argument that extrinsic rewards are unnecessary to motivate students—collaborative learning produces a friendly environment and encourages students to establish egalitarian relationships.

5.2.2 Inclusion, Facilitation, and Zones of Proximal Development

In EUPs 1 and 2, we discussed potential causes of exclusion within group work and together established classroom norms. The students identified three reasons for not participating:

- A self-assigned leader may dominate, resulting in less participation for everyone else;
- Students’ cultural or socioeconomic background might be a reason for being shy or withdrawn;
- Students who perceive their mathematics competence as inferior may choose not to participate.

The students’ recommendation for equal power for each member dissolved the first issue effectively. In all EUPs, students ensured that a dominant attitude was unwelcomed in group work. For the second and third concerns, in each EUP, I facilitated whole-class discussions to clarify that each student in our class is a unique and equal member of our learning community regardless of gender, race, mathematics ability, or social, cultural, or economic background. As the classroom gradually transformed into a community of learners, this point became a routine part of our
classroom practice, rather than an intellectual abstraction. The whole-class discussions went a long way towards making group work inclusive and egalitarian.

As we worked to establish democratic peer relationships and positive interdependence, collaborative learning in small group work became more inclusive. All students were welcomed, regardless of their mathematics skills and knowledge or socioeconomic and cultural background. I noticed that students whose favourite subject was not mathematics were still able to actively participate group work. The inclusive nature of the process seemed to enable these students to learn mathematics with and from each other. Thus, collaborative learning processes helped to transform our classroom into a community of learners. This finding is in accord with Kohn (1992), who suggested that collaborative learning is a necessary element of inclusive group work.

As for my role, whenever I answered questions or provided scaffolding, I attempted to act as another member of their group instead of an authority figure. I answered questions with mathematical and logical justifications, but left space for them to disagree with or question my approach, logic, or mathematical argument. The students noted in journal entries that my facilitative attitude motivated them to learn with and from each other without trying to dominate or compete with one another. This finding further supports the ideas of scholars such as Groundwater-Smith et al. (2003) and Rogers and Freiberg (1994), who suggested that a teacher’s facilitative attitude plays a significant role in transforming a traditional classroom into a collaborative community.

Another way of looking at classroom interactions is to use Vygotsky’s (1978) notion of the ZPD. As I commented in my reflective journal, one must always be wary of asymmetrical power relationships:
Asymmetrical power relation is considered in critical pedagogy as the main obstacle to humanising education, dialogue, and freedom to learn. Then I realised that I needed to be proactive about any dynamics of group work that may potentially produce power relations among students or reinforce existing ones. I should be paying full attention to possible emergence of occurrence such as self-assigned leaders or exclusions, when the students work in each other’s ZPD....Especially I decided to be proactive about situations under which peer tutoring may create a vertical relation between a more competent student and a less competent one. A transmission style of education may occur when peers help each other. (November 18, 2014)

Based on my observations of small group work, I concluded that if ZPD was applied to learning mathematics in the absence of an egalitarian community and dialogic classroom, it would run the risk of reproducing power relations inherent in traditional education. As Wells (1999) argued, changes created by ZPD have may transform or ‘reproduce existing practices and values’ (p. 333). I realised that ZPD, in the absence of a dialogic classroom, would lead to an oppressive and dehumanising education as described by Freire (2013).

Therefore, for each EUP, I facilitated whole-class discussions in which the importance of dialogic peer interactions and egalitarian collaboration was emphasised. Students appeared to act upon these reflections as they worked in each other’s ZPD for any given activity. In other words, the ZPD concept became a catalyst for creating a more egalitarian community of learners. The reciprocal process was also evident: As the classroom became more egalitarian, students worked in each other’s ZPDs more effectively. By building their mathematics knowledge on each other’s ideas, thoughts, and problem-solving strategies, they seemed to make more progress than they could on their own. Regardless of mathematics ability, each student asked questions, made comments, and responded to his or her peers’ ideas.
This finding is consistent with Cesar (1998), who concluded that if dialogic peer interaction is achieved in small group work, a more competent peer is unnecessary to create a ZPD.

5.2.3 Empathy and Respect

The findings in the current study revealed that empathy plays a significant role in fostering democratic peer interactions. Bin’s journal entry for EUP 4 is worth revisiting:

I saw that Jennifer’s set of inequalities was wrong, but I did not say anything at first….You know, I did not want to make her feel bad….Instead, I asked questions to help our group to see one variable was missing in the system….I felt good about myself as I did not pose like a know-it-all jerk.

Bin knew instinctively that if he stated the correct answer immediately, it would make him look like a dominating person in the group, and make Jennifer feel bad. He consciously avoided this situation, choosing an empathic way instead. Implicit in Bin’s reflection is that he rejects the transmission style of education in favour of a facilitative style, which was modelled by the classroom teacher. This brings us to another conclusion about empathy. In a truly democratic community of collaborative learners, the teacher must be an empathetic facilitator, not an all-knowing authority figure. Needless to say, this requires a drastic shift from the assumptions of traditional education (Wells, 1999).

5.2.4 Forming Groups

In some EUPs, students formed their own groups. For example, in EUP 5, the students set their groups based on what they wanted to study in college: Those who wanted to study engineering formed one group; those who wanted to be nurses formed another. However, in EUP 3, I organised all the groups. I explained that to
become a community—as opposed to individuals sitting next to each other in a room—we needed to socialise, work together, and get to know each other.

Years of teaching experience had led me to conclude that students usually socialise with those from their own cultural background. At the planning stage of research, I considered this to be an obstacle to setting groups. I wanted students from diverse backgrounds to learn with and from each other.

Accordingly, at the beginning of each EUP, I explained that what unifies us as human beings is much stronger than what allegedly separates us, such as religion, race, or nationality. The students’ journals and comments indicated their desire to consider our classroom a community. Although the existing literature, such as Kohn (1992; 1999), offers valuable ideas about collaborative learning in general, it provides few practical insights into how to organise group work. I ensured that each group included boys and girls, as well as students from different racial, religious, and ethnic backgrounds. For example, I did not put all the African American students in one group and all the Asian students in another.

These mixed groups seemed to work well. Overall, students worked in harmony; they respectfully listened to each other and responded to each other’s concerns, suggestions, and ideas. Each group independently coordinated its work to complete any given EUP. The groups had some content-based questions, such as setting up or solving linear inequalities, for which I provided scaffolding to enable them to continue independently. I did not assign any member of group a specific task to complete. On the contrary, the groups were free to decide for themselves how to complete the project.

Prior to each EUP, I presented the idea that students need skills such as developing initiatives and democratically working with others to frame an action plan
for their projects. Such cycles of action-reflection reflect a fundamental goal of CME, which aims to help students become critical citizens. As others have noted (Aguilar & Zavaleta, 2012; Ellis & Malloy, 2007; Hannaford, 1998), mathematics education for critical citizenship should help students develop skills and attitudes such as working with others and exercising collective initiative.

In conclusion, findings from EUPs support the distinction between the collaborative learning of mathematics, which draws on communicative rationality, and traditional approaches, which draw on instrumental rationality. Based on classroom discussions, student journals, and my reflective journal, it can be concluded that there is a dialectic connection between the collaborative learning of mathematics and being an egalitarian community of learners. Collaboration is characteristic of an egalitarian community of learners. Collaborative learning grows along with facilitative teacher attitude, dialogic peer interactions, and inquiry-based learning. One cannot be separated from the others. In the micro community that the classroom became, as students developed collective initiatives to complete each EUP, they experienced nothing less than democracy.

5.3 Critical Mathematics Education and Inquiry-Based Learning

Each EUP included inquiry processes that had two interrelated dimensions: content-based inquiry and critical mathematics literacy. I elaborated on inquiry in my reflective journal:

I noticed that there is a dialectic connection between collaborative inquiry and the dialogic classroom. In this research study, the inquiry processes took distinctive forms based on concerns that CME brings to mathematics education. EUPs connected functional literacy (content knowledge) to critical mathematics literacy. Inquiry processes were integrated into each EUP in two ways, such as inquiry with mathematics and inquiry into mathematics (Staples,
Themes of EUPs that were derived from students’ life seemed to enable the students to connect their personal experiences of sociopolitical issues to macro level sociopolitical problems. (April 14, 2015).

Manconi et al. (2008) suggested that inquiry can be framed within the domains such as process, content, strategy, and context. I analyse the findings with respect to these four properties and then discuss obstacles to inquiry-based mathematics education.

