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RESEARCH ARTICLE

An Investigation of the Application of the Golden Ratio and Fibonacci Sequence Associated with the Chart of Nuclides

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Abstract:

The aim of this investigation was to examine the relationship between the golden ratio and atomic structure. The chart of nuclides was evaluated for evidence of a true golden ratio. That is, the neutron number divided by the proton number needed to equal 1.618 and the atomic number (neutrons plus protons) divided by the neutron number needed to equal 1.618. Across the chart of nuclides, this ratio approximated the golden ratio to a varying degree of accuracy for some nuclides but rarely for both calculations. This relationship (golden ratio in the chart of nuclides) has not been previously reported. It was determined that specific nuclides exhibit the golden ratio between protons and neutrons, and that these produce a trend line on the chart of nuclides. The Fibonacci sequence is mathematically evident and this information can be used to postulate new nuclides or elements.

Keywords: Golden ratio, Fibonacci sequence, Chart of nuclides, Super-heavy nuclei

Introduction

In 1869 Dimitri Mendeleev organized the 63 known elements at the time into logical groupings that produced the first periodic table (1). With the discovery of new elements, the periodic table has been expanded and now boasts 118 elements with the discovery of Oganesson in 2002, although it was not officially named until 2016 (1). The search continues for elements 119 and 120. Producing heavy nuclei is challenging with 250 days of bombardment of element 97 with element 48 producing 6 atoms of element 117, whose half-life is measured in milliseconds.

While the search beyond element 118 continues, this requires bombardment with heavier projectiles (eg. elements 50, 54, 58 or 64) and higher flux accelerators. The motivation lies in the theory that there is an "island of stability" among the super-heavy nuclei (super-heavy being elements 104 and above) (2). Indeed, this mysterious island of stability was postulated in the 1960s by Glenn Seaborg and comes with the promise of super-heavy nuclei with half-lives measured in year rather than fractions of a second (figure 1). The theoretical target zone for stability is elements 114 to 120 with a neutron number in the order of 184.

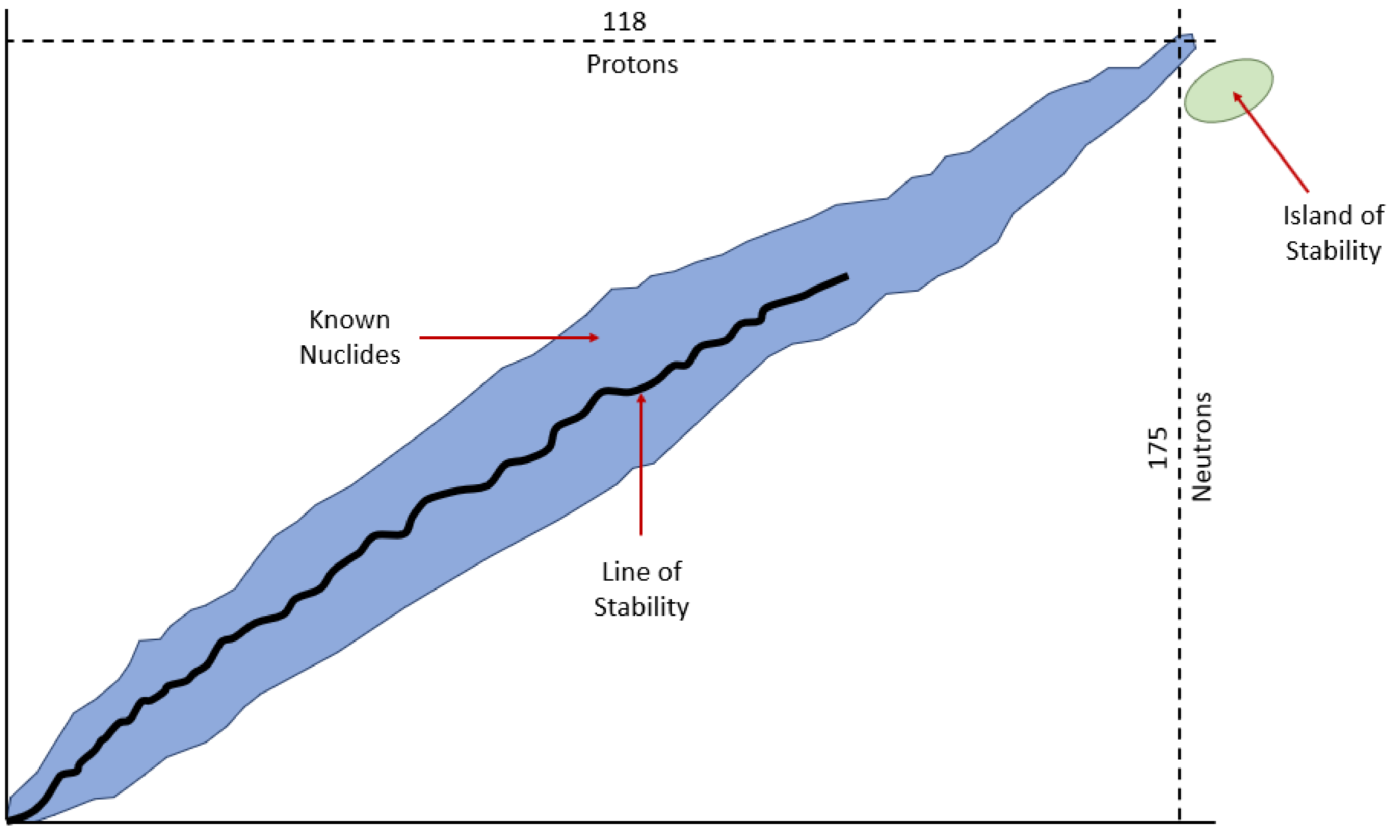


Figure 1: Schematic representation of the chart of nuclides (blue) and the island of stability (green).

