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ORIGINAL ARTICLE

Early mathematical competencies and later achievement: insights from the Longitudinal Study of Australian Children

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Abstract International research suggests that early mathematical competence predicts later mathematical achievement. In this article, we explore the relationship between mathematical competencies at 4–5 years, as measured by teacher ratings, and later results on Years 3, 5, 7 and 9 *National Assessment Program – Literacy and Numeracy* (NAPLAN) numeracy tests. Data from a nationally representative sample of 2343 children participating in the Longitudinal Study of Australian Children (LSAC) are examined. In line with international studies, we report moderate correlations between preschool-entry mathematics and later NAPLAN numeracy test results. However, analysis of individual growth trajectories indicates that early mathematics predicts the initial (Year 3) level, but not subsequent growth. This suggests that early mathematical competencies are important for enhancing achievement in early schooling, but that the quality of mathematics education provided in the schooling years is critical for future development.

Keywords Early childhood · Assessment · Competencies · NAPLAN · National testing

Introduction

Both Australian and international research has established that young children engage with a range of mathematical concepts and processes prior to starting school (e.g. Gervasoni and Perry 2015; Sarama and Clements 2015). Recent research has also identified a link between children's early mathematical skills and their later mathematical achievement (e.g. Watts et al. 2014), with research noting, in particular, the predictive power of mathematical knowledge at school entry for later mathematical

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achievement (e.g. Duncan et al. 2007). An opportunity to explore the relationship between early mathematical competencies and later mathematical achievement has been afforded through the Longitudinal Study of Australian Children (LSAC) (Sanson et al. 2002). LSAC utilises a cross-sequential design to follow two cohorts of children: a Birth cohort of approximately 5000 children aged between 6 and 12 months; and a Kindergarten cohort of approximately 5000 children aged between 4 years 6 months and 5 years. This study focuses on children from the kindergarten cohort of LSAC when they were aged 4 to 5 years and examines both their early mathematical competencies and their later mathematical achievement, as measured by their results on the Years 3, 5, 7 and 9 *National Assessment Program – Literacy and Numeracy* (NAPLAN) numeracy tests. The aim of this study is to examine the extent to which, and how, children's mathematical competencies at age 4/5 years are related to their mathematical achievement later in school.

Background

Importance of early childhood education

The importance of early childhood education is well established. Conclusive international evidence demonstrates that early childhood is a vital period in children's learning and development (Department of Education, Employment and Workplace Relations [DEEWR] 2009), and that what happens in early childhood affects later development (Council of Australian Governments [COAG] 2009). There is consistent evidence (e.g. Karoly et al. 2006; Sammons et al. 2002) showing that high-quality early childhood education has a positive impact on young children's cognitive and social outcomes and adjustment to school (Elliott 2006). With approximately 87% of Australian children spending time in early childhood education and care settings before starting school (Baxter 2015), the educational significance of children's experiences and outcomes increases (Elliott 2006). In 2009, Australia's first national early childhood curriculum framework, *Belonging, Being and Becoming: The Early Years Learning Framework for Australia* (EYLF) (DEEWR 2009), was established "to assist educators to provide young children with opportunities to maximise their potential and develop a foundation for future success in learning" (p. 5). Furthermore, the EYLF contributes to realising COAG's (2009) vision that by 2020 "all children have the best start in life to create a better future for themselves and for the nation" (p. 4). Participation in effective early childhood education has a number of significant benefits, including improved child development, improved school readiness and performance at school and improved educational attainment (Robinson et al. 2011).

Young children's mathematical competencies

Doig et al. (2003) have suggested that the increasing numbers of children participating in early childhood programmes, and the growing recognition of the importance of mathematics in general, provide compelling reasons for understanding children's mathematical development in the early childhood years. Indeed, children begin developing mathematical skills from a very young age, with international research showing that babies and toddlers demonstrate competence in regard to a range of mathematical concepts and processes. In a study of 1003 children aged between 30 and 33 months, Reikerås et al. (2012) found that the toddlers showed mathematical competencies in all areas observed (encompassing number and counting, geometry and problem solving). Similarly, Björklund's (2008) study of children aged between 13 and 45 months demonstrated that toddlers interact with concepts of dimensions or proportions, location, extent, succession and numerosity and use a range of strategies to express their understanding.