5.3.1 Inquiry as Process

My observations and students’ reflections showed that the inquiry in each EUP was a process through which the students seemed to learn ‘generalizable process skills that are specific [to high school mathematics] but that carry broad transferability across many subject-matters’ (Manconi et al., 2008, p. 249). The students were the driving force of the inquiry process in each EUP.

In their journals, students reported that they developed transferable skills such as questioning techniques, listening to each other’s ideas, making suggestions or comments, respecting classmates, trying to be open-minded, being inclusive, interacting without dominating each other, and being empathetic. As a consequence of cycles of reflection on their experiences, the quality of their engagement in inquiry increased. It is encouraging to compare this finding with Staples’s (2007) finding that ‘students’ participation and the opportunities to negotiate meanings seemed to influence their interpretations and understanding of practices in ways that facilitated their participation ‘ (Staples, 2007, p. 37). Across EUPs, I consistently observed that as the students realised that the research was conducted not on them but with them, they took active ownership of the collective inquiry process.
I noted in my reflective journal that the process of collaborative inquiry enabled students to improve their content knowledge by changing their perception of mathematics problems:

The students in each EUP articulated in their journals that working collaboratively enhanced their ability to integrate inquiry into word problems. In the processes of inquiry, as the students worked in the each other’s ZPD, they seemed to exchange ideas and comments and revise their initial approaches to develop and improve a collective investigation. The students indicated that this collaborative inquiry process helped them to recognise some of mathematical misconceptions they had. Some of the students articulated that these cycles of inquiries helped them to realise that there were multiple ways of arriving at similar conclusions or answers and open-ended mathematics problems may have multiple answers. (April 19, 2015)

The students noticed that mathematics problems do not always have to lead to a set of absolute/correct answers; answers may take different shapes depending on one’s ideological, political, and ethical perspective. For example, in EUP 2, the students solved the word problem both from an employee’s perspective and an employer’s perspective, achieving two different answers. EUP 4 was another example: although there was a single optimal numeric solution in EUP 4, each group arrived at it in a different way. I observed this point over time: The inquiry process helped students develop a sense of intersubjectivity in knowledge construction.

5.3.2 Inquiry as Content

EUPs required students to apply their numerical fluency, procedural knowledge, and conceptual understanding of mathematics to solve word problems. In this respect, the inquiry process helped students improve their content knowledge. My reflective journal contains the following note:
I observed throughout EUPs that collaborative inquiry processes improved the students’ conceptual understanding of content knowledge. For example, the students in EUP 4 applied systems of inequality to optimise limited resources. As the students applied procedural knowledge of multipart function to Edward’s job application, they seemed to understand the concept of multipart function better than they did in EUP 2. (April 20, 2015)

Thus, an inquiry-based approach to learning mathematics, along with collaboration and dialogic pedagogy, seemed to help the students develop conceptual understanding of mathematics content. This finding is consistent with that of Manconi et al. (2008): ‘Inquiry provides an opportunity to teach content at a deeper level and to apply knowledge’ (p. 250). The finding also substantiated one of the objectives of CME, which is to ‘honestly and openly address…the instrumental life goals of the learners themselves, both in terms of needed skills and passing exams’ (Ernest, 2002c, p. 6). The inquiry processes in the current research served this goal. EUPs helped students improve their procedural knowledge, numerical fluency, and conceptual understanding of subject matter to be successful on traditional exams. In this sense, EUPs served both functional and critical literacy. The point is underscored in my reflective journal:

At the reconnaissance stage, I anticipated that the students’ lack of prerequisite-content knowledge could be a potential barrier to effective and inclusive collaborative inquiry. Therefore, prior to each EUP, I covered the mechanical aspects of related mathematical content to ensure each student in the class could comfortably solve mechanical problems. For example, we covered linear functions-equations prior to EUP 1; we processed graphical and algebraic analysis of function prior to EUP 2; systems of inequality and optimisation prior to EUP 4, and exponential properties prior to EUP 5. Being proactive seemed to work. No student seemed to struggle with content knowledge in EUPs. (March 18, 2015)
This outcome reflects Aulls and Shore (2008) observation that if students lack prerequisite content knowledge, they will not be able to participate in collective inquiry. I also observed in my reflective journal:

During EUPs I noticed that the students were still improving their content knowledge and correcting their misconceptions. In this sense, the process of inquiry also helped the students to assess their content knowledge. I realised that providing students with prior knowledge is necessary, but this is not a linear process. The students seemed to continue improving their content knowledge while applying their knowledge in the contexts of EUPs. (March 21, 2015)

As my journal indicates, content knowledge and application cannot be separated from each other. Collaborative inquiry created an intellectually stimulating atmosphere where the students were collectively engaged in enhancing their conceptual understanding of the content matter and meaning-making process. The students had to pose questions, explain suggestions, ideas, or arguments, and respond to each other’s conclusions. For example, in EUP 2, finding the intercept point(s) of two functions was a mechanical task. However, as the x-coordinate of the intercept represented the number of hours per week that Edward worked at a part-time job, it became a meaningful process.

5.3.3 Inquiry as Strategy

Although I designed and facilitated each EUP, I did not assign students to specific tasks, nor did I provide them with any prescribed method of inquiry. In small groups, the students organised their own process, discussing how to handle, for instance, the division of labour and presentation of their final work. I observed that students gradually developed three strategies of inquiry. First, as all EUPs were designed as open-ended inquiries that challenged the traditional assumption that there
is always a single and absolute answer, students gradually formed critical and strategic thinking skills to overcome the uncertainty inherent in open-ended problems. Second, the students collaboratively decided what to do next—a proactive approach that improved their organisation and coordination skills. Third, although there was no antagonistic behaviour in class, the students had disagreements from time to time; they appeared to develop strategies to resolve these conflicts. I realised that egalitarian collaboration and dialogic interaction played a vital role in resolving these disagreements and reaching joint decisions (Pine, 2009).

Thus students—instead of following a specified method—developed their own strategies for dealing with uncertainty, timetabling, and conflict, which allowed them to apply mathematics to various assigned problems. The findings in the current research are in line with Barfurth and Shore (2008), Manconi et al. (2008), and Wells (2009), who suggested that inquiry-based education enables students to develop and exercise problem-solving strategies.

5.3.4 Inquiry as Context

My reflective journal addresses inquiry as context as follows:

The data in my research suggests that context of inquiry is another distinctive element of an inquiry-based learning within CME. The context of each EUP enabled the students to make connections between their life in the classroom (life-world) and a larger society (system); they, as a whole class, seemed to develop bottom-up responses to top-down imposed neoliberal ideology and pedagogy. (March 24, 2015)

This finding corroborates Manconi et al. (2008), who concluded that context of inquiry should help students challenge dominant views in a given society and ‘construct a new understanding of the world’ (p. 250). In light of this goal, I designed each EUP as a project-based inquiry with a theme derived from the students’ life-
world to enable them to relate their life in classroom to a larger sociopolitical system. I contextualised each EUP as a socioeconomic and sociopolitical issue. To promote critical literacy, I attempted to combine inquiry into mathematics and inquiry with mathematics. My inspiration was previous scholars who have discussed the importance of critical mathematical literacy Frankenstein (1983, 1990), Gutstein (2006), Skovsmose (2011), Skovsmose and Greer (2012), Alrø, Christensen, and Valero (2010).

CME is unique in linking functional literacy with critical mathematics literacy. For example, in EUP 2, students used their functional literacy such as graphical and algebraic analysis of functions to develop a piece-wise function of Edward’s job offer. However, as they extended the inquiry to determine which part of the function favours whom, they seemed to develop and exercise critical mathematical literacy.

As revealed in their reflections, themes of inquiry in EUPs helped students develop bottom-up responses to top-down imposed neoliberal ideology and pedagogy. I concluded that a connection between the students’ life-world and the system could be constructed through the contextualisation of standardised curriculum content into word problems. Again referring to EUP 2, the students tried to help Edward with the job offer made by his employer. Using mathematical concepts such as linear and quadratic functions and graphs, systems of equations, and piece-wise function, the students developed two multipart functions to model the job offer: one was in favour of Edward as employee, and the other favoured his boss. This context provided the framework for a whole-class discussion to understand our society as a class society. The students realised that we live in a class society; however, if we have critical
mathematics literacy, we can make rational decisions and not be manipulated by others.