While the nuclear forces of nuclides close to the line of stability are well understood and readily modelled, the same is not true for nuclides more distant to the line of stability (3). Super-heavy nuclei, especially if found with longer half-lives, would be a rich source of understanding of the complexity of nuclear forces. Moreover, the origin of heavy elements from iron to uranium on earth remains a mystery so the production of study of super-heavy nuclei may shed light on some of these origins (3). While efforts to synthesise these super-heavy nuclei attract attention, one should consider that natural production of a range of short lived super-heavy nuclei can occur in stellar nucleosynthesis and the pathway from production to heavy elements, including potentially those found on earth, are yet to be charted (3). Indeed, synthetic production of super-heavy nuclei on earth extends insight and understanding into super-heavy nuclei production in rapid neutron capture processes (r-process) in astrophysics (2,3). Nazarewicz (3) identified a number of key questions driving super-heavy nuclei research:

1). What are the heaviest nuclei that can exist?
2). Are there long-lived super-heavy nuclei?
3). Can super-heavy nuclei be produced in stars?
4). What other elements will populate the periodic table?
5). What are the chemical properties of super-heavy nuclei?

The aim of this investigation was to examine the relationship between the golden ratio and atomic structure. The specific objectives include:

1). To determine if specific nuclides in the chart of nuclides exhibit the golden ratio amongst relative proton and neutron number.

2). To determine whether any such nuclides produce a trend within the chart of nuclides (golden line).

3). To determine whether within the chart of nuclides, a group of nuclides follows the Fibonacci sequence.

4). To identify any nuclides that have perfect golden ratio properties (golden nuclide).

5). To determine whether the above, if demonstrated, could be used to postulate the existence of yet to be discovered nuclides.

Applications of the Golden Ratio in Atomic Physics

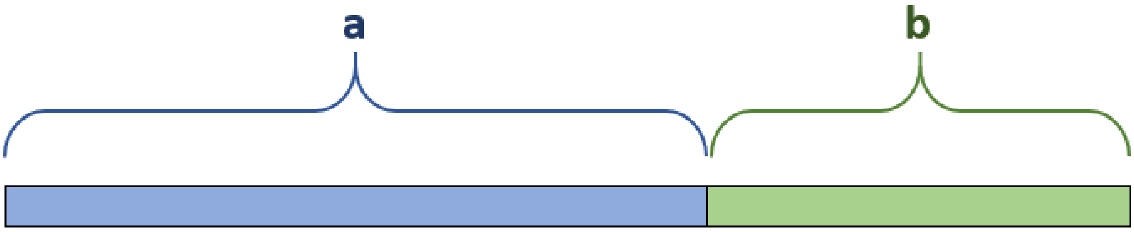
The golden ratio is a mathematical ratio denoted by Phi (pronounced fee) calculated as (4):

$$Phi\ (\varphi) = \frac{(1 + \sqrt{5}\ )}{2} = 1.618 \ldots \ldots$$

The golden ratio is more than the ratio of the long portion to the short portion of a line being equal to 1.618 .... As outlined in figure 2, it requires that both the long (a) and short (b) ratio together with the ratio of the long portion (a) to the total length (a+b) each equal 1.618..... The golden ratio has given rise to numerous other related concepts like the golden angle, golden spiral, golden triangle, golden rectangle, golden proportion and golden mean (5). The golden ratio is also used to refer to the ratio between successive Fibonacci numbers although this only varies after Fibonacci number 8 (5). The Fibonacci sequence is a series of numbers that are the sum of the previous 2 numbers:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610 ...

The ratio for each pair is tabulated in table 1 and the golden rectangles produced by successive pairs can be used to generate a golden spiral (figure 2).



$$\frac{a}{b} = \varphi = \frac{a+b}{a} = 1.618.....$$

Figure 2: Calculating the golden ratio

Table 1: golden ratio for sequential Fibonacci sequence pairs noting that the values are only valid from number 8.

Fibonacci Sequence	Previous Value	Ratio
1	0	-
1	1	1.0
2	1	0.5
3	2	1.5
5	3	1.6667
8	5	1.6
13	8	1.625
21	13	1.615
34	21	1.619
55	34	1.618
89	55	1.618
144	89	1.618
233	144	1.618
377	233	1.618
610	377	1.618

The golden ratio can be found in an array of different applications of mathematics and more broadly in nature. These natural occurrences of the golden ratio are visually appealing and have, therefore, been reproduced in design thinking (eg. architecture, art etc). Despite this, the popularity of the Fibonacci sequence and the golden ratio has been recently driven by the fiction novel “the Da Vinci Code” by Dan Brown. It is worth pointing out that in many cases, the golden ratio is not real with numbers anywhere in the order of 1.618 being promoted as the golden ratio. While it is true that the error associated with small numbers (less than 8) in the Fibonacci sequence are distinct from Phi, true values the golden ratio should be very close to 1.618; perhaps a weak value might range 1.61 to 1.625. Indeed, Phi experts have been critical of false claims of the golden ratio in art and architecture (including pyramids) as fabricated (4). For example, selectively choosing numbers that approximate 1.6 while ignoring other ratios in art and architecture, or by using current values in preference to original values for ancient structures like the pyramids (4). Even the famous nautilus shell has a spiral quite distant to the golden spiral (4). Conversely, strong golden ratios are exhibited in sunflower seeding and DNA (4).

There have been few published works relating to the application of Fibonacci sequences or the golden ratio in atomic physics. In 1963, Wlodarksi (6) published a short summary of the golden ratio in atomic physics suggesting:

1).The proton to neutron ratio approximates 0.6 in near stable nuclei although varies widely;

2).Fission fragments approximate the golden ratio;

3).The distribution of isotopes of even stable elements approximates the golden ratio;

4).The ratio of down to up direction particle emissions from unstable nuclei approximates the golden ratio.