Research with preschool-age children has further demonstrated the ways in which children engage with a range of complex mathematical ideas in the early childhood years. Kinnear and Clark (2014) explored data modelling with a group of 5-year-old children and found that children were able to construct data models which provided evidence of their ability to use existing data, knowledge of the data context and probabilistic reasoning to make predictions. Research by MacDonald (2010, 2012) revealed that children aged 4 to 6 years were able to demonstrate knowledge of measurement concepts such as identifying measureable attributes, comparing attributes such as height or area and using units to describe measurements. A number of studies have shown that young children engage with powerful mathematical processes such as problem solving, argumentation and mathematical reasoning (English and Mulligan 2013; Fielding-Wells and Makar 2015; Makar 2014).

Furthermore, several Australian studies have found that much of the content that forms the mathematics curriculum for the first year of school is already understood clearly by many children on arrival at primary school (Clarke et al. 2006; Gervasoni and Perry 2015; MacDonald 2010), a finding echoed in international studies (e.g. Aubrey 1993; Wright 1994). The seminal Australian study, the *Early Numeracy Research Project* (see, for example, Clarke et al. 2006) investigated the mathematical knowledge of over 1400 children entering primary school and found that most children already understood much of the mathematical content that was to be *taught* in the first year (Clarke et al. 2006). Gould (2012), reporting on data from a standardised interview schedule from 65,000 children entering the first year of school, found that approximately 16% of the children entering kindergarten (first year at school) showed the facility with number expected of children in Year 1 (second year at school). Similar findings were noted by Gervasoni and Perry (2013, 2015) in their longitudinal evaluation of the *Let's Count* early numeracy programme. Approximately 75% of the children were able to demonstrate the *Australian Curriculum – Mathematics* Foundation Standard "Students make connections between number names, numerals and quantities up to ten" (Australian Curriculum, Assessment, and Reporting Authority [ACARA] 2013) before they started school (Gervasoni and Perry 2015).

Differences in early mathematics achievement

Gender Research findings concerning gender differences in early mathematics achievement vary a great deal. As highlighted by Williams et al. (2016), "although some studies have found gender differences in math achievement as early as kindergarten (Jordan et al. 2006), others do not (Ponitz et al. 2009)" (p. 201). Furthermore, "documented gender differences in mathematical ability at 5 years of age have in some instances favoured boys (Jordan et al. 2006), and in other instances favoured girls

(Mensah and Kiernan 2010)" (p. 201). On the other hand, research concerning mathematics achievement in the later schooling years has produced more consistent findings, in favour of boys. Atweh et al. (2012) reported a widening gender gap since the 1990s, which is consistent with more recent research demonstrating gender differences, favouring boys, in school-age mathematics achievement (Carmichael et al. 2014; Weis et al. 2013).

Socio-economic position The influence of a child's socio-economic position (SEP) on their academic achievement, generally, is well established (Sirin 2005), as is the influence upon children's mathematics achievement, specifically Biddle (1997) and Darling-Hammond (1999). Most of the research in this area has focused on children's achievement in the later schooling years, but it has been found that higher family SEP is related to higher teacher-reported mathematics achievement ratings as early as kindergarten (Rimm-Kaufman et al. 2003).