The findings suggest that EUPs were effective in connecting functional literacy to critical literacy. However, it is worthwhile to briefly revisit EUP 3. I had hoped to invite students to an open forum to discuss the formatting power of mathematics (Skovsmose, 1994, 2011) and the historical development of mathematics. However, it became apparent that the context of inquiry and the objective of the lesson were too broad to materialise within two block periods. As a result, we only touched the surface of our objectives. I realised that the history of mathematics must be carefully integrated into each unit throughout the school year. However, the lessons learned in EUP 3 could provide the foundation for future studies to reframe the history of mathematics from the perspective of CME.

5.3.5 Obstacles to Inquiry

In my reflective journal, I described four main obstacles to inquiry-based learning in CME:

First, a lack of ready-to-use curricular materials that are compatible with CME. Second, the inquiry-based approach is time-consuming, which made it difficult to integrate into standardised and scripted curriculum. Third, the standardised curriculum limited the content flexibility of EUPs. Finally, I received no support from admin or my colleagues. (April 15, 2015)

Inquiry-based learning of mathematics is time-consuming. Aulls and Shore (2008) also noted that inquiry-based learning requires much more time than traditional approaches such as direct teaching. As the neoliberal educational changes in the U.S. mandate covering certain curriculum content within a certain time interval, inflexible use of classroom time becomes a real barrier to inquiry in a mathematics class. I could have used a couple of more days for each EUP to expand our inquiry
further. But due to time pressure, we only had two days for each EUP. This finding suggests that mathematics teachers who want to practice CME must allow for time constraints resulting from neoliberal educational impositions.

A lack of materials consistent with the concerns of CME was one of the major obstacles I encountered. Therefore, I had to develop EUPs on my own. As indicated in the literature review, a project-based inquiry in CME should have a certain ethical and philosophical structure, as Freitas (2008) argued. Skovsmose (1994, 2011) also pointed out that critical mathematics education must distance itself from the official curriculum. This was a vital point in my research, as I aimed to make small openings for students to negotiate implications of imposed neoliberal educational implementations and counter neoliberal hegemony.

Recognising this issue, Freitas (2008) suggested that revising word problems in mainstream textbooks could be the solution to the lack of curricular material for CME. Having developed my own EUPs, I concluded that, to some extent, revising word problems in mainstream textbooks could be helpful to integrating inquiry into mathematical content. However, I also realised that CME must be based on the context and themes derived from students’ life-world, and from current social, political, and economic issues at the local, national, and global levels. I made some entries in my reflective journal on this matter:

I wish that there were a network of educators—Internet-based or otherwise—who are dedicated to CME both at the national and international level, so that I could take advantage of teachers’ ideas and lessons that aim at functional and critical literacy in math….It would be great to look at an inquiry project of a math teacher from India, China, or elsewhere in the world. It would be great for me to share my ideas with other math teachers and receive feedback to improve my inquiry projects. (February 12, 2015)
As stated in my reflection, there is a need for ready-to-use inquiry-based projects. The current research provides a solid framework for developing project-based inquiry to counter the hegemony of neoliberal pedagogy. In this sense, EUPs can be considered an original contribution to CME. Furthermore, a network of critical mathematics teachers would enhance the quality of CME practices and provide a forum in which ethical, pedagogical, and philosophical aspects of project-based inquiry could be discussed.

However, the current findings also show that anyone who wishes to develop an inquiry-based project must take certain practical aspects into account. First among these is that mathematics teachers, at least in the U.S., must deal with Common Core State Standards (CCSS). Other contextual issues include student and local community characteristics, as well as current social, political, and economic matters (Schneider, 2015). The advantage of deriving themes of inquiry from students’ social surroundings is that motivation and level of participation may be enhanced (D. L. Ball et al., 2005).

Certain outcomes of the current research resonate with previous studies. For example, Staples (2007) claimed that ‘despite [the] compelling successes [of the inquiry-based approach], traditional models of instruction, particularly at the secondary level, still dominate the educational landscape’ (p. 1). However, the studies mentioned by Staples tended not to probe the education system as the main source of dominance, but instead focused on the dynamics of isolated classrooms. Thus Staples (2007) concluded, ‘Teachers find it very challenging to organise and support student participation in these discourse-intensive practices that centralize students’ ways of thinking…. [The] staying-power of traditional models is mysterious’ (p. 1).
Findings in my study contradict Staples’s (2007) conclusion that the reason behind the dominance of traditional instruction is ‘mysterious’. It is no secret that the transmission style of U.S. education featuring memorisation, recitation, and standardised assessment has resulted from neoliberal educational implementations since the 1980s (Giroux, 2012; Hill, 2008). In her analysis, Staples identified the dominance of direct teaching as an obstacle to inquiry-based learning without probing its root causes and political background: ‘The students had experienced traditional models of teaching and learning in middle school, and many reported that they did not enjoy mathematics nor math class…which also creates challenges for participation’ (p. 34).

My research, however, confirms Staples’s (2007) point that students enter the classroom with already established values and attitudes towards certain ways of learning mathematics within the existing system. However, emergence of this obstacle has not been a spontaneous process. The top-down imposed neoliberal educational implementations across the nation have shaped life in the classroom. At the reconnaissance stage of the current research, I realised that without acknowledging and challenging this fact, it would have been very difficult to implement collaborative inquiry in a mathematics classroom under the current system, dominated as it is by neoliberal ideology.

To cope with this potential barrier to inquiry-based learning, I facilitated whole-class discussions to turn our classroom into a public forum for students to discuss philosophical and educational implications of inquiry-based collaboration and dialogic learning. Through whole-class discussions, the students had opportunities to negotiate traditional education and critical education, based on their experiences in other classes and their experience during EUPs. As the students realised that this
study took a critical approach to mathematics education and that they were equal partners in the process, they enthusiastically participated in EUPs. As an amendment to Staples’s (2007) conclusions, the findings in the current research suggest that inquiry in mathematics education should enable students to critically reflect on their past learning experiences; inquiry-based learning should be collaborative and dialogic in order to cultivate critical literacy and promote critical citizenship.

Findings in the current research revealed that the inquiry process in CME is dialectically connected with egalitarian collaboration and dialogic interactions in the classroom. This makes sense, as inquiry is a collective activity that requires a community of learners. Inquiry can be framed as real-world problems to contextualise content learning of social, political, and economic issues. The context of inquiry enables the connection between functional and critical literacy in mathematics. Themes of inquiry derived from students’ life-world help them understand and challenge oppressive implications of the existing social, political, and economic system on multiple fronts; whole-class discussions can be structured as self-assessments to keep improving the quality of the process.

5.4 Critical Mathematics Education and Citizenship in the Neoliberal Era

This section is divided into three parts. The first focuses on life in the classroom, where the classroom is understood to be a micro community and democracy is experienced in the form of dialogue and collaboration. The second part concerns critical literacy in relation to democracy and citizenship, and the third discusses some limitations of CME in this study. This section aims to address the following question, which was one sub-question of the current research:
In what ways can collaborative and dialogic mathematics education be facilitated to help students to become critical citizens?

5.4.1 Democracy in the Classroom

Inspired by Freire (1998), I developed EUPs with a goal to create a dialogic classroom environment for students to develop skills, values, and attitudes to become critical citizens. Pursuing this goal underscored that life in the classroom must be democratic in order to cultivate democratic values. Results from the reflections revealed that the combination of collaborative learning, inquiry-based approach, and dialogic pedagogy transformed our classroom into an egalitarian community of learners. In the following excerpt from my journal, I reflected on this point:

As I had anticipated, critical participatory action research (CPAR) resonated well with objectives of CME….Exceeding my anticipation, integrating inquiry into collaborative learning and dialogic pedagogy has turned the learning process into a democratic forum and our classroom into a dialogic and facilitative environment where the students established and enjoyed a thick version of democracy within the classroom; the students actively participated in decision-making and inquiry-based collaborative learning process. Egalitarian and respectful peer interaction and horizontal teacher-student relationships became a self-regulatory process over time and turned our classroom into a community of mathematics learners. (February 25, 2015)

The EUPs created a communicative space for the students to push outside their comfort zone through inquiry and build knowledge on each other’s comments, ideas, and thoughts. While learning mathematics, the students seemed to develop and exercise skills and values, such as listening to peers in the group, respecting different points of view, and empathising with classmates. This finding is in line with those of Ellis and Malloy (2007) and Hannaford (1998), who argued that mathematics
education oriented to promote critical citizenship should help students to develop such skills and values.