There are several important points to consider. Firstly, the golden ratio and Fibonacci sequences are more than simply the relationship between 2 numbers. As described above, it requires a 2-dimensional fit for it to be truly a golden ratio. Secondly, perhaps the most interesting golden ratio in atomic physics is that of fission. A fission profile produces a bi-nodal distribution indicating that a larger and a smaller nuclei are produced in each fission reaction (along with neutrons and energy). The sharp edge of the twin peaks has a starting point approximating 89 and end edge of 144; both Fibonacci numbers. Indeed, if the 3 neutrons are accommodated on both sides of the equation, 236U would be adjusted to 233 plus 3 neutrons producing a potential fission pair of 144Cs and 89Se plus 3 neutrons; 89, 144 and 233 are a golden ratio. Nonetheless, this is one of many fission pairs.

Earlier work by Nobel prize winner Francis W Aston in the 1920s reported that the Fibonacci sequence corresponding to missing mass numbers of known elements (7). This is perhaps the first and last citation related to using Fibonacci sequences or the golden ratio to predict undiscovered nuclides of elements. A careful read of pages 110 and 111, however, indicate that the sequence “shows no semblance of regularity” so, at the time, was not recognized as a Fibonacci sequence.

The chart of nuclides

While the periodic table provides a convenient summary of the 118 elements, each element has multiple isotopes or nuclides. The element is defined by the number of protons held in the nucleus but species for the same element may have different numbers of neutrons and, thus mass number. These represent the isotopes of the elements. Some isotopes can exist in different energy states (metastable) which produces the same isotope (proton and neutron number) but a different nuclide (energy state). The chart of nuclides is a representation of all known nuclides and was first introduced in 1935 by Giorgio Fea with just 327 known isotopes for the 92 elements ranging from hydrogen through to uranium (3) (figure 3). By 1958 the chart was expanded with 102 known elements with 1500 nuclides and by 2015 there were 4000 nuclides for 118 elements (3). The chart of nuclides plots neutron number on the X-axis and proton number on the Y-axis. This format means that each horizontal line has the same proton number and thus nuclides of the same element. The chart of nuclides is typically colour coded to indicate half life (or black for stability) and each box can be annotated with physical properties (eg. emissions, energies, half lives).

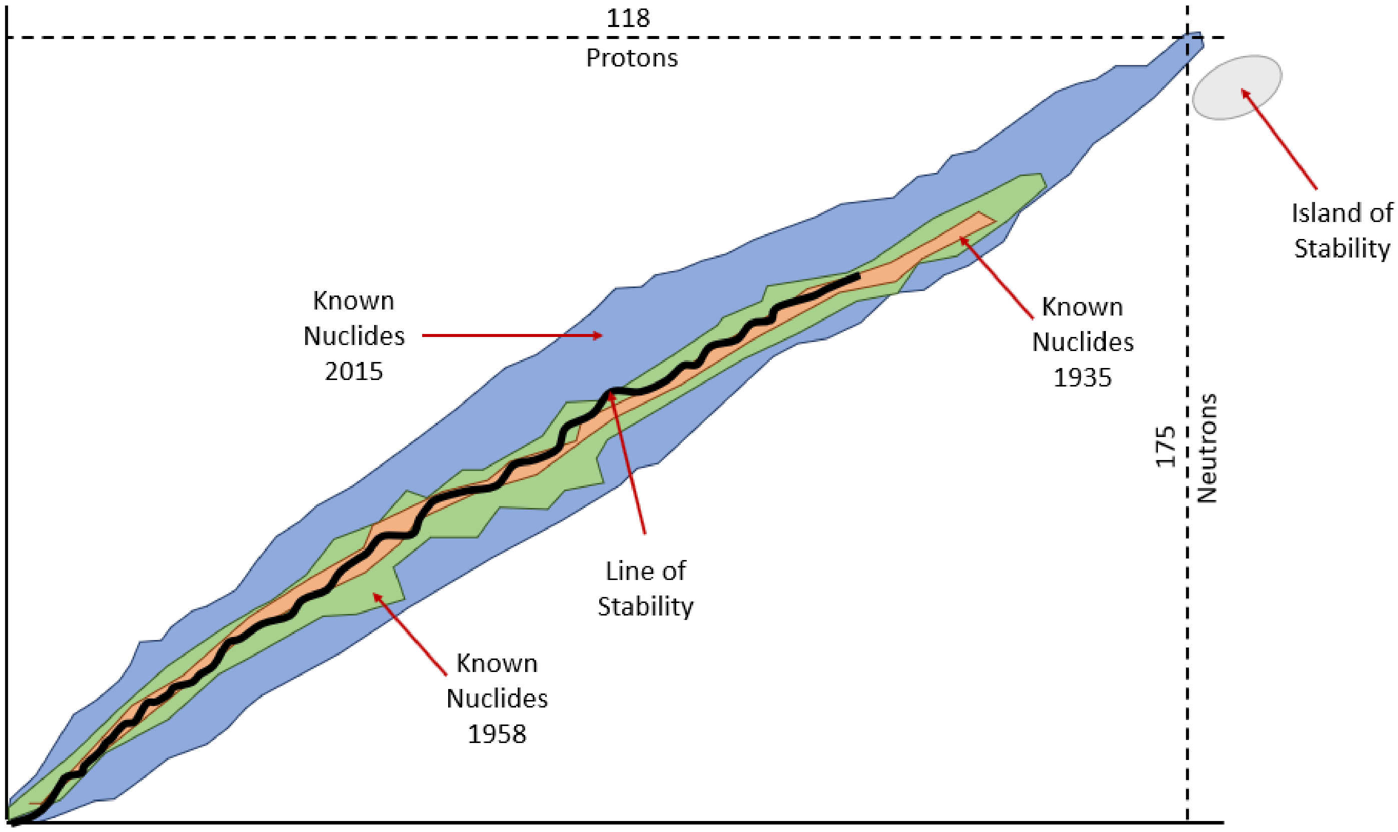


Figure 3: Schematic representation of the chart of nuclides with neutron number plotted on the X-axis and proton number on the Y-axis. The blue zone represents the 4000 nuclides currently known to exist (observed), the green zone known nuclides in 1958 and the orange zone the known nuclides from 1935.