Early mathematical competencies and later achievement

In recent years, studies have emerged which indicate the predictive power of early mathematical knowledge for later achievement, both in mathematics, specifically, and in terms of more general academic achievement. Watts et al. (2014) conducted a tracking study of 1364 children from 4.5 to 15 years and found that there were statistically significant associations between preschool mathematical ability and adolescent mathematics achievement, even after accounting for early reading, cognitive skills and family and child characteristics. Furthermore, gains in mathematical knowledge from preschool through first grade were even more predictive of mathematics achievement at age 15 than preschool knowledge. Geary et al. (2013) conducted a longitudinal study of 180 children from kindergarten (first year of school) through to age 13 and found that early number system knowledge predicted functional numeracy more than 6 years later, controlling for intelligence, working memory, in-class attentive behaviour, mathematical achievement, demographic and other factors. Duncan et al. (2007) examined six longitudinal data sets to identify links between school-entry academic, attention and socio-emotional skills and later school reading and mathematics achievement. Across all six studies, the strongest predictors of later achievement were school-entry mathematics, reading and attention skills. A meta-analysis of the results showed that early mathematics skills have the greatest predictive power. This finding was consistent for boys and girls, and for children from high and low socio-economic backgrounds. Similarly, Claessens et al. (2009) used data from a nationally representative sample of children from the Early Childhood Longitudinal Study – Kindergarten Cohort (ECLS-K) to estimate the predictive power of school-entry academic, attention-related and socio-emotional skills for reading and mathematics achievement in first, third and fifth grades. School-entry mathematics skills were consistently predictive of fifth grade achievement. Early mathematics skills were not only highly predictive of later mathematics achievement, but of later reading achievement as well. Mathematics skills were the single most important set of kindergarten-entry skills emerging from the analyses.

Given the growing body of research which suggests a relationship between early mathematics and later school achievement (Levine et al. 2010), it is important to ascertain the extent to which children's early mathematical competencies predict later mathematical achievement.

The current study

Most of the studies cited in the last section reporting associations between school-entry mathematics achievement and later achievement are based on US data, and none have specifically analysed this association in an Australian context. National variations, however, exist in school structures and mathematics curricula, suggesting US results may not be replicated in other jurisdictions. As noted by Duncan et al. (2007), for example, a British study that they analysed did not report any association between school-entry level mathematics and later academic achievement. Further, these studies have tended to base their findings on bivariate relationships between early and later mathematics achievement, ignoring growth in mathematics achievement during the intervening period. Such analyses, however, have long been regarded as inadequate for the systematic study of change (e.g. Rogosa and Willett 1985; Bryk and Raudenbush 1987), primarily because they do not fully utilise the available information. The current study seeks to build on the studies reported earlier by analysing Australian longitudinal data from a large representative sample and studying change through the analysis of individual growth trajectories, as recommended by Bryk and Raudenbush (1987). As such, it considers the research question: To what extent, and how, are competencies at age 4/5 related to mathematical achievement in Years 3, 5, 7 and 9?

Method

Sample

The study is based on 2343 students from the LSAC K-cohort for whom mathematical competency assessments were available at age 4/5 and for whom at least three NAPLAN numeracy test results were available when these students were in Years 3, 5, 7 and 9. Details of the design and implementation of LSAC are available in Sanson et al. (2002). The size of this sample is considerably smaller than the original 4983 contained in the LSAC K-cohort. At least three numeracy test results were available for just 3500 of the children in the K-cohort. Many (55%) of the 1483 children without these results, however, were no longer participating in the LSAC study. Further, only 3176 children in the K-cohort had teacher reports of their mathematical competency at the age of 4/5. Some of the children ($n = 221$) without these data did not attend school at that age, and parents of others ($n = 96$) declined to give permission for their child's teacher to be contacted. It can be assumed, however, that for the majority of cases without these data ($n = 1490$) teachers declined to provide this information about their students.

Variables

Outcome Our measure of mathematics achievement, the main outcome variable in the study, are the scores in the NAPLAN numeracy tests held when the students were in Years 3, 5, 7 and 9. These tests contain a mix of multiple-choice and short response items that reflect the *Australian Curriculum – Mathematics* [AC-M] (ACARA 2013). This alignment with the AC-M adds to the validity of these tests as measures of mathematics achievement. It is acknowledged, however, that as with all formal tests there are concerns regarding the validity of the NAPLAN numeracy tests (e.g. Harris et al. 2013). Scores from these tests are developed using a Rasch measurement paradigm, in that irrespective of his or her year level at school, each child's score is placed on a scale ranging from 0 to 1000. This feature of the scoring allows for meaningful longitudinal comparisons and because of this our reporting of results is based on this scale. Sample means and standard deviations for the outcome are shown in Table 1, which also includes population figures.

