THE LINE AND THE NUMBER ARE NOT NAKED IN PAPUA NEW GUINEA

A RETA E O NÚMERO NÃO ESTÃO DESPIDOS EM PAPUA-NOVA GUINÉ

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ABSTRACT

Two key mathematical concepts, the line and the number, are considered in terms of the cultural context of several examples from the many cultures of Papua New Guinea. This paper highlights how the continuous line in a topological sense is significant diverse in cultural practices, which are relatively common in the nation though with cultural variations with each language group. In particular, it looks at the significance of the line in art, string-bag making, and in coverings. Number is also considered with evidence of invented ways and borrowed ideas in which people developed systems for large numbers of objects indicating a strong sense of number. Evidence is given from both Austronesian and Non-Austronesian languages. The implications for using children's home background in terms of new starting points for education together with new understandings of mathematics in a cultural context in this country are also highlighted.

Keywords: Papua New Guinea, Systems for Large Numbers, Lines in Geometry and Art, Cultural Contexts and Beliefs.

RESUMO

Dois conceitos matemáticos importantes, a reta e o número, são considerados em termos do contexto cultural de vários exemplos oriundos de muitas culturas da Papua Nova Guiné. Este artigo destaca como a reta contínua, em um sentido topológico, é culturalmente significativa nas práticas culturais, que são relativamente comuns na nação, embora com variações culturais em cada grupo linguístico. Em particular, este artigo examina a importância da reta na arte, nas mochilas de corda e em seus revestimentos. O número também é considerado com evidências de formas inventadas e ideias emprestadas por meio das quais as pessoas desenvolveram sistemas para um grande número de objetos que indicam um intenso sentido numérico. A evidência é dada a partir de ambas as línguas austronésias e não-austronésias. As implicações para a utilização da bagagem cultural adquirida no lar pelas crianças em termos de novos pontos de partida para a educação em conjunto com os novos entendimentos da matemática em um contexto cultural neste país também são destacadas.

Palavras-chave: Papua Nova Guiné, Sistemas para Números Grandes, Retas em Geometria e Arte, Contextos Culturais e Crenças.

1. Introduction

What is a line and how is it perceived? In school mathematics it may be the joining of two faces or a single dimension or a set of points of infinite number represented in one- to three-dimensions depending on whether you accept curved lines as lines. It is the last definition that best represents the intersection of the meaning of a line for Papua New Guinea societies and school mathematics.

What is a number and how is it perceived? Perhaps I should actually use the term numerosity as that is closer to the thinking of many in Papua New Guinea when it comes to number. There are, however, some systematic patterns for different language groups for counting but the mathematical understanding of the communities in terms of number is far greater.

Since there are over 850 different oral languages and hence societies and cultures in Papua New Guinea, it is not possible to generalise or attribute just one system to all its people. Rather, this article will delve into only a few of the available records to indicate the theme of this paper. (For more systems, see Lean (1992), Owens (2001), and Owens, Lean, With Paraide, and Muke (forthcoming, 2016).)

The title of this article derives from my reading of a book called Writing Never Arrives Naked (van Toom, 2006) which is about how Australian Aboriginal people began writing and using English at its introduction in various parts of Australia at the time of colonisation/devastation by non-Aboriginal, mostly English invaders. There is no doubt that many who were responsible for Aboriginal groups to learn to read and write English had good intentions including being able to write letters of complaint to authorities, but the context of the learning was inevitably for ulterior purposes such as “providing civilisation” and “reading the Bible”. The loss of culture at the expense of this action is monumentally saddening even when the Aboriginal persons used writing and reading for their own purposes. The same happens with school mathematics that has come with colonisation and neo-colonial governments and governments’ decrees (policies) all trying to do the “right thing”. What is lost?

However, in this article, I am looking at how mathematics never came naked, but developed within the cultural context of the community. There were purposes for lines and representations of numbers. The aim is to set out some exemplars to widen people’s eyes of possibilities of mathematics in cultural contexts. The line in these examples is generally represented by some form of drawing or string. The numbers are represented orally by words that can be understood in terms of a relationship in a systematic way and by systematic uses of objects or body parts.

2. Examples from Other Places around the World

Two examples suffice to show that the Line and its representations are significant in other Indigenous cultures around the world. Though significantly different, these stories set the scene for realising the importance of the Papua New Guinea stories for mathematics in a cultural context.

2.1. The Centre and Intersecting Lines in Yu’pik Mathematics

Lipka et al. (2015) have provided a key concept, the centre of everything, from the Yu'pik Elders in Alaska and shown how this centre is marked by intersecting lines at right angles and is key to understanding the person and his place. A fold can set up parallel lines, and halving by folding the other way can provide right angles. The centre is the beginning of everything; it is not only a critical centre for the worldview of the Yu'pik but it is a central concept in Yu'pik mathematics. A related key concept is that of symmetry. Two other central concepts and processes are that of halving and of measuring by comparing. Lipka et al. (2015) also note the conceptualisation of scaling is developed through experience and in the head. Starting with these key concepts from culture, the teacher can easily develop systematically many useful mathematics concepts, albeit from a different starting point to Euro-Asian school mathematics.