Across EUPs, through the cycles of action-reflection, the students seemed to make conscious attempts to transform the class to a dialogic classroom. Dialogic and collaborative learning were the defining elements in the process of the class becoming a community. This finding supports other research which has shown that education oriented towards the thick version of democracy and critical citizenship should promote dialogic learning and egalitarian collaboration (Gandin & Apple, 2002; Hyslop-Margison & Thayer, 2009). However, as my reflections and students’ journals indicate, the collective learning in our class was not a matter of collective compliance. Each member in the group—actively and with equal power—participated in decision making.

The EUPs together revealed three elements that enabled our classroom to become a democratic community. First, students developed a sense of belonging to the classroom community as well as a sense of responsibility and collectivism. This was an important step in countering neoliberal pedagogy, because, as discussed in Chapter 2, neoliberal ideology advocates a market-based citizenship that promotes individual consumerism and omits communal solidarity (Hyslop-Margison & Thayer, 2009; Westheimer, 2015).

Second, collaborative learning enabled students to actively participate in knowledge construction and the meaning-making process. The students became agentic participants instead of passive consumers of knowledge. This way of participation can also be considered a means of countering neoliberal pedagogy and citizenship. This result is consistent with that of Freire (2000) and Gandin and Apple
who argued that an educational process that envisions students as passive receivers of knowledge is oppressive and anti-democratic.

Third, in all EUPs there was a dialectical connection between individual transformation and transformation of the whole classroom. As the students did not see their peers as obstacles to their learning and success, they genuinely strived to learn from and with each other, resulting in both individual and communal growth. The individual growth in the current research can be considered a negation of the self-concerned individualism portrayed in neoliberal pedagogy (Abdi & Carr, 2013; Kohn, 1992; Rogers, 1995; Wells, 1999). In other words, the findings here show that if a classroom is democratic, then individual students’ growth and the growth of the entire classroom resonate with one other.

In addition to these three implications of EUPs, the fact that small group work was inclusive contributed to the process of the class becoming a community of mathematics learners. The students’ proposal for non-dominating peer interactions in EUP 1 materialised inclusiveness as practical wisdom instead of an intellectual abstraction. On this point, Dewey (1916) indicated that attitudes and values could not be imposed: ‘The required beliefs cannot be hammered in; the needed attitudes cannot be plastered on’ (p. 11). Complementing Dewey’s point, Kohn (1992) suggested that students should be guided to internalise certain ethical values in order to make moral-based decisions. Whole-class discussions seemed to provide the communicative space necessary for students to negotiate and consciously embrace certain values and virtues oriented towards being an egalitarian community of learners.

Hyslop-Margison and Thayer (2009) elaborated on this point: ‘Unless the desired behavioural traits are fully internalised into the value system of students, the character development objectives of thick democratic citizenship are unlikely to be
realised’ (p. 115). Based on student reflections, I realised that our whole-class discussions helped them revise and substantiate their views and values to make group work inclusive in a deeper, more meaningful way. As I wrote in my reflective journal,

I realised that our emphasis on values like inclusiveness, equality, non-dominating peer interactions brought about some practical results. In my teaching experiences prior to this study, I had always had students with grades of Fs and Cs, who seemed to be alienated and outcast. However, after the second unit (EUP 2), the students had either As or Bs….I concluded that egalitarian collaboration and inclusiveness notably increased equal distribution of mathematics education and that reflected on the students’ grade in traditional assessment as well. I think this result itself can be regarded as one indicator of our classroom being a democratic space. (February 24, 2015)

It can be concluded that the class’s being an egalitarian and inclusive community was a strong indicator that the students and I established necessary conditions for thick democracy; thus our classroom became a micro society. Our reflective actions (praxis) created small openings to democratise classroom life.

5.4.2 Democracy in a Form of Dialogue

In EUP 5, I co-constructed an exponential model to calculate interest rate. As noted in my reflective journal, the process was organised as follows:

I facilitated this learning process by making numerical patterns visible for the students to make generalisations to construct the formula. The students built on each other’s comments and ideas and made most of the calculation by their graphing calculator. I structured the process based on mathematical axioms, properties, and theories. I did not continue to the next step without receiving the students’ approval. In other words, when I justified my steps in this process, I drew not on my authority as the teacher, but on mathematical and logical verification. (April 12, 2015)
The quotation reveals that high school mathematics content can be dialogically structured and that such structure promotes critical citizenship. I was particularly inspired by Almeida’s (2010) connection between mathematics and democracy, and Habermas’s (1987) theory of the ‘ideal speech situation’. According to Almeida (2010), just as government in a democracy must justify its actions by referring to the Constitution, mathematics teachers must justify their teaching through mathematical logic, axioms, and principles.

Habermas (1987) made a similar but broader case that non-dominating and democratic communication requires a verification process based on better argument, not greater power. As I attempted to apply this proposal throughout EUPs, I embraced my role as facilitator rather than as authority. Students’ journal entries indicate (Section 4.2.6) that they considered the classroom a place where peer relations were egalitarian, the learning process liberating, and the classroom environment democratic.

5.4.3 Mathematical Literacy and Citizenship

One of the main objectives of EUPs was to promote critical mathematical literacy by giving each student an opportunity ‘to examine one’s own and others’ lives in relationship to sociopolitical and cultural-historical context’ (Gutstein, 2006, p. 5). Being engaged in a dialogic pedagogy enabled students to interrogate social, economic, cultural, and political matters that affect their lives. In my journal, I reflected:

EUPs seemed to help us to create communicative spaces for the students to discuss standardised assessment, in EUP 1, and develop a bottom-up response to top-down neoliberal impositions. In EUP 2, we challenged the dominant view of mathematics that problems in mathematics have always a single correct answer and mathematics is politically neutral. In this context, we also
discussed the importance of critical mathematical literacy for working-class citizens to make educated decisions to identify and protect their best interests. In EUP 3, we held an open forum to discuss collaboration and competition in terms of their implications for the common good and well-being of world citizens. In EUP 4, we approached community service and volunteer actions to counter neoliberal tenets in those domains. In EUP 5, we questioned student loan debt crisis resulting from neoliberal educational policies and came up with foundational and short-term proposals to remedy the situation. (April 26, 2015)

The quotation above reveals a point that is consistent with the ideas of Nagda, Gurin, and Lopez (2003), who suggested that through action and reflection, ‘Students can develop a more abstract understanding of social life….The permeable boundaries between the classroom and the larger world can allow students to continually reflect on their in-class learning in relation to the outside, and vice versa’ (p. 169). Students connected their life in the class to that of a larger society by developing bottom-up responses to neoliberal top-down impositions. In this sense, EUPs served critical mathematical literacy and thus emancipation: each EUP, in addition to content-based empowerment, provided small openings to question the root causes of injustice and oppression.

EUPs provided evidence of the connection between critical mathematical literacy and critical thinking. For example, EUP 2 encouraged students to discuss labour relations and socioeconomic inequality in class-based societies. The students concluded that critical mathematics literacy is needed to know, communicate, and defend your best interests. Otherwise, you would be vulnerable to distortion and manipulation, and possibly at the mercy of employers or others who are in a power position. To arrive at those conclusions, critical mathematical literacy involved critical thinking.
However, the term ‘critical thinking’ is itself problematic, as it is also part of neoliberal discourse. Therefore, it is important to distinguish between critical thinking based on communicative rationality and critical thinking based on instrumental rationality. According to Giroux (1994), neoliberal ideology drew on instrumental rationality to empty and redefine critical thinking in market terms. In neoliberal pedagogy, critical thinking comprises a set of analytic thinking skills but no ethical or political orientation (Giroux, 1994). Critical thinking is considered only as individual action. Hyslop-Margison and Thayer (2009) explained: ‘These [neoliberal] critical thinking constructs promote technical [instrumental] rationality by encouraging students to address problems from a limited perspective that ignores wider workplace, labour market, and socioeconomic issues’ (p. 27). In contrast, critical pedagogy takes a wider perspective, effectively transforming critical thinking into emancipatory praxis.