Explanatory Teachers were asked to answer "Yes" or "No" to whether their student had the ability to achieve each of five early mathematical competencies: (C1) sort and classify objects by shape or colour; (C2) count the number of a few objects accurately; (C3) count to 20; (C4) recognise numbers; and (C5) do simple addition with concrete materials. For this sample, 97% of children attained C1, 96% attained C2, 65% attained C3, 77% attained C4 and 63% attained C5. The majority of students (62%) gained four or more of the five competencies. Hereafter, these students are referred to as high competency. The total number of competencies achieved by the student (Ncomp) was the main explanatory variable in the study and ranged from 0 to 5 ($M = 3.7$, SD = 1.2); however, we also use each of the individual competencies as explanatory variables in separate analyses.

Demographic variables A number of student and family variables were also considered because they are known to be predictors of mathematics achievement. Of the 2343 students in the sample, one half were male and their ages[1] when their data were first collected ranged from 4.3 to 5.6 years ($M = 4.8$, SD = 0.20). Students undertook an initial test of academic ability, as measured by the *Peabody Picture Vocabulary Test*, 3rd Edition (PPVT-III) (Dunn et al. 1997). The PPVT-III is a measure of children's receptive vocabulary which is given verbally. The examiner shows the child a page with four pictures and speaks a word, and the child is required to identify the picture that represents the word. A shortened version was used in LSAC as a screening test of verbal ability (Rothman 2005). PPVT-III scores for this sample ranged from 31 to 85 ($M = 65.1$, SD = 5.8). Finally, the SEP of the child was considered, using a standardised index of socio-economic status based on the unweighted mean of the following: the family's income, both parents' educational attainments and the status associated with each parent's employment. This index, described in Blakemore et al. (2009), ranged from −2.3 to 3.0 ($M = 0.2$, SD = 1.0).

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[1] This was the age when children's parents completed the survey. Teacher data were obtained once parental permission was obtained.

Table 1 NAPLAN means and standard deviations for sample and population

	Year 3, 2008	Year 5, 2010	Year 7, 2012	Year 9, 2014
Sample	426 (73)	506 (71)	559 (74)	606 (73)
Sample size	$n = 1769$	$n = 2292$	$n = 2272$	$n = 1863$
Population	397 (70)	489 (70)	538 (74)	588 (71)

Analysis

Investigation of bias Initially, we investigated possible bias in our sample by comparing K-cohort children included and excluded from our study across a number of key variables including sex, age, socio-economic background and scores in the PPVT.

Bivariate relationships We then undertook simple bivariate analyses between key explanatory variables and the outcome variable. Correlations between scores in each of the NAPLAN numeracy tests and the total number of competencies achieved were calculated. Mean differences between high- and low-competency children in subsequent NAPLAN tests were also compared. Finally, we examined mean differences in later NAPLAN tests between children who had gained and did not gain each of the separate competencies.

Multivariate analyses Given that demographic factors such as children's sex and socio-economic background are known to impact their mathematical achievement, it was necessary to control their influence through regression techniques. In particular, multilevel regression models were used because we sought to incorporate all of the achievement data available for these students during the period 2008 through to 2014. The violation of independence between an individual's achievement scores during the period is overcome by modelling individual growth trajectories at one level, and explanatory variables at a second level. The models also accommodate missing data on the outcome variables. In this study, estimates were obtained using R (R Development Core Team 2011) and in particular the Multilevel package (Bliese 2012), as described in (Faraway 2006). Our analysis commenced with the total number of competencies as the key predictor variable. We then used each competency separately as the key predictor in a series of models.

Results

Analysis of possible bias in the sample

Given the substantial reduction in sample size, we investigated possible bias between students in the included sample ($n = 2343$) of the K-cohort and those in the excluded sample ($n = 2640$). There were no significant differences between the mean ages of children included in the sample and those excluded. Similarly, there were no significant differences in the proportions of boys in the included and excluded samples. However, there were significant differences in the mean socio-economic status (SEP) scores of children in these samples, with those included in the study more likely to come from wealthier family backgrounds than those excluded from the study ($d = .32, t = 11.3, p < .01$). Further, K-cohort children included in the study were more likely to score higher on the PPVT than those excluded from the study ($d = .29, t = 9.4, p < .01$). Thus, our sample of children was more likely to come from slightly wealthier families and score higher on the PPVT than those excluded. These results are consistent with the NAPLAN means reported in Table 1, which shows that our sample performed slightly better than the population on each of the four tests.