The chart of nuclides is characterised by the line of stability (connection of stable nuclei), the line where the number of protons and neutrons are equal, magic numbers and the lines connecting them (figure 4), the proton drip line and the neutron drip line (2). Magic numbers represent those number of protons or neutrons which produce a complete shell in the nucleus (eg. 2, 8, 20, 28, 50, 82, 126) and where a nuclide has both proton and neuron number as magic numbers they are considered doubly magic (eg. 48-calcium with 28 neutrons and 20 protons) (8). For protons 114, 122, 124 and 164 are also magic numbers while 184, 196 and 236 are magic numbers for neutrons. Beyond the known limits of the chart of nuclides lies the island of stability, a region of yet undiscovered nuclides where nuclei deformation for neutron numbers of 162 to 184 produce greater nuclei stability (2). The focus of the island of stability has been 298Fl because it has 114 protons and 184 neutrons (double magic numbers). Theoretical predications suggest elements beyond element 118, up to element 172 are waiting to be discovered (2).

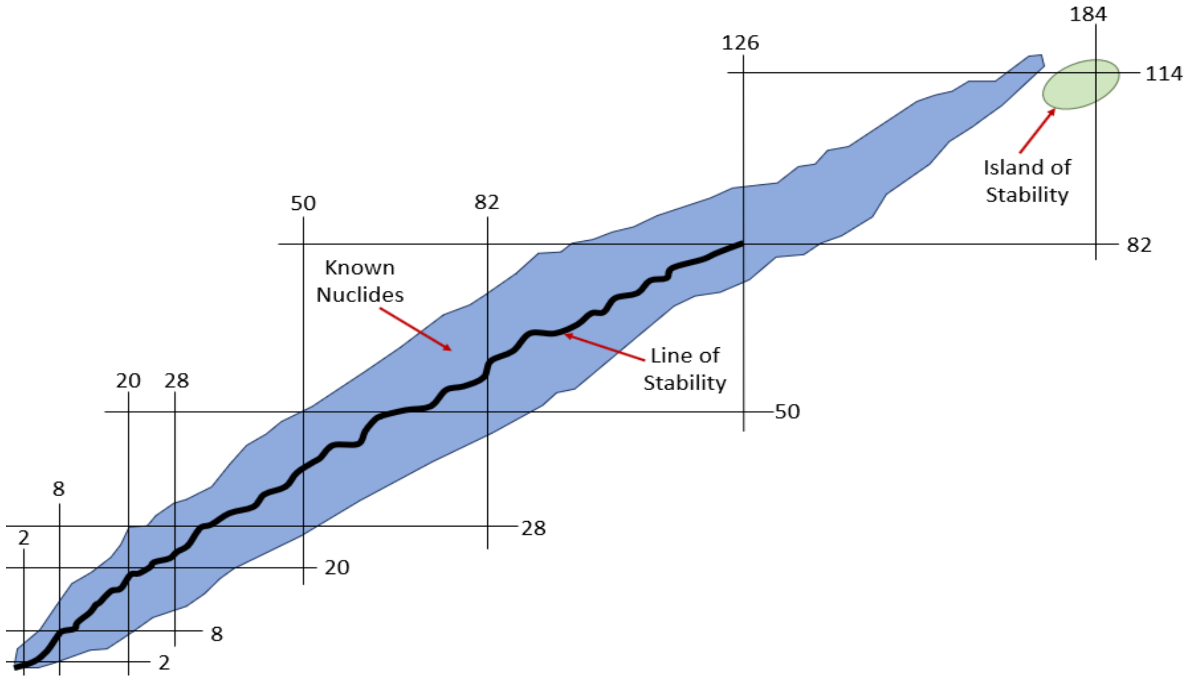
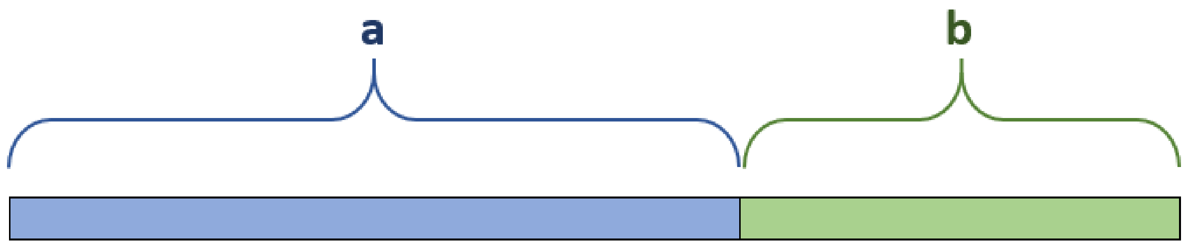


Figure 4: Schematic representation of the chart of nuclides with neutron number plotted on the X-axis and proton number on the Y-axis with the magic number matrix overlaid.