2.2. The Line as a Path in Navajo

Pinxten and François (2011) tell the beautiful story of a boy traversing the land and points out the alternative conception and visualisation that the child has of a line in space, a continuous line. In particular, he is aware of its position relative to the land and the significance of the various landmarks he passes. Experience of walking the path and noticing features of the landscape provides knowledge of the path.

2. The Line in Papua New Guinea

As in many societies, the curved line has special roles and line lengths can be recognised in curved fixtures. The line may curve, but its continuity and connectivity are paramount. This is particularly the case in the stories associated with the walking of paths in Papua New Guinea. The path may belong to an Elder who has traversed it for hunting. The place is full of spirits who keep the place, its features, plants and animals. Some places are more important as they represent an activity of importance associated with the Elder or the story and order of events. Thus, the line as a path of continuity across space and time has relationships to place and people that are important in the education of the child.

3.1. The String as a Measuring Tool for Proportions

A long leaf, a piece of soft bark or a stick, or any number of other long objects are used to measure for comparison and equality. They are also commonly used for halving and finding halfway. The long pieces will be halved and quartered or if needed into thirds to find these points between two end points. This will be done for planting posts for a house if there are to be three, four or five lines of points. The string is halved and hence the floor space if there is just a middle row of posts together with the outside walls. The string is divided into three by estimating the halfway of the folded string. The string is divided into four by halving and halving again. If the house is half as much again then half the length is used to extend the house and an additional row of posts placed. The floor space and all additional requirements like roofing and walls are also proportioned.

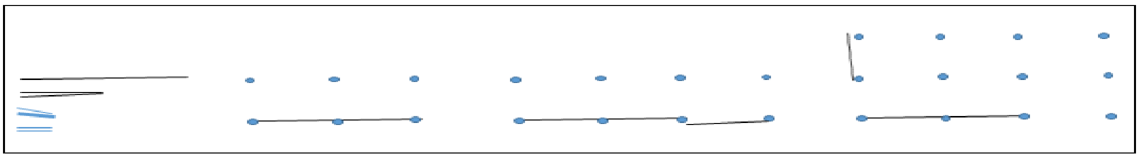

Figure 1. The house plan and "half as much again" plans at Malalamai, Madang Province

To find halfway between two points, it is also possible to use a stick to mark points closer together that are equidistant from the end points. When this is done, it is easier to guess the halfway point or the task can be repeated with a shorter stick.

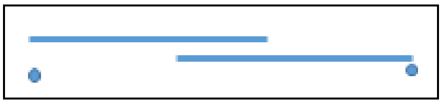

Figure 2. Finding halfway in Kopnung

Parallel and perpendicular lines are determined in a variety of ways. It was not unusual to have a right angle made by folding a flat leaf or paper in half-and-half the other way but it is more likely that they were making a line by lining up posts by eye in both directions and experience told the person at the corner of a house that the lines were "square". In some areas people checked for a shape to be a rectangle by not only having the opposite sides equal but also the diagonals (personal communications, e.g. Henry Atete from Enga, 1997), knowing that if these were not equal then the shape would be a "poor rectangle", a parallelogram.

Parallel lines were often marked off by using an equal length along one line. This was used, for example, to space the roof leaf "tiles" (*morata).* Another use of parallel lines was in building ditches as they do in highlands Provinces such as Jiwaka and Hela. With the use of string for the straight line, first one side of the ditch is marked and then the other, ensuring it is parallel. More importantly, the slope of the ditch is carefully constructed despite the reliance on eye to decide although the spade itself is used for comparison.

A student also noted that two people could walk in the same direction keeping themselves equidistant in order to mark two parallel lines. If they were walking from one path to another that intersected then the student noted they would form a trapezium with the paths. This action together is typical of people working together to create in any village activity.

Lines represented by posts have names such as 'mother' and 'father'. The lines joining the poles are also significant in terms of the positioning of parts of the house and people relationships (from East Sepik and Madang Provinces). Weaving of perpendicular and parallel lines also brings meanings of knowledge being joined together to make the whole. The person who weaves and whom he/she weaves for are also important. The gender of the weaver, like many other activities such as sago making, varies from region to region and to differing degrees of demarcation of roles. Nevertheless, weaving and other objects are also consumable objects sold for profit as well as giving in reciprocity exchanges.

2.2. The String as a Container

Papua New Guineans did not weave cloth from string although most communities did weave leaves, canes, and split bamboo for baskets of various kinds and could make hats, mats, and walls in this way. Men did this weaving in some cultures, women in others,

and for different objects and purposes they may have been woven by men or women or both. They wove ornaments, symbols, containers, and bindings such as for spear heads onto the shafts. *Tapa* made by smashing flat the inner bark of the *tulip* (two leaves together) tree provided some clothing. Continuous string was made into a very flexible expanding container called a *bilum* and for front skirts, toys, displays and musical instruments.
The “stitching” (usually they use a strong needle with the string and put it through the figure-of-eight loop) or looping varied to make open loops, strong neck openings for the *bilum*, or for fishing nets or tight small containers for precious objects. Is the *bilum* significant? Yes. Mackenzie (1991) wrote a book on the way in which the *bilum* was an androgynous object for Telefol women with a mythical origin, indications of open lives portrayed by the open loop and a strong *bilum* being a symbol of a good woman and their social value. It also mattered in the relationship between the maker and receiver of a *bilum* of specific kinds.