In EUP 2, students commented that Edward’s story was interesting but unrealistic. Reflecting on their part-time job experiences, the students concluded that no employer would give such a choice to an employee, for several reasons: First, there is an asymmetrical power relation between workers and employers, and there is no democracy at workplace that would lead to such an option for workers. Second, if we compare how much money an employee’s labour brings to a company and the employee’s salary, we realise that workers receive a small fraction of the money earned by the company. Therefore, from the students’ perspective, the weekly salary in Edward’s job offer was incongruent with reality of the current labour market. In other words, the students were engaged in critical thinking that draws on communicative rationality.
Based on the students’ reflections, I realised that it is important to identify the neoliberal version of community volunteer service in order to design open-ended projects to counter individually responsible, consumer-based citizenship. Therefore I designed EUP 4 by putting the concept of ‘false generosity’, into perspective to reclaim community service volunteer involvement for thick democracy and justice-oriented citizenship. As I wrote in my journal,

As I was designing this project, I exercised caution to avoid neoliberal pitfalls. I realised dealing with the notion of community service volunteer in educational context was a delicate issue. In EUP 4, the goal was to provide the students with an opportunity to negotiate democratic values such as solidarity and generosity in the context of helping homeless people and debate the root causes of poverty and homelessness in the U.S. The students’ arguments and emotional responses against neoliberal tenets were way beyond my anticipation. In the context of Edward’s story, the students commented on participation in voluntary organisations to help homeless people and contributed a reflective discussion to unpack social, economic, and political structures that produce homelessness and poverty in the first place. (February 25, 2015)

The students in EUP 4 used functional literacy—mathematics content knowledge—to calculate the optimal number of rentals. They also debated Edward’s possible motivation to participate in homeless shelter organisation. Had we finalised the project at that point, EUP 4 would have served only functional literacy and promoted personally responsible participatory citizenship. In that case, the project would not have avoided the pitfall of false generosity.

However, as noted in Section 4.2.4, we continued the project by discussing, as a whole class, questions such as ‘Why do social problems such as socioeconomic inequality, homelessness exist in our society? Is success/failure an individual or a social matter? Do we need values such as caring for others and solidarity or self-
concerned individualism to establish and sustain a democratic and just society?’ In their response, students radically challenged the premises of neoliberal ideology. The questions seemed to create a vivid intellectual and communicative ambiance that resulted in a dialogic classroom and promoted justice-based participatory citizenship (Orlowski, 2012; Westheimer, 2015).

EUP 4 was aimed at serving critical literacy and promoting democratic citizenship to counter consumer-based neoliberal citizenship. High school students in the U.S. are often involved in community volunteer services. However, they tend to consider such service as a personal investment to help build a strong resume for their college application. A consumer-based neoliberal pedagogy discourages students from perceiving community service in terms of citizen responsibility and solidarity (Brown, 2015; Lund & Carr, 2008).

Similarly, Westheimer (2015) pointed out that participation alone does not lead to a justice-oriented voluntary involvement. Instead, Westheimer emphasised the significance of critical literacy for justice-oriented involvement in community volunteer services. Freire (2000) also indicated that a community volunteer service or any civil organisation aimed at helping people in need should avoid the pitfall of false generosity. Westheimer’s (2015) example of students’ involvement in a food drive clarifies this concept. If students participate in a food drive but are not provided with the necessary tools to investigate the root causes of poverty and hunger, it actually helps reproduce the existing system while giving students the illusion that something beneficial is being done.

5.4.4 Neoliberal Hegemony and Mathematics Education for Critical Citizenship

The potential promise and limitations of CME in the classroom were presented in Sections 5.1, 5.2, and 5.3. However, the findings from the current
research also identified certain obstacles to CME. Two stand out. First, neoliberal educational implementations affect the entire school atmosphere. The second is a limitation that concerns the community of critical educators.

During EUP 3, the principal visited my classroom to do a formal evaluation, and expressed his concern that my lesson was not aligned with the standardised curriculum. I responded to the principal’s question by explaining the importance of the students developing insight into the historic dimension of mathematics. The principal later told me in person that I should have covered nothing but standardised curriculum in my lessons. This particular situation did not result in any serious consequences—only a mild warning. However, it could have negatively impacted my professional evaluation and even threatened my job security. This event made me anxious for the rest of the project: I kept asking myself, What if the principal shows up during a discussion of poverty, homelessness, or the student loan debt crisis? I would have to provide a standardised learning target for the lesson! I had no short answer for the question, as the Common Core State Standards do have not a single reference to democracy or citizenship. In that case, I would have violated the administrator’s expectation twice, and a more serious consequence would be likely.

I took the risk, but limited whole-class discussion to one day for each project. My journal entry reveals my anxiety:

Practising CME in a classroom colonised by top-down imposed neoliberal pedagogy necessitates a course of action to create small openings where bottom-up responses could be initiated. I realised that creating small openings was more risky than I had anticipated at the very beginning. Neoliberal education system has its own version of pedagogy, democracy, and citizenship that shrinks space for any other alternatives. Under these circumstances, practising CME became more difficult and risky than ever. On the one hand, my commitment to CME provided me with moral energy and hope to say, 'I
should continue subversive teaching no matter what.’ On the other hand, I needed to keep my job to be able to teach. This is a more delicate and challenging issue than I thought. (April 12, 2015)

A year after writing this journal entry, I still teach full-time at the same school, but these management and control activities have intensified. Now we have an instructional coach, who is supposed to teach all teachers how to apply ‘best practices’ to develop and teach standards-based lessons. This finding is in line with Apple’s (2000a) observation: ‘Not following these specified appropriate methods puts the teachers at risk of administrative sanctions’ (p. 70). The current research confirmed the difficulties of practising subversive teaching in the neoliberal era.

While I was conducting this project, I attempted several times to discuss my experiences during staff meetings and professional development days. Each time, the school administrator rejected my request by claiming that the topics of discussions were inconsistent with their agenda and, therefore, would be disruptive. This situation was demoralising and discouraging, as I stated in a journal entry:

I was so disappointed that I was not allowed to share my research with my colleagues! Our administrators were engaged in public relations of ‘classroom-based research’ sponsored by big corporations and think-tanks. But they were not interested in a research study conducted in their own school. I cannot blame individuals in the admin positions; they were expected to follow a scripted agenda mandated by the system, which exploits the concept

36 As part of neoliberal educational changes (running schools like business), every public school in the U.S. now has instructional coaches, a position which mirrors the managerial position in a business. Further information can be found on many websites, including http://edsource.org/2015/classroom-coaches-critical-as-teachers-shift-to-common-core/73730. Practising CME a year later is much more difficult and risky as the colonization of life-world has infused every sphere of school life. For example, if I were doing my research this year, 2016, I would have had to justify whole-class discussions to my instructional coach, who has already visited my class 12 times so far this year to ensure that I am following the scripted curriculum and doing test-solving sessions to prepare students for standardised assessment.
of research as an apparatus to justify market-driven changes in education. (March 12, 2015)

I was aware of the situation that educational ideas and thoughts that are critical of neoliberal educational policies and implementations are not welcomed. However, my original thought was that given that standardisation movement in education has been presented as a research-based ‘best practice,’ the school district or at least the school administrator would be interested in the initial findings of my research as it was conducted in our school. On the contrary, I was not allowed to share my research. My reflections confirmed Apple’s (2000a) conclusion that the neoliberal managerial approach is ‘not based on trust, but on a deep suspicion of the motives and competencies’ (p. 70). I received no feedback or recognition for inquiry-based collaborative and dialogic teaching of mathematics in my class.

As indicated earlier, my research was hampered by a lack of curricular materials and professional development opportunities. Designing EUPs on my own was a time-consuming activity that intensified my workload. This issue cannot be blamed on neoliberal colonisation. However, the CME community as a whole has to face the problem as it limits the practicality of CME in classrooms. As I commented in my reflective journal,

Studies in CME and critical pedagogy do offer very limited practical insight for their classroom application….I wish that there were similar studies done by other math teachers or researchers…that I could use to develop my own projects….What can be done? I think there could be a network of critical math teachers perhaps internationally to exchange ideas and classroom experiences….This would challenge critical educational theorists who undertake theoretical studies in critical pedagogy and CME to focus on life in classrooms. It would address this internal issue by restoring the dialectical

37 By CME community, I mean educators and writers who are committed to critical pedagogy and produce academic work on critical (mathematics) education.
connection between theory and practice as well as opening up critical teachers’ vision to find ways of practicing CME and critical pedagogy under restrictions of standardisation movement. (March 10, 2015)

In the process of developing my own EUPs, I noticed that this issue reflects more than a lack of curricular material. Having reviewed literature in critical pedagogy and CME, I realised that researchers from these communities have no clear information about teaching and learning practices in high school classrooms in the U.S. For example, as indicated in Chapter 2, the CME literature includes very few studies about CME for democracy and citizenship—most of the research is theoretical. As these theoretical studies are produced without feedback from life in classrooms, they become, by necessity, theories about theories.