The Golden ratio, Fibonacci sequence and the chart of nuclides

The chart of nuclides was evaluated for evidence of a true golden ratio. That is, the neutron number divided by the proton number needed to equal 1.618 and the atomic number (neutrons plus protons) divided by the neutron number needed to equal 1.618. Across the chart of nuclides, this ratio approximated the golden ratio to a varying degree of accuracy for some nuclides but rarely for both calculations. This relationship (golden ratio in the chart of nuclides) has not been previously reported. To properly fit the golden ratio, the nuclides have to satisfy both tests of the golden ratio (figure 5). Atomic mass numbers (A) from 1 through to 377 were divided by absolute φ to identify nuclides that satisfy 1 of the golden ratio criteria. The resulting neutron numbers (N) which were rounded to the nearest whole number and subtracted from A to provide the proton number (Z). The resulting ratios of N to Z and A to N were calculated and tabulated. The results were then subtracted from absolute φ and squared to negate negative values. The squared differences for the 2 ratios was then subtracted with the square root of the result providing a numerical error from the true golden ratio. The error term developed for this experiment is outlined below:

$$error = \sqrt{(\varphi - \frac{N}{Z})^2 - (\varphi - \frac{A}{N})^2}$$



$$\frac{a}{b} = \varphi = \frac{a+b}{a} = 1.618.....$$

$$\frac{\text{Neutrons}}{\text{Protons}} = \varphi = \frac{\text{Neutrons+Protons}}{\text{Neutrons}} = 1.618.....$$

$$\frac{N}{Z} = \varphi = \frac{A}{N} = 1.618.....$$

Figure 5: Calculating the golden ratio for the atomic nucleus.

The initial observation was that mass numbers (A) associated with the Fibonacci sequence produced ratios of neutrons to protons and mass number to neutrons very close to the golden ratio (1.618): elements 5, 8, 13, 21, 34, 55, 89, 144, 233 and 377. These are a Fibonacci sequence (table 2) and will be referred to as Fibonacci nuclides.

Table 2: The nuclides that provided the most accurate correlation to the golden ratio, the 'Fibonacci Nuclides'

Mass Number (A)	Neutrons (N)	Protons (Z)	Nuclide
1	1	0	Free Neutron
3	2	1	^{3}H
5	3	2	^{5}He
8	5	3	^{8}Li
13	8	5	^{13}B
21	13	8	^{21}O
34	21	13	^{34}Al
55	34	21	^{55}Sc
89	55	34	^{89}Se
144	89	55	^{144}Cs
233	144	89	^{233}Ac
377	233	144	^{377}?

Based on the data in table 2, multiple trends that relate to the golden ratio can be seen. Each column corresponding to mass number (A), neutron number (N) and proton number (Z) follows the Fibonacci sequence. For each nuclide in this group of Fibonacci nuclides the A, N and Z numbers are all sequential numbers in a Fibonacci sequence; a Fibonacci triad. For example, (0, 1, 1), (1, 2, 3), (2, 3, 5), (3, 5, 8).......... (144, 233, 377). Among the Fibonacci nuclides (errors less than 0.1%), a more critical examination revealed that among the known elements, 233Ac provided a zero error fit to the golden ratio making it the golden nuclide. A linear fit on the chart of nuclides of the Fibonacci nuclides (figure 6) produces the golden line which corresponds to all the nuclides that approximate the golden ratio in the initial analysis, the error reducing as mass number increases. Below atomic mass of 89, errors of 1% or better were associated nuclides with gaps of 2 or 4 nuclides between. Beyond 89, the error margins significantly reduce.

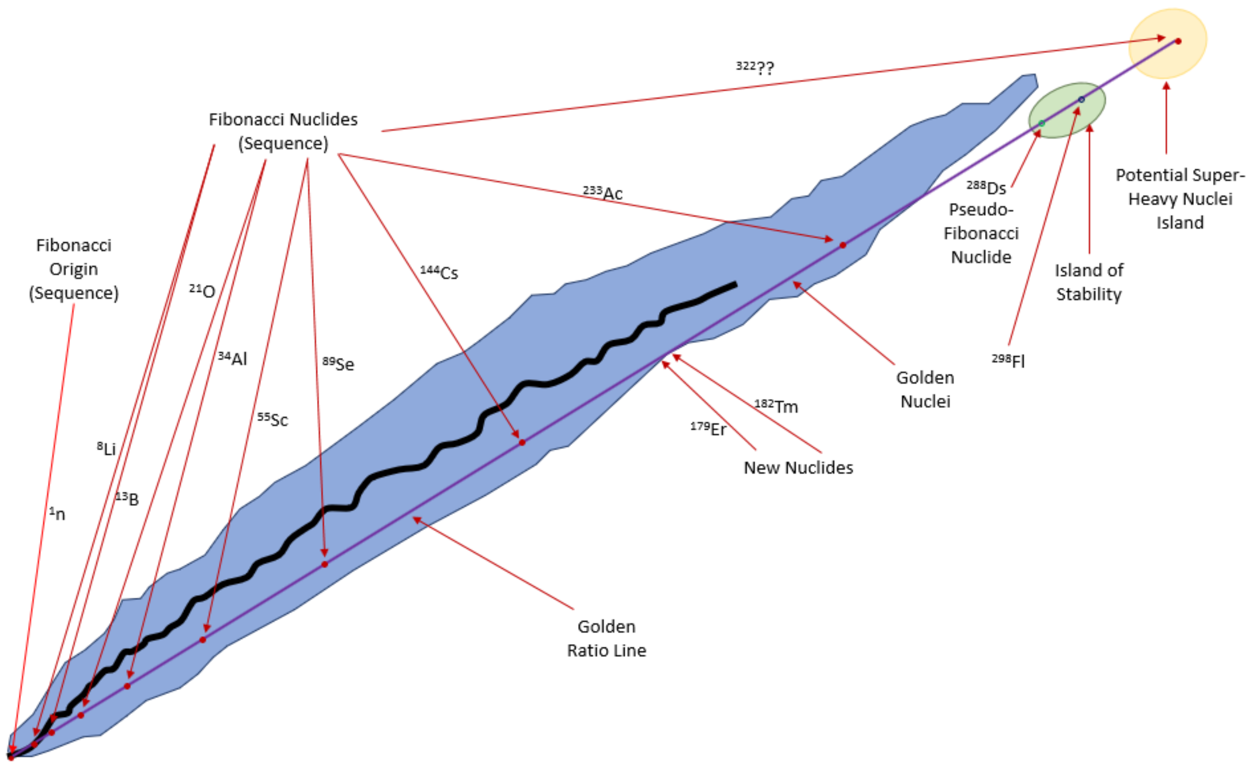


Figure 6: Schematic representation of the chart of known nuclides (blue), the island of stability (green), the potential super-heavy nuclei region (yellow) and the golden ratio line (purple).