Bivariate analyses

The results of bivariate analyses are reported in Table 2, which lists relationships between each of the key explanatory variables and subsequent achievement. As is seen from the table, the total number of competencies (Ncomp) gained at age 4/5 was moderately correlated with mathematics achievement in Year 3 ($r = .3, p < .01, \mathrm{df} = 1767$), Year 5 ($r = .3, p < .01, \mathrm{df} = 2290$), Year 7 ($r = .3, p < .01, \mathrm{df} = 2270$) and Year 9 ($r = .3, p < .01, \mathrm{df} = 1861$).

As is also seen in the table, high-competency students, those with four or five competencies, achieved significantly higher than their low-competency peers in Year 3 ($d = .6, t = 11.0, p < .01$), Year 5 ($d = .5, t = 12.0, p < .01$), Year 7 ($d = .6, t = 13.0, p < .01$) and in Year 9 ($d = .6, t = 12.0, p < .01$). These students achieved on average more than one half a standard deviation higher than their peers and appeared to maintain that advantage throughout their schooling years.

Each of the individual competencies was assessed against later NAPLAN performance. The students who were able to sort and classify (C1) at age 4/5, on average, achieved significantly better than their peers in Year 3 ($d = .6, t = 4.2, p < .01$), Year 5 ($d = .5, t = 3.6, p < .01$) and Year 7 ($d = .6, t = 4.2, p < .01$), but not significantly better in Year 9. Similar results were evident for students who were able to count objects (C2). Interestingly, those students who could count to 20 (C3) at age 4/5 achieved significantly higher than their peers in Year 3 ($d = .6, t = 11.3, p < .01$) and broadly maintained this advantage in Year 5 ($d = .5, t = 12.2, p < .01$), Year 7 ($d = .6, t = 13.4, p < .01$) and Year 9 ($d = .6, t = 11.5, p < .01$). Students competent with recognising numbers (C4) and doing simple additions (C5) reported similar results.

Students' SEP was moderately correlated with achievement in Years 3 and 5 ($r = 0.3$), and more strongly in Years 7 and 9 ($r = 0.4$). As is seen in Table 2, students' measures of verbal ability at age 4/5 (PPVT) were correlated with mathematics achievement in each year that it was measured. Also reported in the table is a slight advantage for boys, who achieved 0.13 standard deviations higher than girls in Year 3 and increased this advantage slightly in the remaining years.

Multivariate analyses

The modelling process applies lines of best fit to the three or four NAPLAN outcomes for each of the 2343 students. The intercepts and gradients from each of these 2343 lines, in turn, form two response variables that are predicted by the explanatory and

Table 2 Bivariate relationships between key explanatory variables and subsequent achievement