Due to a lack of dialectical connection between theory and practice, the language of critical pedagogy and CME becomes abstract and to some extent opaque. I noticed that critical educational researchers argue that neoliberal hegemony has eradicated possibilities of an education that would promote a thick version of democracy and critical citizenship. However, at the same time, they enthusiastically champion CME and critical pedagogy as more important than ever. But if neoliberal ideology has eliminated the possibility of critical pedagogy and CME, the recommendation to practise it would seem to be nothing other than empty rhetoric.

A leading figure in critical educational research, Michael Apple (2013), recognised this problem, arguing that critical scholars ‘have not been sufficiently connected to the actual realities of schools and classrooms’ (pp. 50–51). A researcher in CME, Ernest (2010) used stronger language when he questioned academic work. He argued that scholars who write articles on CME without having a connection to classroom experiences actually are ‘complicit with the [neoliberal] system’ that they are so critical of (p. 82). Blacker (2013) agreed, claiming that scholarly articles
written by critical educators and theorists may improve the writers’ careers, but they have no impact on resisting neoliberal colonisation in education. Thus, there is widespread agreement that the gap between theory and practice presents a serious limitation for CME. I hope that the current research goes some way to bridging it.

To the extent that it does, this study opens new domains for research aimed at expanding CME from the classroom to school and community to counter neoliberal hegemony and generate hope in the possibility of a more just and rational world.

5.5 Conclusion

CME can be implemented through inquiry-based collaboration and dialogic teaching and learning. Dialogic pedagogy, inquiry, and collaboration should be integrated to promote critical mathematics literacy and help students experience a thick version of democracy. The current research suggests that CME needs to be a counterhegemonic educational practice, as the contemporary U.S. classroom has been colonised by top-down neoliberal policies. But implementation will be sustainable only if CME is oriented to transform the classroom into a community of learners. Taken together, results from EUPs revealed that classroom teacher’s facilitative stance is central to such a transformation. As the classroom became a community, dialectically, life there became more democratic.

Findings also indicated obstacles to practising CME in the high school context. Lack of ready-to-use curriculum materials obliges teachers to design learning materials themselves. Lack of support from administrators and fellow teachers make the practice of CME a lonely undertaking. Nevertheless, the present results show that materials such as open-ended word problems can indeed be integrated into the standardised curriculum, thereby helping to resist the control and management policies of the neoliberal hegemony.
Chapter 6: Conclusions

This study aimed to investigate the scope and limitations of critical mathematics education in the neoliberal era. CME is a relatively new but growing research field. As indicated in the review of the literature, CME to date comprises mostly theoretical studies—there is a serious shortage of classroom-based research. Indeed, the present study is the first classroom-based critical participatory action research in CME in a high school context. Its aim is to bridge the gap between theory and practice in CME.

The study was conducted in a low-income area, in a public high school mathematics classroom in the U.S. to answer the central question:

- What are the potentials and limitations of critical mathematics education in terms of classroom teaching in the neoliberal era?

The findings led to the conclusion that despite an educational environment resulting from the market-based standardisation movement, CME can be implemented through the interconnected dynamics of collaborative learning, dialogic pedagogy, and inquiry-based practice (see especially Sections 5.1, 5.2, and 5.3). When these elements were oriented towards promoting critical citizenship and a ‘thick’ version of democracy, students began to take on democratic values, critical mathematical literacy, and critical citizenship. Such a transformation is aligned with theoretical studies by Aguilar and Zavaleta (2012), Ellis and Malloy (2007), and Hannaford (1998).

The main conclusion of the current study concerns three domains. First, the cycles of plan-act-observe-reflect gradually turned the classroom into an egalitarian community of mathematics learners. A facilitative pedagogic ambiance was created
where the students experienced mathematics learning in the form of a dialogue. This finding is consistent with the studies of Kennedy (2009), Kohn (1992), and Rogers and Freiberg (1994). Second, lessons presented as end-of-unit projects (EUPs) created a communicative space for students to develop and exercise critical mathematics literacy, democratised the learning process, and initiated bottom-up responses to counter the hegemony of neoliberal ideology in education. Third, certain practical limitations of CME must be acknowledged, given the overwhelming neoliberal colonisation of education in the U.S.

6.1 Dialogue, Collaboration, and Inquiry

As indicated in Section 5.1, this study aligned with the findings of existing research concerning the importance of a dialogic approach to mathematics education. Further, the study provided data allowing us to reinterpret the existing literature (see Section 2.8) to distinguish dialogic pedagogy in CME that draws on communicative rationality from other dialogues that draw on technical (instrumental) rationality. Dialogic pedagogy in CME is egalitarian, which is crucial for an inclusive process of teaching and learning. In this connection, recall that students’ first proposal was to insist on non-dominating peer relationships in group work (Section 4.2.1). Dialogic pedagogy in CME is structured through inquiry-driven learning materials to create communicative space in the classroom. To be motivational, these materials must reflect students’ lived experiences; because they focus on significant issues, they act as catalysts to open spaces for unpopular ideas to be considered.

As indicated in Section 2.8, there is currently a gap in the CME literature concerning classroom practice. The present findings (Sections 4.2.6 and 5.2) show that collaborative learning provided students with a more equal access to mathematics knowledge and skills, and also helped them develop a sense of belonging necessary to
promote a thick version of democracy. As the collaborative process contradicts neoliberal pedagogy, it requires a radical shift from market-driven, individualist, competitive learning to egalitarian and facilitative learning processes. My students and I materialised this shift through whole-class discussions. After reflecting on previous experiences with group work, students unanimously decided in favour of egalitarian peer relationships. Thus they rejected the political, pedagogical, and ethical consequences of individualistic competitive learning.

The inquiry process in CME, as indicated in Section 5.3, must be in harmony with dialogic pedagogy and collaborative learning. Across EUPs, students became more agentic participants. They collectively negotiated and constructed knowledge; nothing was imposed on them. This way of learning mathematics subverted the transmission style of education and served as our response to top-down imposed neoliberal pedagogy. Freire (2000) elucidated such an approach when he noted, ‘Projecting an absolute ignorance onto others, a characteristic of the ideology of oppression, negates education and knowledge as processes of inquiry’ (p. 58).

Confirming Skovsmose’s (1994, 2011) argument that inquiry in CME replaces the exercise paradigm and rote memorisation, the current study showed that CME can be realised through open-ended word problems, where the goal is for students to develop transferable skills, improve content knowledge, and contextualise mathematics into sociopolitical and socioeconomic issues. While we were engaged in a practice of CME, the students still had to pass standardised tests. The inquiry process, therefore, cannot neglect the content knowledge that students will need for the test.

6.2 The Mathematics Classroom as a Micro Society

Although some theoretical studies emphasise the importance of making the classroom a community (Kennedy, 2009; Murphy & Fleming, 2010), none of them
concerns mathematics. Put differently, the CME literature is silent on classroom-based approaches. The most important original contribution of my study to CME, therefore, is that it is firmly rooted in an actual U.S. high school mathematics classroom. The present study bridges the gap (Section 2.8) between theory and practice, because a mathematics classroom was transformed into a community.

A central tenet of CME is that all classroom practices must be oriented towards creating an egalitarian community of learners. The basic elements of CME in the classroom—dialogic pedagogy, collaborative learning, and inquiry-based lessons—are unsustainable if they are not dialectically structured to establish and maintain an egalitarian community. The following three interconnected sub-conclusions substantiate the central conclusion.

First, the present findings show that mathematical concepts can be taught through dialogic pedagogy—authoritarian teaching is not the only way. As reviewed in Section 2.8, the CME literature distinguishes between dialogical and nondialogical teaching of mathematics. I posed the following question: Can CME completely avoid nondialogical (authoritarian) teaching? Disproving Mortimer and Scott (2003) claim that the authoritarian approach is inevitable when mathematics and science teachers introduce a new topic, the findings here indicate that dialogic teaching is effective for teaching mechanical aspects of mathematics. However, as shown in Sections 4.2.5 and 5.1, introducing a topic through dialogue is not attainable in a traditional classroom driven by vertical student-teacher relations: It requires instead an egalitarian community.

Second, this study confirmed that students’ learning improved to the extent that they were able to learn from and with each other to materialise their full potential (Vygotsky, 1978); there was no need for more competent students in small group
work. This process of egalitarian peer collaboration also helped me as the classroom teacher to become a facilitator (Wells, 1999). These findings have an important implication for the notion of ZPD: In order to apply ZPD as part of CME practice, the classroom must be an egalitarian community of learners. In the absence of a facilitative classroom environment and egalitarian peer interactions, the ZPD process could instead produce power relations among peers, thus reproducing transmission-style education (see Section 5.2).