Predictions

The value of identifying a robust trend line is using it to extrapolate the line. In this case, mathematically, the golden line offers a near perfect linear fit (figure 7). Gaps in nuclides on the fit provide string support that they exist and yet to be discovered. In atomic physics, a number of approaches have been adopted to predict the presence of new nuclides and new elements but the golden ratio has not been part of that process. Based on data analysed in this investigation, it can be postulated that the 4 nuclides outlined in table 3 exist and await discovery. Interestingly, many currently known nuclides would have been predicted by the golden line when they were undiscovered in 1958 (figure 8).

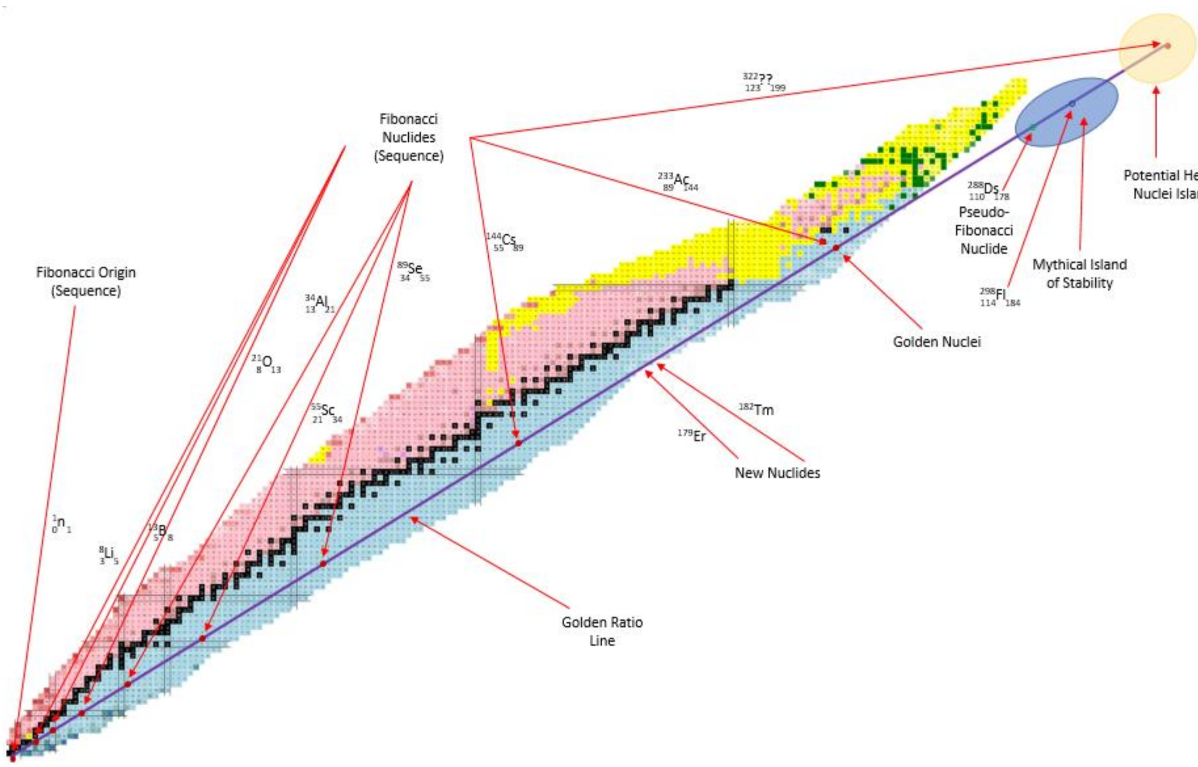


Figure 7: Annotated nuclide chart with predicted nuclides and Fibonacci nuclides.

Table 3: 4 postulated nuclides based on filling gaps on the golden line.

Name	Symbol	Mass Number	Neutrons	Protons
Erbium	Er	179	111	68
Thulium	Tm	182	113	69
Darmstadtium	Ds	288	178	110
Flerovium	Fl	298	184	114