Each group also had their own way of making traditional *bilums* and their own designs. Even within the one language area, there could be different designs of looping and often a design pattern on a particular *bilum* was associated with a particular maker or subclan. Nowadays, modifications have been made that may be shared by *bilum*-makers with their friends and they are regularly given as gifts. Part of bride-price ceremonies was the giving or display of *bilums* full of food so their nature and size was also valued.

What do these loops look like and is this continuous string significant as a continuous line? Figure 3 shows some of these loops but there are many kinds. The women make a new loop and pull the rest of the string through winding it onto the thumb and small finger of the other hand until the length is finished and then they join by rubbing on their leg (in the same way as they make the traditional string itself) to make a continuous string. This continuity is significant too and can go around and around the *bilum* or be made by starting a new row. With today’s designs, the woman may have up to 20 needles making up the design (the needle helps to put the string through the loops).

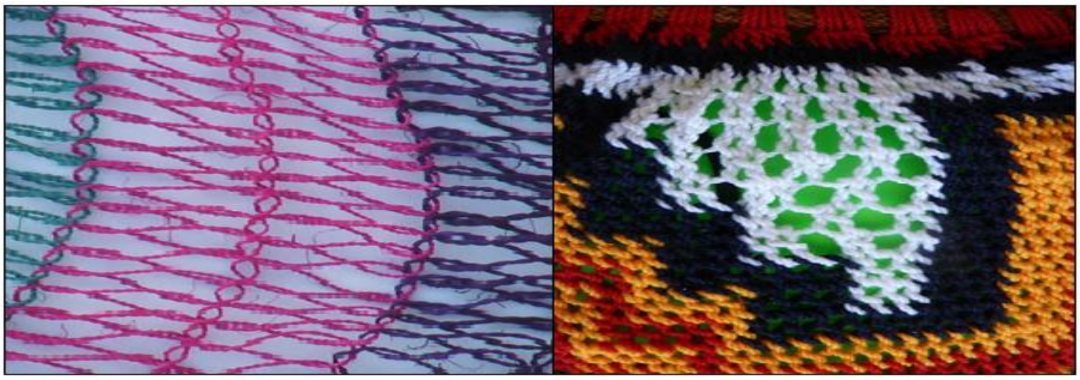

Figure 3. Continuous line looping of *bilum*

As part of early mathematics, this could be used even if traditionally made by girls. Some sharing of mathematical ideas and of roles has varied over time. The fathom length usually used on the needle then becomes a certain size when looped into the finished product depending on the colours and design and the size of the loop. Familiarity with loops and knots is also part of the children’s cultural background with them noticing how the loop is made in terms of in and out of the loop and the previous loops. At the same time, they follow the number of loops before changing direction or colour in the overall design. However, this early topology is rarely developed even in high school or academic mathematics. White lines running through the Abelam *bilum* just as in their paintings of one panel to the next (see §2.4) is a significant thread of continuity. Thus, the continuous line as it curves is a place to start with lines.

2.3. The String as a Story-Maker

String figures or cat's cradles seem to be played around the world especially among Indigenous communities. Extensive recording of string figures in one area of Papua New Guinea can be found in Haddon (1930, reprinted in 1979). Recently Vandendreissche (2007, 2016) described the mathematics in terms of polynomes and relevance to various theories. One important review of our approach to space and geometry would result from considering the subprocedures (Vandendriessche's term) that can combine and make procedures and their links to end products.

This alternative beginning about lines is compared to the idea of Euclidean shapes with three or more straight lines forming the polygons and the various relationships (sets and subsets of polygons) that we usually start with in school. Vandrendriessche notes that at least two subprocedures on Goodenough Island, Papua New Guinea, had local technical terms and that some designs were memorised by the series of local technical terms. Nevertheless, much of the memory was actually held in the fingers and remembered only by the practitioner beginning to make the series of figures.

Furthermore, the string figures can be transformed into a new order of procedures or mini-sequences that go from one relative "normal" set of procedures to the next. These intermediary steps could be considered as a paragraph in communicating. There is an opening, a series of subprocedures (sentences in a paragraph) and a finished state, then the next paragraph begins until the story concludes (Jenness, 1920). Practitioners worked out how to transform one figure into a new one (Vandendriessche, 2016). The transformation from one figure to the next is a continuity, a theme often found in connecting lines with culture in Papua New Guinea.

While we can look at the mathematics of the finished design such as the number of rhombus between the fingers, and how an even or odd number can be made by releasing strings on both or one hand respectively, they can also be used for storytelling. Vandendreissche (personal communication, 2006) noted this about the Trobriand Islanders, Papua New Guinea where he was researching string figures but he needed the translation of the stories from Kilivila. Storytelling is prolific in Papua New Guinea.

The Senfts (1986) recorded many of the Kilivila stories noting that many were about plants with significance to them but the changes in the figures would often end in an unexpected way, a simple figure, and often associated with risqué storytelling. However, in one highland area, the string figures evolved together with the story of travel to a mountain, Kambea Peak (teacher from Kagua/Erave quoted in Owens (2015, Figure 5.3, pp. 147, 174).