Third, this study revealed that the ‘ideal speech situation’ outlined by Habermas (1990, 2005) can be attainable only if the classroom is an egalitarian community of learners. After four cycles of planning, acting, observing, and reflecting—EUPs 1 to 4—our classroom had visibly become an egalitarian community. As reported in Sections 4.2.5 and 4.2.6, creating conditions for the ideal speech situation was a time-consuming process and required a radical change in power dynamics. However, we were rewarded with qualitative changes in peer interactions and student-teacher relationships.

The classroom transformed into a much more inclusive and a more open communicative space, especially in EUP 4 and EUP 5, where no coercive forces were in place to motivate students. Each student had equal power to question, speak, and act in small group work and whole-class discussions. There was no secret agendas or distorted communication; the driving forces of interaction among the students were mutual trust, honesty, and respect. Nothing was imposed on students: Meaning making was driven by argument, not by rank. Findings also demonstrated that empathy is an effective and significant element in creating grounds for the ideal speech situations. Empathy in the classroom cannot be generated in the absence of love and hope, as articulated by Freire (2000).
6.3 Citizenship and Mathematics Education

One of the sub-questions of the study was:

- In what ways can collaborative and dialogical mathematics education be facilitated to help students to become critical citizens?

As there is no previous classroom-based research on CME linking mathematics education to democracy and critical citizenship, this study provides the first response to the question. The answer can be framed in four domains.

First, inquiry-driven collaborative learning and dialogic pedagogy democratised life in the classroom. In EUPs, the students experienced mathematics learning as a democratisation of classroom life. We experienced a ‘thick’ as opposed to a ‘thin’ (neoliberal) version of democracy (Orlowski, 2012; Westheimer, 2015). In agreement with Freire (1998), I found that teaching critical citizenship necessitates democratising life in the classroom. As the classroom became a democratic space, we had a solid ground to relate mathematics to larger social, economic, and political issues.

Second, the study revealed the significance of making small openings in the classroom colonised by neoliberal (and neoconservative) educational implementations. The openings created by EUPs allowed me to incorporate critical mathematical literacy and critical thinking into the standardised curriculum. The students discussed some premises of neoliberal ideology and questioned irrational and unjust implications of market-driven educational policies. Through whole-class discussions, the students developed a collective, bottom-up response to neoliberal hegemony. In their view, education is a human right and a social investment, not an individual commodity and personal investment. As they embraced inquiry-based
collaborative and dialogic learning, they rejected the competitive, authoritarian, and rote aspects of neoliberal pedagogy.

Third, the process of developing bottom-up responses entailed critical thinking as part of critical literacy. Engaging in the structural analysis of society and imagining a better one, the students objected to corporations’ involvement in education and made proposals to make society at both micro and macro levels more just, equal, and sustainable. In this sense, the students were engaged in critical thinking that draws on communicative rationality and that recognises the ethical and political dimensions of critical thinking. This version of critical thinking differs radically from the neoliberal version that draws on technical rationality to solve business problems. As discussed in Section 5.4, EUPs prompted critical mathematical literacy, through which the students developed ability to question authorities and keep them accountable. Therefore, this study promoted a thick version of democracy and a participatory, social justice–oriented citizenship.

Fourth, the students clearly opposed neoliberal policies and implementations. When communicative space was made in the classroom, students raised their voices against the neoliberal world view. Students’ journals indicated that they did not consent to neoliberal ideology as a dominant discourse. As Habermas (1975) articulates in a broader sense, the system colonises the life-world and prevents free public debate, which makes legitimacy of the system questionable. The legitimacy issue applies to educational policies as well. The results here show that a start can be made by creating small openings in the classroom, where students can develop bottom-up responses to counter neoliberal colonisation.
6.4 Learning Materials in CME

Five EUPs provided solid data to answer one of the sub-questions of this study:

- In what ways can mathematics lessons and projects be developed to practise CME without disrupting the process of preparing students for standardised assessment?

As described in Section 2.9.2, there is a gap between theory and practice of CME in terms of developing word problems and projects to counter neoliberal pedagogy. The findings from the current study clearly begin to bridge the gap. In relation to the elements of CME, five aspects of potential projects and word problems can be identified.

First, as noted in Section 5.1, a distinctive element of dialogic pedagogy in CME is alternative learning materials; notably, *open-ended* problems and projects. In other words, projects must be in harmony with the principles of dialogic learning. The present findings corroborate Skovsmose’s (1994) suggestion that problems in CME should create a ‘landscape of investigation’. Problems must be a forum where students can relate their learning to a larger society in order to negotiate social, political, cultural, and economic issues that affect their lives. I concluded that CME projects must not be limited to the exercise paradigm or solving a modelling problem.

Second, word problems and projects in CME must be multilayered, so that students working in small groups can negotiate implications of the problem and build on each other’s contributions.

Third, problems must be inquiry-driven for the students to learn mathematical skills and knowledge that are transferable to different domains of study. Word problems must help the students improve their conceptual understanding, procedural
knowledge, and numerical fluency in order to pass standardised tests and be successful in the conventional sense.

Fourth—and the most important element of problems oriented towards CME—is that they must be built on clear ethical and political grounds to be able to counter neoliberal hegemony. It is worthwhile here to revisit EUP 4. As noted in Section 4.2.4, we contextualised the community volunteer service issue to counter neoliberal hegemony in education. I noticed the significance of the political and ethical ground on which I designed the project to distinguish the notion of helping others—‘false generosity’ as posited by Freire (2000), in a thin version of democracy from solidarity in a thick version. The same distinction can be applied to the notion of critical thinking: To promote critical mathematics literacy, word problems and projects should help students distinguish between critical thinking based on communicative rationality and one based on technical rationality.

Fifth, the findings serve as a reminder that critical mathematics teachers need to be aware of risks to their job security and be proactive about them. As reviewed in Sections 2.2 and 2.4, the learning targets in the U.S. standardised curriculum are part of the management and control process in public schools. A mathematics teacher, therefore, must find ways of linking word problems and projects to these standards. Otherwise, they could face disciplinary consequences. In my case, each EUP, with one exception, was connected to a specific learning target outlined by the school district. However, I could not link EUP 3 to any learning target, as the standards do not mention the history of mathematics. As described in Section 4.2.3, the principal’s classroom visit at that time put a question mark on my evaluation. This caused only a minor problem for me, but it could have turned into a much more serious issue.
The conclusion is that for the sustainable practice of CME, word problems and projects must be linked to learning targets in the standardised curriculum. This is a new contribution of my study to the existing CME literature. However, I do not claim that my conclusion is the final answer. There is a need for more classroom-based research from different parts of the country—and from other countries—to provide political and pedagogical insight into integrating word problems and projects into the standardised curriculum without punitive consequences.

6.5 Management and Control

This study has also shown that a mathematics teacher who wants to practice CME should allow for some possible consequences. CME is not welcomed in schools colonised by neoliberal pedagogy. Although one may succeed in creating an egalitarian community of learners in the classroom, life in other classes is mostly organised by market-driven educational discourses. This situation could demoralise students and teachers alike. Therefore, a practice of CME must openly negotiate these kinds of situations with students through whole-class discussions.

My research study shows that it is possible to practise a humanising education that sides with students as human beings and citizens against the imperatives of the neoliberal system; however, such a practice is accompanied by certain political challenges. It is sustainable only if the classroom is treated as a democratic community. Because market-driven objectives currently colonise classroom life, it is imperative to create small openings in which students can develop critical mathematical literacy, reclaim their voices, and thereby subvert neoliberal hegemony.
6.6 Participatory Action Research and Critical Mathematics Education

This study drew on critical participatory action research, which enabled us to open ‘communicative spaces’ (Kemmis, 2008, p. 126) in the classroom to initiate bottom-up responses to top-down educational policies. Cycles of plan-act-observe-reflect were in harmony with the natural flow of classroom teaching; students connected learning to sociopolitical and socioeconomic problems as equal participants in the research process.

The current study supports Skovsmose and Borba (2000), who suggested that participatory action research that focuses on changes in the classroom resonates with the concerns of CME. This study showed that critical participatory action research resonates well with an inquiry-based collaborative and dialogic pedagogy. In this sense, critical participatory action research enabled us to democratize the classroom life and therefore can be an effective instructional model to practice CME.