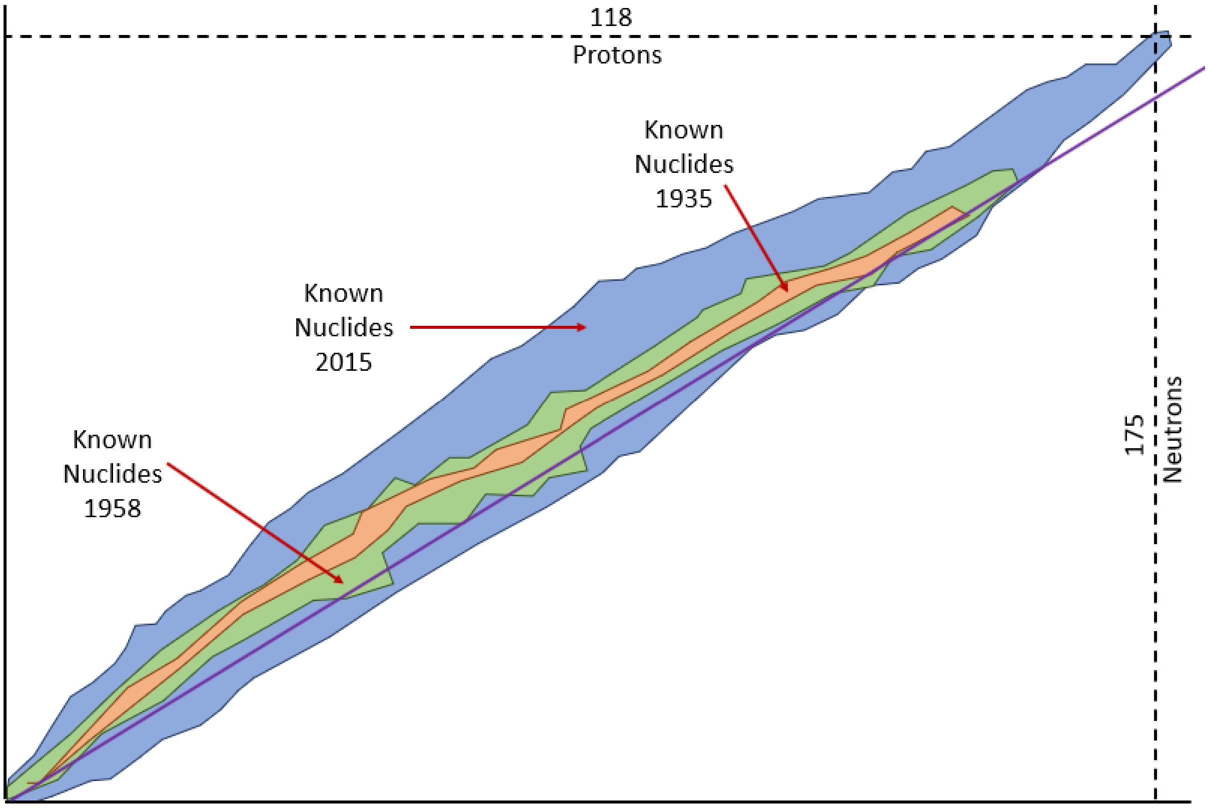


Figure 8: Schematic representation of the chart of nuclides with neutron number plotted on the X-axis and proton number on the Y-axis. The blue zone represents the 4000 nuclides currently known to exist (observed), the green zone known nuclides in 1958 and the orange zone the known nuclides from 1935. The purple line is the linear fit of the golden ratio line showing that the line would have predicted a large number of currently know nuclides unknown in 1958.

Physicists are generally excited about the island of stability that represents super-heavy nuclei. They postulate that new nuclides may be discovered with long half-lives (hours to years) than those typical of the heavy end of the chart of nuclides (seconds or less). The focus generally appears to be in the region of 185 neutrons and 114 protons (298Fl) but this research uncovered an interesting nuclide elsewhere in the island of stability (figure 9). 288Ds has 178 neutrons and 110 protons, fits very tightly to the golden line with an error of less than 0.001%. While it is not a Fibonacci nuclide, its position on the golden line between two Fibonacci nuclides (element 233 and element 377) is defined by the golden ratio:

$377 - 233 = 144$

$377 - 288 = 89$

$288 - 233 = 55$

$89 / 55 = 1.618$

$144 / 89 = 1.618$

A known element, this new nuclide positioned within the island of stability with numerous golden ration and Fibonacci characteristics may be encouragement to target this super-heavy nuclei for synthesis.

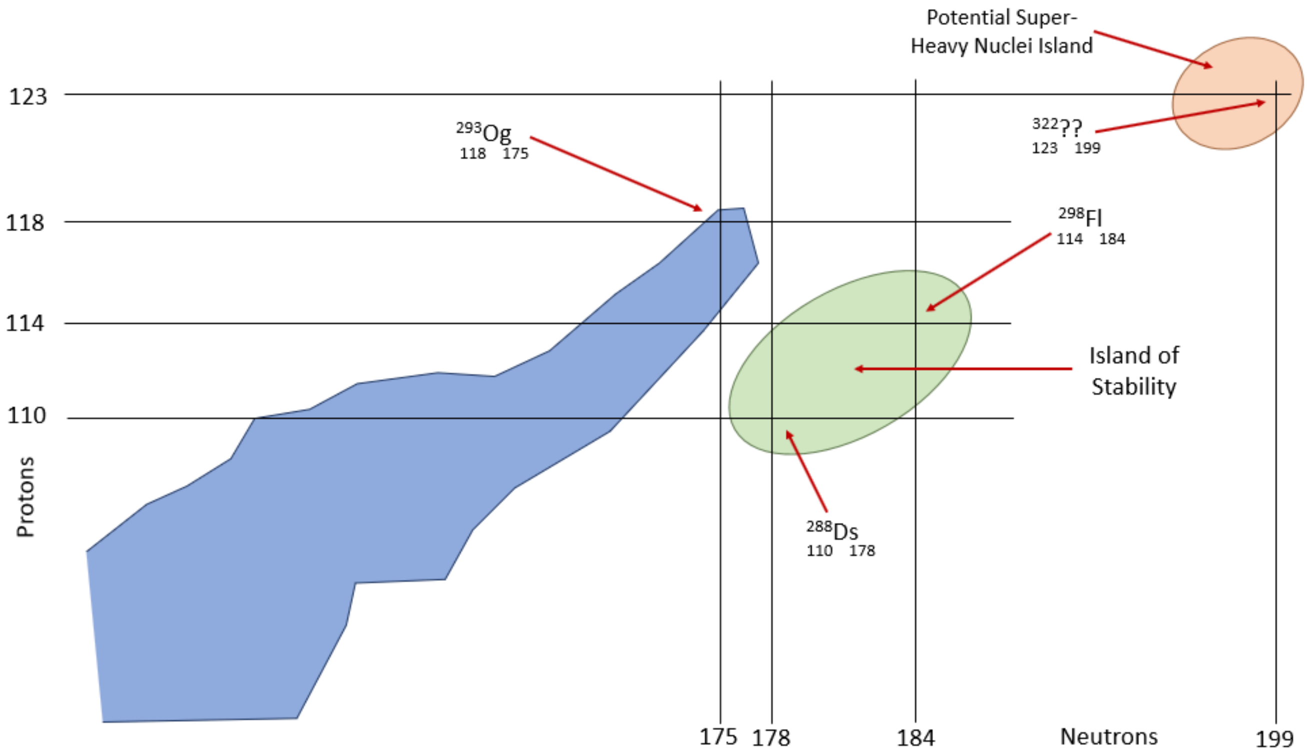


Figure 9: Schematic representation of the super-heavy nuclei region of the chart of nuclides.