2.4. The Created Continuous Line and the Combination of Strips of Lines

Hauser-Schäublin (1996) says

> (…) in Abelam [East Sepik, PNG] art and aesthetics the line, the strip, and fronds are conceived as the basic constituents of designs. All patterns are perceived from the perspective of the line, or 'visual open-work' rather than from that of the homogeneous plane so abundantly displayed and represented in cloth [in Indonesia] (Hauser-Schäublin, 1996, p. 82).

When painting designs on a large house façade or wooden carving, the white line is the main aspect of the design, which is followed and decorated in other colours. The white line is continued into the second row of the design. When painting the design, the master artist will bring the white to a point so that it can be continued into the next panel. This connection is more mystical than efficient. The continuous white line has meaning just as the *bilum* is made of a continuous piece of string.

"The white string that unites and holds even an extremely large and complex painting together like a web or a net. (…) The white continuous line is called *maindshe*" (Hauser-Schäublin, 1996, p. 89). The line has the same importance in strips of leaves laid on froth found on the top of water, or hanging from the walls of a spirit house to hide the initiates, or in the *bilum*. The designs are assembled rather than being a continuous plane and therefore they may be in different dimensions. The white design stands out also in the woven mats of white bamboo through sago-leaves. Coverings are also assemblages of strips of lines.

In other cultures, the lines might be created in tattoos that are repeated as if in steps to create the desired lengths. The lines would form patterns with the background then being blacked out. Each line, pattern and combination had a story and meaning. For example, the tear tattoo of the Motuans symbolised the tears by each woman as she waited for the safe return of her husband travelling on the seas to collect sago and trade pots (Owens, 2015, Figure 5.4, p. 148). The tattoos also had an origin story associated with them as in the Mekeo (Opu, nd).

2.5 The Line as a Part of a Pattern

Patterns and designs in societies of Papua New Guinea had ownership related to wealth, spiritual power, and relationships. These may be shared but the copyright remained with the original owner. Examples of these perspectives are recorded among the Abelam of the Sepik (Harrison, 2006) and on New Ireland (Were, 2010). Frequently the teacher/observer with a Euro-Asian mathematical background might see a triangle or rhombus but the line is that of a zigzag or a continuing curve crossing over the other zigzag or curve with some connection to a central point as found on the New Ireland discs.

Parts of designs might be shared with others (Kuchler, 1999) and the connecting "path" traceable between the origin and the group with whom it was shared in a relationship. Again, the idea of an assemblage is noted. This applies to house designs also where designs are "copyrighted" and "borrowed" (Coiffier, ~1990). The connection then of a line with the overall pattern or design is significant to culture. The Kwoma of the Sepik had few, styled shapes and designs. Nevertheless, the artists would pride themselves in being able to embellish and modify clan totems and other items (Bowden, ~1990).

The way in which continuous lines are created as in weaving leaves or split bamboo emphasises the interconnection of clan knowledges. Both knowledges are needed for development. Clans are joined through marriage and exchanges. The result in the weaving is often of a continuous line or zigzag.

Figure 4. The line in a Sepik painted carving, on a Sepik bilum and in three common weaving designs

2.6. Summary

Children come to school observing different aspects of the spatial world related to the continuous lines depicted in their communities' arts. If a series of names of shapes is thrust upon them at school, it will reduce the ability of children to notice some key complexities of lines especially in their topology and interconnectedness. The continuous line and its meaningfulness should be noticed and valued.

The way in which a line can be intertwined to create a loop as in the *bilum*, a loop as in binding, a knot in binding or making connections, or a stylised form of various parts of people and nature often have a localised mathematical system that children in their early years at school should value. Teachers' thinking may need to be decolonised (Smith, 2005) to recognise these aspects and desire to incorporate them into their teaching and curriculum. There is time later for children to make connections between these geometries and that of the school system with its Euro-Asian basis.

3. The Number

When people steeped in Euro-Asian number systems learn that a group in Papua New Guinea use only one and two to count, they may consider their understanding of number as poor. However, these same people may have made extraordinarily large exchanges of material objects in their reciprocal relationships with neighbours. In fact, their numerosity was often very good and they had ways of using their counting systems to keep track of this number size.

3.1 The Body-Part Numbering Systems

Tallying was also sometimes regarded as a non-system but again this would be to misunderstand the ways in which this tallying of objects to body point is being made. There were some claims that people could not see an ordered system or do operations but that also does not seem to be the case as children used the body parts to add in school (Saxe, nd). They also used the 10 mark to assist with adding and place value as if recognising it as a group of 10 in the English language.

The informants in Saxe's study may have truncated to 20 or extended to 30 the number of body-parts to adjust to currency, employment or contact with people beyond the village area. Furthermore, the number of body-parts varied with the language group and in some cases adapted to alternative systems to accommodate interactions between two language groups. There were other social reasons for truncating the body-tally systems too (Owens, 2015; Owens et al., forthcoming, 2016).