6.7 Limitations and Suggestions

My study provides a solid framework for CME in relation to market-driven educational changes. However, I do not claim that this is a complete frame. I conducted research in one class in one high school. Although neoliberal educational policies have been widely implemented across the U.S., educational changes may have impacted states—even school districts within states—to different degrees. Therefore, further research in other settings should be undertaken to develop a more comprehensive picture of the scope and limitations of CME.

6.8 Final Word

I am aware that the transformative changes in our classroom may not mean so much at the macro level. Nevertheless, this study created small openings in a high
school classroom and initiated an egalitarian community of mathematics learners. By doing so, it showed that a classroom *could* be transformed into a community and thus neoliberal pedagogy could be countered. With the creation of many more small openings, these promising results could be expanded to show that a dialogic teaching of mathematics and a more democratic education is possible, even within educational conditions that are contradictory to the larger emancipatory vision of critical mathematics education.
References


*Schools, curriculum and civic education for building democratic citizens* (pp. 7-17). Rotterdam: Sense Publisher.


Appendix A

Student-produced multiple-choice quest

Question 1:
Given that the intercept point of the functions $f(x)$ and $g(x)$ is $(1, 4)$.
Find the value of $b$.

\[ f(x) = 2x - 1, \quad g(x) = \frac{5}{6}x - b \]

\[ b = ? \]

A) 7  B) 8  C) 9  D) 10  E) 11

Question 2:
Find the shortest distance between the line and point $A(2, 6)$.

A) 7  B) 8  C) 9  D) 10  E) 11

Question 3:
Given that functions $f(x)$ and $g(x)$ are perpendicular to each other.
And the expression \( \beta = \frac{k}{2m} - \frac{3}{5} \).
Find the value of $\beta$.

\[ f(x) = mx + b, \quad g(x) = kx + c \]

A) -0.5  B) 0.5  C) 1  D) 1.5  E) 2

Question 4:
Solve the linear system given below.

\[ 3x - 6y = 12 \]
\[ 4 - 5y - 2 = 0 \]

A) $x = ?$  B) $y = ?$  C) $x = ?$  D) $y = ?$  E) $x = ?$  F) $y = ?$
Appendix B

Part-Time worker: Edward’s Job offer

Final Work from The Group

To solve the problem for Stuart we put the functions into a multipart function that best explains his maximum profit. If Edward works 1-9 hours the first function is most beneficial. If Edward works an exact 10 hours then either function would give him the same profit. But if Edward worked 11-20 hours then the second function would give him the biggest profit outcome for his work.

Example:

9 hrs.  \[ 16(9) + 20 = 144 + 20 = \boxed{164} \]

10 hrs.  \[ 16(10) + 20 = 160 + 20 = \boxed{180} \]

11 hrs.  \[ 16(11) + 20 = 176 + 20 = \boxed{196} \]

\[ f(x) = \begin{cases} 
16(x) + 20 & \text{if } 1 \leq x \leq 10 \\
(8/5)(x^2) + 20 & \text{if } 10 < x \leq 30
\end{cases} \]

\( \left(\frac{8}{5}\right)(10^2) + 20 \times 32 \approx \$75,920 \) per year assuming income tax is 30%.

\[ \$75,920 \times 3 = 227,760 \rightarrow 75,920 \rightarrow 22,776 \approx \$53,144 \text{ per year} \]
This choice is the best for Edward because when he works between 0 and 10 hours, he uses the linear equation to make the most money, and when he works from 10 to 30 hours, he uses the exponential equation because he will make more money.
Appendix C

History of Mathematics

The History of Infinity ∞

The Beginning:
- No record of earlier civilizations regarding ∞, we know so far as ancient Greeks.
- Greeks used specific terms like: ‘infinite’, ‘infinitesimal’. They thought of it as negative thing, passion, not perfection, infinite = evil.
- Everything had a number, unknown counted them.
- Were not curious about time having no end, life temporary.
- Died one, then died.
- Used with terms of prime without bounds.
- Aristotle tried to avoid actuality of infinity with minimal infinity.
- Defined as ‘without a limit’.
- Been used for mathematics in modern mysticism and literature.
- Symbolically: A name, magnitude of x grows beyond any assigned value.
- Different forms: ordinal & cardinal.
- Georg Cantor: developed system of transfinite numbers in late nineteenth century.

In Greece:
- Modern Society:
History of

Infinity

To INFINITY and BEYOND - BUZZ LIGHTYEAR

History of Mathematics

Egyptians:
Use of pi to help build the Pyramids of Giza

Greeks:
Mathematicians, such as
Archimedes, found that 
(22/7) ≤ π ≤ (157/53)

Before the 16th Century, 156
mathematicians studied
Archimedes and his use of
dimensional geometry, which led
to the calculation of the value of π,

A Brief History

Computers, such as 
Champlain, 
Cramer, 
Gauss, 
Hanna Scholl 
Euler 
Babbage 
William Jones

Discovered 
that π ≈ 3

India:

3.1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253421170679

Struggles to understand the nature of π
Appendix D

Community Volunteer Service

Project #4: Spirit of Community

Graph:

1. \[(0,0)\]
2. \[(20,0)\]
3. \[(10,15)\]
4. \[(10,10)\]

\[2,000x + 3,500y = \text{Profit}\]

\[
\begin{align*}
1. \quad & (0,0) \quad \quad \quad \quad P = 0 \\
2. \quad & (20,0) \quad \quad \quad P = 40,000 \\
3. \quad & (10,15) \quad \quad \quad P = 52,500 \\
4. \quad & (10,10) \quad \quad \quad P = 55,000
\end{align*}
\]

\[
\text{Max} = (10,10) \\
10 \text{ Individual units} \& \\
10 \text{ Family units}
\]
Appendix E

Student Loan Depth Crisis

Table I

<table>
<thead>
<tr>
<th></th>
<th>1st scenario</th>
<th>2nd scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loan for college tuition</td>
<td>$14376</td>
<td>$46576</td>
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<tr>
<td>Loan for extra expenditures</td>
<td>$58872</td>
<td>$29472</td>
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<tr>
<td>Interest rate</td>
<td>6.7%</td>
<td>6.7%</td>
</tr>
<tr>
<td>Total money he is supposed to</td>
<td>$243190.512</td>
<td>$173719.94</td>
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<tr>
<td>pay back</td>
<td></td>
<td></td>
</tr>
<tr>
<td>His monthly payment</td>
<td>$2043.25</td>
<td>$1489.33</td>
</tr>
<tr>
<td>His future job, %10 of which</td>
<td>$2043.25</td>
<td>$1489.33</td>
</tr>
<tr>
<td>will be equal to the monthly</td>
<td></td>
<td></td>
</tr>
<tr>
<td>loan payment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>His future job, %20 of which</td>
<td>$1048.65</td>
<td>$7478.45</td>
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<td>will be equal to the monthly</td>
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<tr>
<td>loan payment</td>
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<td></td>
</tr>
</tbody>
</table>

1. \(17344 \times 4\)
2. add 911 x 6!
3. \((1 + r)\)
4. \((19576 + 58872)(1.06)^4\)
5. \((108448)(1.06)^4\)
6. \(2043.25 \times 10^3\)
7. \(20432.5\)
<table>
<thead>
<tr>
<th>Predictable monthly expenditures for Edward as a single person</th>
<th>1st scenario</th>
<th>2nd scenario</th>
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</thead>
<tbody>
<tr>
<td>Rent</td>
<td>$500</td>
<td>$2000</td>
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<tr>
<td>Cell phone</td>
<td>$65</td>
<td>$65</td>
</tr>
<tr>
<td>Internet or TV, cable or communication</td>
<td>48</td>
<td>48</td>
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<tr>
<td>Transportation or gas and insurance</td>
<td>150</td>
<td>125</td>
</tr>
<tr>
<td>Food, kitchen and household expenses</td>
<td>150</td>
<td>860</td>
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<tr>
<td>Bill #1 Utilities</td>
<td>110</td>
<td>160</td>
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<tr>
<td>Bill #2 Gym Membership</td>
<td>10</td>
<td>26</td>
</tr>
<tr>
<td>Entertainment</td>
<td>50</td>
<td>300</td>
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<td>Other-1 Clothes</td>
<td>20</td>
<td>500</td>
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<tr>
<td>Other-2</td>
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<td></td>
</tr>
<tr>
<td>Other-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...?</td>
<td></td>
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</tr>
<tr>
<td>Total</td>
<td>3,56</td>
<td>3,981</td>
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