Beyond the island of stability may be another zone of relatively stable (longer half-life) super-heavy nuclei. This region is well beyond the current limits of element 118 (figure 9). Element 322 has 199 neutrons and 123 protons, fits very tightly to the golden line with an error of less than 0.001%. While it is not a Fibonacci nuclide, its position on the golden line between two Fibonacci nuclides (element 233 and element 377) is defined by the golden ratio (indeed, the mirror of the previously discussed 288Ds):

$377 - 233 = 144$

$377 - 322 = 55$

$322 - 233 = 89$

$89 / 55 = 1.618$

$144 / 89 = 1.618$

An unknown element, this new element / nuclide positioned within the super-heavy nuclei island with numerous golden ration and Fibonacci characteristics may also be encouragement to target this new super-heavy nuclei for synthesis.

While bold to predict multiple new nuclides, and especially a new element, in recent decades significant changes have been made to the chart of known nuclides (figure 8). Not only have more nuclides of each element been discovered, but more elements have been discovered, expanding the chart both horizontally and vertically. An interesting and aesthetically appealing way to depict the golden line, Fibonacci nuclides and identify potential new nuclides is by presenting the golden line as a golden spiral (figure 10).

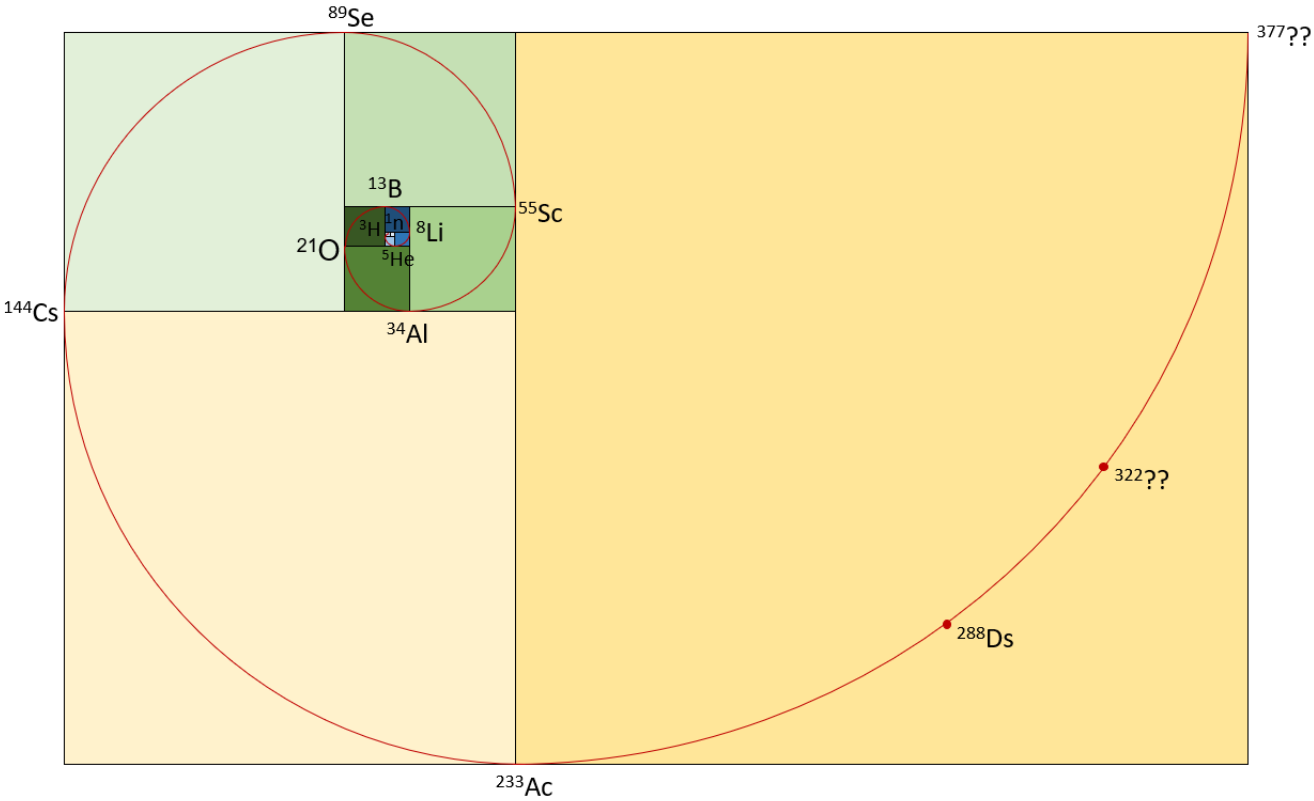


Figure 10: The first Fibonacci nuclides, golden line and prediction of new nuclides (288Ds) and new elements (elements 322 and 377).

Conclusion

While element 144 (atomic mass 377) is a Fibonacci nuclide and has very accurate correlations with the golden ratio, the element is a long way from where our modern human knowledge and capabilities (for synthesis) are. Nonetheless, theoretical predications suggest elements beyond element 118, up to element 172 are waiting to be discovered (2). The pseudo-Fibonacci nuclide (element 123 with atomic mass of 322) is a more realistic target for synthesis over coming decades. In keeping with the golden ratio line, elements 123 and 144 are postulated.

The aim of this investigation was to examine the relationship between the golden ratio and atomic structure. It was determined that specific nuclides exhibit the golden ratio between protons and neutrons, and that these produce a trend line on the chart of nuclides. The Fibonacci sequence is mathematically evident and this information can be used to postulate new nuclides or elements.

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