3.2 Large Numbers

However, data has shown that body parts have also been used to tally, for example, 10s, 100s and higher powers of ten. Thus, the initial body-tally system or a secondary body-tally system was superimposed on a 2, 5, 10 or 20-cycle system. This might have been as a product, e.g., in *Iqwaye.*

3.2.1 Powers of 20 in *Iqwaye,* Border of Eastern Highlands and Morobe Provinces

The deliberate use of digits when counting indicates that the fingers and toes are important representations of number that links counting to their whole bodily existence and the relationship with others when using the fingers of another. Even the numbers can incorporate this deictic of "this" and "that" or "mine" and "yours" or "another". Nevertheless, this close link with the physical body parts does not prevent the Iqwaye of thinking of numbers more abstractly. The initial 20 and a second 20 (mine and yours) are useful for counting purposes, the Iqwaye are also able to use the set of fingers and toes to denote not only ones but each group of 20 as they are being counted and even to a third level, thus making it possible for the Iqwaye to think of large numbers (powers of 20) appropriately (Mimica, 1988).

They might only use the large numbers for counting cowrie shells but they can explain their system. They also use cowrie sticks so when counting these, for example, 3 or 7 *ungye* or *hilyce,* they are referring to 15 or 35 individual shells. A rope of shells may also be used for comparison in bride price. The sense of wholeness of a rope and a sense of the body, namely the digits, as embodiments of the counting scheme provide relationships between numbers that are significant for different purposes.

Quantifying is not necessarily the end point but the decision required for which counting or its equivalent objects might assist. The objects and digit assist with sense of number size and comparison. Thus "the background of number use [is evident in] (…) the Iqwaye perception of quantities in their appearances as deemed equivalent and different, substitutable, exclusive or commensurate" (Mimica, 1988, p. 18). Further details are given also in (Owens et al., forthcoming, 2016).

3.2.2. Large Numbers in Yu Wooi, Mid-Wahgi, Jiwaka Province

A common practice in digit-tally systems that marked 1 to 20 using the fingers and toes was to then take another man's fingers and toes. Some people would also put objects in groups of ten and then use their fingers, folding one at a time to reach a hundred. These practices were evident in *Yu Wooi* or Mid-Wahgi (Muke, 2000). Some people used the set of hands, "this hand and this"- *angek yem yem* "this one" - *elsi* repeated six times pointing to six of their fingers one at a time to mean 60. This could be continued using hands and toes for each group of ten and thus reaching 200. Thus, 600 pigs would be the hands and legs of three men. Muke also detailed the use of the body parts that were also used for large numbers: the head, ear, nose, mouth, right arm, left arm, back, front, right leg, left leg to assist with the 1 to 10 hundreds respectively so a way of tallying to a thousand. Thus *angek yem yem simb daro* – "both hands – left leg) means nine 100s or 900. This tallying helped people remember the number of pigs, say, given during an exchange. Using the body parts of the whole person was *hi end sim angek begenj* and if for two persons, it represented 2 000 etc.

Another tallying method was by use of small banana fruit used to match each item and these were grouped. This would assist with the agreement on payment as each of the members of the tribe would pick what he would offer and place in the payment group. Fractions were also symbolised by a food parcel of cooked sweet potato wrapped in banana leaf. Four parcels were in one *bilum* symbolising a pig so the distribution of parcels gives quarters and halves. Comparisons of numbers was also shown by two sub-clans lining up pigs matching one-to-one so the longer line was obvious and brought pride.

The tying of knots, according to Muke's elderly father, only occurred after Europeans arrived. Knots were tied on a rope, an indicator tied to separate different kinds of wealth (e.g., pigs, feathers, and shells). They were then taken to the girl's family to communicate the amount of bride price they would pay. The girl's family then indicated what they would payback. "It was also possible to see a form of subtraction in practice (…) as this is the net gain of the girl's family" (p. 143). Thus tallying was an important part of methods of communication and agreement.

3.3. Large Numbers on the New Guinea Island Region and Milne Bay Islands

These two groups were both non-Austronesian languages from the Highlands but separated by around 300km. Large number counting methods also occur around the coast and on the islands in both Austronesian and non-Austronesian languages where they have a basic base 10 system with words for 100, 1 000, and 10 000. Quantitative classifiers are used as a prefix or suffix. For example in *Uisai* on Bougainville *–ku* is a suffix for decades and *–egi* for hundreds. *Uisai* also has a word for 1000 and 10 000. The former means "domestic fowl" – *kukurei* is also found in two other Non-Austronesian languages (*Nasioi* and *Siwai*) having been borrowed from the Austronesian Solomon Island Cluster where it is another power of 10 (Owens et al., forthcoming, 2016).

Yele, a Non-Austronesian Isolate language of Rossel Island in the Milne Bay Province, has borrowed some neighbouring Austronesian words for small numbers especially from 4 to 8 but they count to 50 in a relatively systematic way in terms of tens but then they start from one again until they have two lots of 50. *Yele* also has different words to express 1 000, 2 000, up to 10 000 but interestingly a 4-cycle operates in that the words for 1 000 to 5 000 are repeated in the words for 5 000 to 8 000 with the prefix *mwa-* and then in 9 000 and 10 000 with other words. The words are given in Table 1.