Explanatory variable	Effect size (*d*)	Group size (*n*)	Test statistic	*p*
Relationship with Y3 (2008) achievement				
Total number of competencies (Ncomp)		1769	$r = 0.3$	< 0.01
High competency vs low competency	0.6	1084/685	$t = 11.0$	< 0.01
C1 (sorting and classifying)	0.6	1718/51	$t = 4.2$	< 0.01
C2 (counting objects)	0.7	1688/81	$t = 6.2$	< 0.01
C3 (count to 20)	0.6	1147/622	$t = 11.3$	< 0.01
C4 (recognise numbers)	0.5	1339/430	$t = 9.6$	< 0.01
C5 (simple additions)	0.4	1173/596	$t = 8.2$	< 0.01
SEP		1768	$r = 0.3$	< 0.01
PPVT		1629	$r = 0.3$	< 0.01
Male	0.1	906	$t = 2.7$	< 0.01
Relationship with Y5 (2010) achievement				
Total number of competencies (Ncomp)		2292	r = 0.3	< 0.01
High competency vs low competency	0.5	1433/859	$t = 12.5$	< 0.01
C1 (sorting and classifying)	0.5	2233/59	$t = 3.6$	< 0.01
C2 (counting objects)	0.4	2193/99	$t = 3.5$	< 0.01
C3 (count to 20)	0.5	1488/804	$t = 12.2$	< 0.01
C4 (recognise numbers)	0.5	1756/536	$t = 9.3$	< 0.01
C5 (simple additions)	0.4	1448/844	$t = 9.2$	< 0.01
SEP		2290	$r = 0.3$	< 0.01
PPVT		2122	$r = 0.2$	< 0.01
Male	0.2	1147	$t = 4.9$	< 0.01
Relationship with Y7 (2012) achievement				
Total number of competencies (Ncomp)		2272	$r = 0.3$	< 0.01
High competency/low competency	0.6	1420/852	$t = 12.5$	< 0.01
C1 (sorting and classifying)	0.6	2211/61	$t = 4.2$	< 0.01
C2 (counting objects)	0.4	2173/99	$t = 4.1$	< 0.01
C3 (count to 20)	0.6	1473/799	$t = 13.4$	< 0.01
C4 (recognise numbers)	0.5	1741/513	$t = 10.2$	< 0.01
C5 (simple additions)	0.5	1433/839	$t = 10.4$	< 0.01
SEP		2270	$r = 0.4$	< 0.01
PPVT		2102	$r = 0.3$	< 0.01
Male	0.2	1133	$t = 4.7$	< 0.01
Relationship with Y9 (2014) achievement				
Total number of competencies (Ncomp)		1863	$r = 0.3$	< 0.01
High competency/low competency	0.6	1220/643	$t = 11.6$	< 0.01
C1 (sorting and classifying)	0.3	1824/39	$t = 2.1$	> 0.05
C2 (counting objects)	0.2	1795/68	$t = 1.4$	> 0.05
C3 (count to 20)	0.6	1253/610	$t = 11.5$	< 0.01
C4 (recognise numbers)	0.5	1478/385	$t = 8.3$	< 0.01
C5 (simple additions)	0.4	1145/718	$t = 9.1$	< 0.01
SEP		1861	$r = 0.4$	< 0.01

Table 2 (continued)

Explanatory variable	Effect size (*d*)	Group size (*n*)	Test statistic	*p*
PPVT		1723	$r = 0.2$	< 0.01
Male	0.2	912	$t = 3.5$	< 0.01

demographic variables. More formally, the model assumes that, at the student level, growth in NAPLAN numeracy is linear and expressed as:

$$Y_{ti} = {}_{0i} + {}_{1i}t + {}_{ti}Y_{tij} = {}_{0ij} + {}_{1}\mathrm{t} + {}_{tij} \tag{1}$$

where, Y_{ti} is the NAPLAN score of the *i*th student at time $t = 0$ (2008) to 3 (2014), β_{0i} the random intercept, β_{1i} the random slope and ϵ_{ti} the error term. The terms β_{0i} and β_{1i} then form the response variables for two simple linear regression equations, each with their own random component, which is explained using predictor variables. The variance associated with the error term (ϵ_{ti}) is thus partitioned into the variance associated with the random intercepts (u_{0i}), the variance associated with the random slopes (u_{1i}) and the residual variance (ε_{ti}).

As recommended by Faraway (2006), the outcome variable was modelled against time prior to the inclusion of predictor variables. During this stage, the error structure was assessed to be appropriate and a likelihood ratio test confirmed that the inclusion of a random slope significantly improved model fit. The results of this model are reported as Model A in Table 3.

With results from all 2343 students considered, this model predicts that the mean achievement result should be 437 in Year 3 and 614 in Year 9, both figures higher than

Table 3 Results of model predicting growth from total number of competencies

Variable	Model A		Model B	
	Estimate	SE	Estimate	SE
Fixed effects				
Intercept	437	1.5	379.0	4.6
Time	59	0.4	58.3	0.5
Ncomp			12.7	1.1
SEP			15.9	1.5
PPVTz			15.4	1.4
Male			17.5	2.5
Time × *SEP*			2.7	0.5
Time × PPVTz			−2.2	0.5
Random effects				
Intercept variance		4003		2871
Slope variance		112		107
Residual variance		1188		1175

those reported in Table 1. This null model also provides estimates for each of the variances of the random effects, shown at the bottom of the table.