Number	Yele	Number	Yele
1 000	*yili*	6 000	*mwadwong*
2 000	*dwong*	7 000	*mwateme*
3 000	*teme*	8 000	*mwadab*
4 000	*dab*	9 000	*mwadi*
5 000	*mwayili*	10 000	*mwadi mwadab*

Table 1. Yele number words for thousands

The shell money used on the island was fairly large and counted individually within a closed system with no new shell money being added or taken away as a complex system of rules governed borrowing and repayment of loans. However, the people required large numbers for their cultural purposes. Amstrong (1928) observed that:

> A curious feature of this last series of terms [Table 1], the combination of the terms for 8 000 to 9 000 to express 10 000 is explained in the legend, which attributes the invention of counting to Wonajo, who wished to count the *nko* (shell money) that he had made. Having counted up to 9 000 he grew weary, and, unable to think of a fresh word for 10 000 adopted the novel, if unmathematical, device of using in juxtaposition the words for the last two thousands (Amstrong, 1928, p. 78).

In contrast, the Tolai joined their small shells together and used length measures to determine their numerosity. They had several systems, one based on groups of 12, which could be made by groups of 3 or groups of 4 in systematic ways with words to establish the grouping. They then established large numbers by combining lengths of fathoms into circles (Owens et al., forthcoming, 2016; Paraide, 2009). Figure 5 indicates these large numbers. Importantly, there were ways of representing the large numbers required and also ways of dividing up the total with family members. The smallest unit when divided is usually a string of shells about a foot long with around 12 shells.

Figure 5. Tolai bride price with 10x10x3 fathoms of shells, a very large number (Photo: Paraide)

In contrast, with the *Dobu* language, an Austronesian language spoken by the people of Loboda on the Northeastern tip of Normanby Island, Milne Bay, they can count but choose to show large amounts spatially. Thune (1978) noted that in cultural activities where exchange of shell-money, time, and objects might require large numbers,

> (…) they invoke alternative ways of describing their world which make use of relative rather than absolute scales or in which the qualitative aspects of objects are inextricably bound up with their

> quantification, and thus an abstract system of enumeration disassociated from the objects to be quantified is, on the whole, unnecessary and irrelevant. For example, in the ritual exchange of yams, a group giving yams should eventually receive an amount equivalent to what they gave. In this case, it is the overall size of the total pile of yams to be given which is significant rather than the number of individual yams in the pile. The size of a yam pile is recalled for purposes of repayment in terms of the names of the people who received parts of it. Other categories of goods to be exchanged: pig, betel nut, stove goods, etc are treated in the same way. This form of name accounting obviates the use of a precise enumeration of the items in a given category (Lean, 1992, Appendix on Milne Bay comments on *Dobu*).

3.3. The Pairs Numbering Systems

One of the systems of counting in *Yu Wooi* (Mid-Wahgi) involved counting in pairs, two fingers together then another two on one hand, two on the other hand, and another two, then the two thumbs saying 2, 4, 6, 8, 10 (Muke, 2000). A similar system is used in the two dialects *Hagen* (Strathern, 1971) and *Gawigal,* Western Province, PNG (Owens, field notes). These groups of 10 are then counted. Also for the Hagen groups, their counting system is a (2, 4, 8, 16) cycle system with multiples of 16 for larger numbers (Vicedom & Tischner, 1948, cited in Lean 1992). One Engan also suggested this was a system for his clan which he described as powers of two (Benedict Yaru, personal communication, 1997) and Lean explored that the *Mae* dialect of *Enga* had groups of four so that each group had a special name such as "dog", "pig", "fire" for the last group up to 60 (Lean, 1992).

However, the most extensive use of paired counting in terms of counting words is from the Austronesian languages especially in Central Province such as *Nara, Roro, Keapara, Gabadi,* a dialect of *Sinagoro,* and *Motu* (Owens et al., forthcoming, 2016). While the numbers 1 to 5 are distinct, compounds are formed for the second pentad. In some cases, there are still special words e.g. for 7 in *Motu*. Otherwise, the numbers are 6 = 2x3, 7 = 2x3+1, 8=2x4, 9=2x4+1. 10 is often addressed by "group of" and varies depending on the group of items. However, multiplicative compounds of 10, there is a classifier or base-suppletion. For example in *Motu* it is the suffix *–ahui* and for *Nara, na vui.*

3.4. The 4- and 6- Cycle Numbering Systems

Above there are some references to 4-cycle systems. Another is *Skou* which has a discrete word for 5 and 6 is 5 + *n* but then 7 is 4+3, 8 is discrete and 9 is 8+1 with 12 being discrete although 17 is 12+5 but 19 is again 12+5+3 with 20=12+8, plus *n* (*n* = 1 to 3) until 24. Thus the idea of a 4-cycle is present (Donohue, 2008). It has as a basis (4, 8, 12, 24) cycle system. The 6-cycle systems also tend to have developed from groupings. Three of these are actually in West Papua close to the PNG border on Kolopom island. The numeral systems of each of these languages possesses a primary 6-cycle; *Kimaghama* has a secondary 20-cycle while *Ndom* has a (6, 18, 36) cyclic pattern. There are some on the mainland straddling the border with PNG. One is *Kanum*, which has a simple, moderate, and complex counting system. By having the different systems, the people immediately know whether they are dealing with small or large numbers. Donohue (2008) provides the data as shown in Table 2.

It is evident that the *Kanum* speakers had developed their own system to cater for large numbers using exponential powers of 6 but there are also variations. Donohue also notes that there is some confusion with the meaning of *ntamnao* (1296) as 1000, which is the lowest valued Indonesian banknote. The systems share numerals especially for four and so not imported either from the dominant local language *Marind* or Indonesian. Furthermore, decimal systems have not influenced it. The simpler system was not lost as it had its place such as in giving the number of children. Furthermore, terms for 12, 18 and 24 are reused as higher powers. There are cognitive advantages in keeping all three systems as they simplify the language for memorising, for example, payments.

3.5. Summary on Number

These few examples from the language groups of Papua New Guinea indicate that, even with simple counting techniques, these ancient cultural groups are not necessarily restricted by their counting system, but they were able to adapt to meet their cultural and spiritual values as well as social activities requiring a systematic mathematical numerosity. It is evident that counting systems can be diverse and not necessarily restrictive, but surprisingly able to account for large numbers in many different ways. Interestingly exponentials and multiplicative or divisive comparisons develop in line with cultural practices. Exponentials are commonly found among the Austronesian Oceanic languages of Island Melanesia, Micronesia and Polynesia (Owens et al., forthcoming, 2016). The links to body are not just as a tally system, but also for purposes of spiritually linking numbers to the body and people together. Thus, not only the exchanged goods, but also the numerosity of those goods is part of the cultural relationship building. However, the quality of the exchanged goods is also very important.

Simple	Moderate	Complex
1 *naempr aempy aempy*		
2 *yempoka ynaoaempy ynaoaempy*		
3 *ywaw ylla ylla*		
4 *eser eser eser*		
5 *swabra tampwy tamp*		
6 *'swy traowao ptae*		
7	*psymery aempy* '6'+1	*aempy ptae* 1+6
8	*psymery ynaoaempy* '6'+2	*ynaoaempy ptae* 2+6
9	*psymery ylla* '6'+3	*ylla ptae* 3+6
10	*psymery eser* '6'+4	*eser ptae* 4+6
11	*psymery tampwy* '6'+5	*tamp ptae* 5+6
12	*psymery traowao* '6'+6 or *yempoka traowao* 2×6	*tarwmpao* 12
13		13 *aempy tarwmpao* 1+12
14		14 *ynaoaempy tarwmpao* 2+12
15		15 *ylla tarwmpao* 3+12
16		16 *eser tarwmpao* 4+12
17		17 *tamp tarwmpao* 5+12
18		18 *ntamnao* 18
19		19 *aempy ntamnao* 1+18
20		20 *ynaoaempy ntamnao* 2+18
		. . .
24		24 *wramaekr* 24
25		25 *aempy wramaekr* 1+24
		. . .
30		30 *ptae wramaekr* 6+24
31		31 *aempy ptae wramaekr* 1+6+24
		. . .
36 (6^2)		(*ntaop*) *ptae* (big) 6
37		37 *aempy* (*ntaop*) *ptae* 1+(big) 6
50		*ynaoaempy tarwmpao* (*ntaop*) *ptae* 2+12+36
100		*eser wramaekr ptae ynaoaempy* 4+24+(36×2)
		. . .
216 (6^3)		*tarwmpao* 216
1296 (6^4)		(*ntaop*) *ntamnao* (big) 18
7776 (6^5)		(*ntaop*) *wramaekr* (big) 24

Table 2. Simple, moderate, and complex counting systems in Kanum
Source: Donohue (2008)

4. New Thinking

By taking these studies of line and number, an exploration of philosophies of mathematics education emerge. It is evident that the teacher in PNG can use the child's home knowledge and cultural practices to establish systematic mathematics in schools. This is in line with a critical mathematics education perspective (Andersson, 2010) as well as ethnomathematics (Francois & Stathopoulou, 2012). Valuing the child and his/her culture strengthens cultural diversity and the child's identity as far as culture and mathematical thinking are concerned (Owens, 2007/2008, 2012, 2015).

This study also establishes that there were cultures that have continued to the present from more than 15 000 years BCE that were using large numbers before Euro-Asian school mathematics or contact with people from Europe. They also had cultural ways of interpreting lines and had established a strong recognition of the continuous line that was used in art, ceremony, and in mathematical reasoning that linked the line to relationships with parts and the whole both of space and community.

These two key concepts of school mathematics have already existed in the cultures discussed in this article but they did not develop naked. Their development and abstraction are entrenched in cultural purposes, values, and aesthetics. Is school mathematics ever devoid of its cultural roots?

5. References

Amstrong, W. (1928). *Rossel Island*: an ethnological study. Cambridge, UK: Cambridge University Press.

Andersson, A. (2010). Making sense of critical mathematics education. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *MERGA33 - Shaping the Future of Mathematics Education* (pp. 37-44). Fremantle, Western Australia: Mathematics Education Research Group of Australasia.

Bowden, R. (~1990). Kwoma ceremonial houses. In N. Lukehuas, W. Kaufmann, D. Michell, L. Newton,L. Osmundsen, & M. Schuster (Eds.), *Sepik heritage*: tradition and change in Papua New Guinea. Proceedings of 1987 conference in Switzerland. Durham, NC: Carolina Academic Press.

Coiffier, C. (~1990). Sepik river architecutre: changes in cultural traditions. In N. Lukehuas, W. Kaufmann, D. Michell, L. Newton, & L. Osmundsen (Eds.), *Sepik heritage*: tradition and change in Papua New Guinea. Proceedings of 1987 conference in Switzerland (pp. 491-497). Durham, NC: Carolina Academic Press.