Following the fitting of Model A, the total number of competencies (Ncomp), together with demographic variables, was entered into the model as predictors of the random intercept. These variables were also tested as predictors of the random slope. Significant predictors are shown as Model B in Table 3. Those interacting with time, in this case SEP and the standardised PPVT score (PPVTz), are also predictors of the random slope.

As shown in Table 3, after controlling for demographic variables, the number of competencies students gained at age 4/5 was a significant predictor of the random intercept but not of the random slope. Thus, the model suggests that mathematical competence at age 4/5 is an important predictor of achievement in Year 3, but not of subsequent growth in achievement. The model also predicts that boys will achieve 17.5 points higher than girls in Year 3, but that gender will not influence subsequent growth.

Interestingly, SEP predicted both the random intercepts and slopes. Students from homes with an SEP one standard deviation higher than average (SEP = 1) are predicted to achieve 15.9 points higher in Year 3 than students from homes with an average SEP and then increase this advantage in Year 9 ($t = 3$) to 24.0[2] points. Similarly, the standardised PPVT-III score of students at age 4/5 (PPVTz) predicted their initial score and also their growth. Interestingly, the effect of PPVTz on growth was negative, suggesting a waning effect. Though not reported in Table 3, age differences when data were first collected did not impact later achievement.

Dichotomous variables representing each of the five competencies were entered separately into the model instead of Ncomp; each of the corresponding models is reported in Table 4. As reported in the table, results for each competency were fairly similar, with the exception of C2 (counting objects), which had a significant negative interaction with time. Children with this competency at age 4/5 were likely to score 27.5 points higher in the Year 3 NAPLAN test than their peers without this competency; but by Year 9 ($t = 3$), this difference is predicted to have reduced by 18.6 points[3] to just 8.9 points. The models suggest that, with all other competencies, success predicted a positive impact in the Year 3 NAPLAN test but not on subsequent growth.

Discussion and limitations

Our analysis clearly demonstrates the importance of mathematics in early childhood for later achievement on the Year 3 NAPLAN numeracy test. Our bivariate analyses confirmed results from earlier studies (e.g. Duncan et al. 2007; Watts et al. 2014) that the achievement of mathematical competencies in early childhood is moderately associated with later mathematical achievement, even as late as Year 9. Moreover, our results show that students gaining four or more of the five competencies in early childhood, on average, achieved 0.5 standard deviations higher in Year 3 NAPLAN numeracy results and maintained this gap over their schooling years. In regard to the mechanism by which early mathematical competencies predict later achievement, our

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[2] Using the interaction term in Model B, with Time = 3, SEP = 1 and $\beta = 2.7$, giving an increase of 8.1 points.

[3] Using the interaction term in Model C2, with Time = 3, Competency = 1 and $\beta = -6.2$.

Table 4 Results of models predicting growth from each separate competency

Variable	Model C1		Model C2		Model C3		Model C4		Model C5	
	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
Fixed effects										
Intercept	402.3	8.1	400.5	6.9	408.5	2.6	408.7	3.0	419.6	2.1
Time	58.3	0.5	64.2	2.3	58.3	0.5	58.3	0.5	58.3	0.5
Competency	25.1	8.0	27.5	6.9	27.2	2.7	23.3	3.0	20.1	2.7
SEP	18.2	1.5	18.2	1.5	16.5	1.5	17.1	1.5	17.3	1.5
PPVTz	18.5	1.4	18.2	1.4	16.5	1.5	17.5	1.4	17.1	1.5
Male	15.7	2.5	15.6	2.5	17.3	2.5	16.4	2.5	15.7	2.5
Time × SEP	2.7	0.5	2.8	0.5	2.7	0.5	2.7	0.5	2.7	0.5
Time × PPVTz	− 2.3	0.5	− 2.1	0.5	− 2.3	0.5	− 2.3	0.5	− 2.3	0.5
Time × competency			− 6.2	2.3						
Random effects										
Intercept variance		3061		3048		2923		2969		3002
Slope variance		108		107		107		107		108
Residual variance		1174		1174		1174		1174		1174

analysis of growth trajectories indicates that early achievement, in terms of the total number of competencies gained by the children, predicted performance in Year 3, but not subsequent growth. Whereas early childhood mathematics is important for launching children's growth trajectories in mathematics achievement, subsequent learning is equally important in maintaining growth.