Donohue, M. (2008). Complexities with restricted numeral systems. *Linguistic Typology, 12*(3), 423-429. doi: 10.1515/LITY.2008.043.

Francois, K., & Stathopoulou, C. (2012). In-between critical mathematics education and ethnomathematics. A philosophical reflection and an empirical case of a Romany students' group mathematics education. *Journal for Critical Education Policy Studies, 10*(1), 234-247.

Haddon, K. (1930, reprinted in 1979). *Artists in strings*. New York, NY: AMS.

Harrison, S. (2006). *Fracturing resemblances*: identity and mimetic conflict in Melanesia and the West. New York, NY: Berghahn Books.

Hauser-Schäublin, B. (1996). The thrill of the line, the string, and the frond, or why the Abelam are a non-cloth culture. *Oceania, 67*(2), 81-106.

Jenness, D. (1920). Papuan cat’s cradles. *Journal of the Royal Anthropological Institute of Great Britain and Ireland, 50*, 299-326.

Kuchler, S. (1999). Binding in the Pacific: Between loops and knots. *Oceania, 69*(3), 145-156.

Lean, G. (1992). *Counting systems of Papua New Guinea and Oceania.* (Unpublished PhD Thesis), PNG University of Technology, Lae, Papua New Guinea. Retrieved from http://www.uog.ac.pg/glec/.

Lipka, J., Andrew-Ihrke, D., Koester, D., Zinger, V., Olson, M., Yanez, E. E., & Rubinstein, D. (2015). *Indigenous knowledge provides an elevant way to teach the foundations of mathematics.* Paper presented at the PME-NA 37 Annual Conference of the International Group for the Psychology of Mathematics Education - North America, Boston, MA.

Mackenzie, M. A. (1991). *Androgynous objects*: string bags and gender in central New Guinea. Chur, Switzerland: Harwood Academic Publishers.

Mimica, J. (1988). *Intimations of infinity: the mythopoeia (cultural meanings) of the Iqwaye counting and number systems*. Oxford, UK: Berg Publishers.

Muke, C. (2000). *Ethnomathematics*: Mid-Wahgi counting practices in Papua New Guinea. (Unpublished MEd thesis), University of Waikato, Waikato, New Zealand.

Opu, L. (nd). Mekeo designs *Mathematics, language and culture* Goroka, Papua New Guinea: University of Goroka.

Owens, K. (2001). The work of Glendon Lean on the counting systems of Papua New Guinea and Oceania. *Mathematics Education Research Journal, 13*(1), 47-71.

Owens, K. (2007/2008). Identity as a mathematical thinker. *Mathematics Teacher Education and Development, 9*, 36-50.

Owens, K. (2012). Identity and ethnomathematics projects in Papua New Guinea. In D. Jaguthsing, L. P. Cheng, & S. F. Ng (Eds.), *Mathematics education: expanding horizons*, Proceedings of 35th annual conference of Mathematics Education Research Group of Australasia. Singapore: MERGA.

Owens, K. (2015). *Visuospatial reasoning*: an ecocultural perspective for space, geometry and measurement education. New York, NY: Springer.

Owens, K., Lean, G., With Paraide, P., & Muke, C. (forthcoming, 2016). *The history of number*: perspective from Papua New Guinea and Oceania. New York, NY: Springer.

Paraide, P. (2009). Number in the Tolai culture - Occasional paper.

Pinxten, R., & François, K. (2011). Politics in an Indian canyon? Some thoughts on the implications of ethnomathematics. *Educational Studies in Mathematics, 78*(2), 261-273. doi: 10.1007/s10649-011-9328-z.

Saxe, G. (nd). Cultural development of mathematical ideas. Retrieved from http://www.culturecognition.com/.

Senft, G., & Senft, B. (1986). Ninikula fadenspiele auf den Trobriand Inseln Papua New Guinea. *Baessler-Archiv, 34*, 93-69.

Smith, L. T. T. R. (2005). *Decolonizing methodologies*: research and indigenous peoples. New York, NY: Zed Books.

Strathern, A. (1971). *The rope of moka*: big-men and ceremonial exchange in Mount Hagen, New Guinea. Cambridge, UK: University Press.

Thune, C. (1978). Numbers and counting in Loboda: an example of a non-numerically oriented culture. *Papua New Guinea Journal of Education, 14*, 69-80.

van Toom, P. (2006). *Writing never arrives naked*: early aboriginal cultures of writing in Australia. Canberra, Australia: Aboriginal Studies Press.

Vandendriessche, E. (2007). Les jeux de ficelle: une activité mathématique dans certainess sociétés traditionnelles (String figures: a mathematical activity in some traditional societies). *Revue d'histoire des mathématiques*, *13*(1), 7-84.

Vandendriessche, E. (2016). *String figures as mathematics*: an anthropological approach to string figure-making in oral traditional societies. Dortrecht, The Netherlands: Springer.

Vicedom, G., & Tischner, H. (1948). *Die mbowamb*: die kultur der hagenberg-staemme in oestlichen zentral-neuguinea. Hamburg, Germany: De Gruyter and Co.

Were, G. (2010). *Lines that connect*: rethinking pattern and mind in the Pacific. Honolulu, HA: University of Hawai'i Press.