With regard to the actual competencies gained by these students at age 4/5, our findings were consistent with those of Geary et al. (2013) in that students who were able to count to 20, recognise numbers and do simple additions were more likely to score higher in Year 3 NAPLAN numeracy, even after controlling for individual and family factors. However, our analysis does not agree with that undertaken by Watts et al. (2014), in that there was no evidence to suggest that these early childhood competencies were predictive of mathematics achievement in adolescence, except via their contribution to performance in early primary school. This finding could be due to the differences in measures of early competencies and later mathematics achievement, with Watts using standardised tests on both occasions. However, Watts et al. (2014) also used average growth models, with research suggesting that individual growth models are better able to model the variance/covariance structure in longitudinal studies (Kwok et al. 2007). Interestingly, our analysis suggests that children who could not count objects at age 4/5 appear to slowly catch up with their peers as they age; a time interaction that is not evident for the other competencies. This may be due to the fundamental nature of this competency in mathematics, with most children mastering this before they begin school or shortly thereafter (Gervasoni and Perry 2015).

The only factor that positively predicted growth was socio-economic position, which is of concern as it suggests that children from wealthier families have an initial advantage and that this advantage increases during the school years. This finding is consistent with Sirin's (2005) meta-analytic review of research pertaining to socio-economic status (SES) and academic achievement, which found a medium to strong SES-achievement relation and concluded that school success is greatly influenced by students' family SES. With respect to gender, our analyses show that boys have an initial advantage and maintain that advantage. However, gender did not contribute to subsequent growth, suggesting that reported gender gaps are evident in the early years but may remain constant for each cohort. This result does not support the widening gap identified by Atweh et al. (2012); rather, it suggests that any gaps are established prior to starting school and that such gaps may be maintained—but not necessarily widen—during the schooling years. Interestingly, children's receptive vocabulary (PPVTz) was positively associated with Year 3 mathematics achievement but negatively associated with subsequent growth, suggesting that it too is a fundamental skill achieved by most children in the early years, and thus of less importance in later years.

Of course, it is important to note that our analysis has been undertaken within the limits of the LSAC study design and its measures, including the use of NAPLAN data as a measure of mathematics achievement. A limitation of the LSAC assessment of early mathematical competencies is that it was based on educators' judgements only and indeed was restricted to children enrolled in formal early childhood programmes. Furthermore, the early mathematical competence scale used in LSAC is very limited in its consideration of mathematical skills, and there may be more appropriate measures elsewhere; however, the analysis could only include data from the existing LSAC study. We also acknowledge the bias of the sample, and recognise that a more representative sample may see an independent effect of early competencies on later achievement due to the benefits of early childhood education for children from low-SEP families. Our study points to the need for a new, large-scale study that utilises a representative sample and more comprehensive assessment of early mathematical ability to further interrogate the relationship between early competencies and later achievement. This is consistent with Peter-Koop and Scherer's (2012) call that research leading to the development of a detailed competency model that goes beyond number and integrates the different content areas of mathematics is still needed. Moreover, such a study needs to explore the growth in mathematical competencies in the crucial early childhood years.

Conclusion

Our study identified a reasonable correlation between preschool-entry mathematics and Year 9 NAPLAN results. However, looking at growth trajectories suggests that early mathematical competence predicts the initial (Year 3) level results, but not subsequent growth. In turn, Year 3 then predicts Year 5, Year 5 predicts Year 7 and so forth. This is important to note, because it emphasises that while early competency in mathematics is predictive of children's mathematics achievement at Year 3, it is what happens afterwards that contributes to growth. This reflects the discourse around "early childhood education getting children off to the best start", but also emphasises that a good start alone is not enough—what happens in the schooling years is critical.

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