Noticing and visuospatial reasoning

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In this article Kay Owens explains how a real-world context can be used to encourage student noticing and deeper analysis of geometric structures when students are asked to draw what they saw. Kay presents examples of students' drawings made during a school trip to the zoo which followed a series of inquiry learning experiences in geometry.

Visuospatial reasoning

Reasoning has become a focus for learning through mathematical inquiry experiences. Reasoning involves some form of argument or explanation (Stacey, 2012). Reasoning is generally considered in terms of verbal learning as children place conceptual ideas and ways of solving problems into words. However, people also reason visually and spatially. We use it when travelling, assembling furniture, gardening, organising our classrooms, and in creative pursuits and problem solving. Visuospatial reasoning incorporates visual imagery, spatial capabilities, and spatial or embodied imagery from gestures and movement.

Visual imagery

Mental visual imagery stores information and may be used for recognising, learning and being responsive in inquiry. Learners make comparisons visually with present and past imagery. This imagery might be a picture image in the mind but research is increasingly showing it may be more complex (Owens, 2015). Visual imagery may be mentally manipulated, and linked with verbal knowledge (Goldin, 1987). It is developed through incidental experiences (van Hiele, 1986) and planned experiences, through practice and discussion (Towers & Martin, 2014). It is generated from verbal input and imagination especially linking various imageries and experiences together in a motivating situation (Owens, 2015; Sinclair et al., 2016). Thus visual imagery is more than a picture image in the mind but research has shown that it may also be dynamic. For example, rotating, flipping, or stretching a triangle in one's mind. This imagery involves movement and may occur naturally in students' thinking or be encouraged in the classroom (Presmeg, 1986, 1997, 2014). Inquiry involves planning and reasoning about changes in the imagery created in the mind and may be associated with external tools that mediate the thinking (Goos, Galbraith, Renshaw, & Geiger, 2003; Sinclair et al., 2016). This dynamic imagery may be assisted through experiences such as using a loop of string or elastic or a computer program (e.g., Geoboard app or GeoGebra) that allows for movement from one triangle into another triangle but always with three straight sides. Students even remember the visual imagery with their own actions such as making different pentomino shapes (made from five squares with sides joined) and visualising different pentominoes as they move and check each piece around three or four squares in a row (Owens, 2015).

Spatial capabilities depend on what a person notices and to what they attend. While some spatial capabilities are classified as rotating shapes in the mind, it is likely that people also focus on certain features to make decisions and recognise similarities of shapes or hidden figures (Shepard & Metzler, 1971). Experiences with jigsaws, paper folding, and fun activities with shapes will encourage these capabilities (Sinclair et al., 2016). Spatial capabilities include seeing a shape within a shape or completing a shape, rotating a shape, and seeing a shape from another perspective (see Tartre, 1990 for an effective categorisation of these skills).

Spatial imagery comes from movements but may be instantaneous and not necessarily controlled by the brain (Sulistyawati, Wickens, & Chui, 2011). Spatial imagery is part of the body's muscular control system. Spatial imagery often results from embodied learning in gestures such as showing a slope with a hand. A child might use fingers to mark the sides of a shape that form an angle and thus compare it with another angle moving the fingers to mark it (Owens, 1996) and thus the conceptual and visual knowledge about angles increases. The interaction of embodied learning and visuospatial learning is significant for students with alternative learning paths (Healy & Fernandes, 2011). Drawing may be one of the spatial imagery activities often used in learning about shapes and objects that also provides a visual imagery. Some of the on-screen modifications of shapes, for example in GeoGebra, using a mouse or touch screen provide both spatial and visual input for learning. However, spatial imagery may also focus attention.

Noticing and attention

Whether comparing images of concepts or reasoning using patterns or dynamic changes, noticing becomes a particularly relevant process in reasoning. In the 1990s, Mason (1992) provided examples of the role of noticing in imagery in activities such as walking around the corner of a building looking at it and keeping the dynamic imagery in mind. Mason (2003) noted that gazing, that is looking steadily for some time, may result in seeing an object differently and noticing different properties, e.g., a regular hexagon comprised of equilateral triangles may appear as a three-dimensional cube.

Importantly, in developing conceptual structures, people first have a primitive knowing and then make an image about which certain aspects are noticed. As the concept is structured and used the person may fold back to the initial phases that are so reliant on imagery (Pirie & Kieren, 1991). Noticing or attending to aspects of visuospatial experiences plays an important role in starting and developing visuospatial reasoning. Visual memories of past experiences and importantly cognitive schemes focus attention (Owens & Clements, 1998). Wright and Smith (2017) show this important aspect of noticing and anticipating with students selecting nets for objects. In problem-solving, the student notices and pays attention to the arrangement of concrete materials, words of others, or their own verbal and imagistic thoughts. Then the student responds and moves forward to take another step in problem-solving which in turn influences the others in the learning environment or the learning tools are modified, and a new cycle begins. In other words, responsiveness is a key to students solving problems through their noticing and reasoning (Owens & Clements, 1998).

Visuospatial reasoning and context for learning

A person's utilisation of visuospatial reasoning is context specific. The immediate surroundings of a shape or object (e.g., optical illusions), colour and words can influence what is noticed. It depends on what one is expecting. The wider context includes cultural relevance, impact on what is being noticed and how a person responds to the situation (Owens & Clements, 1998). Across many different cultures, there is evidence of visuospatial reasoning in cultural activities (Owens, 2015; Sinclair et al., 2016). Teachers need to encourage students to discuss what they notice and how they plan to manipulate objects for collaborative problem solving. Collective property noticing occurs (Towers & Martin, 2014) as the immediate and wider cultural experiences of children will impact on their learning (Owens, 2015).

Examples of students' noticing during a zoo activity following a series of inquiry learning experiences on 3D geometry

With large 3D objects at the zoo, examples indicate diversity in noticing and responding to solve novel problems.

Figure 1. a. Two students concentrating on what they are to draw and name. 1.b. A group of students with the viewing platform behind.

Parallel and oblique lines

Students were asked to sketch the top and front views of the viewing platform from a distance (Figure 1b). The roof was made of corrugated iron with a short central ridge. For the top view, most students began by drawing the vertical and then horizontal parallel lines for the rectangle then the configuration of oblique lines (Figures 2b, 2d, 2e, 2g, and 2i). However, Figure 2h was started by drawing the four oblique lines separately and then the scalloped rectangle. Others attempted to draw each of the shapes that formed the roof such as the trapezium (Figure 2c, 2f and 2g). In every case they were imagining and reasoning

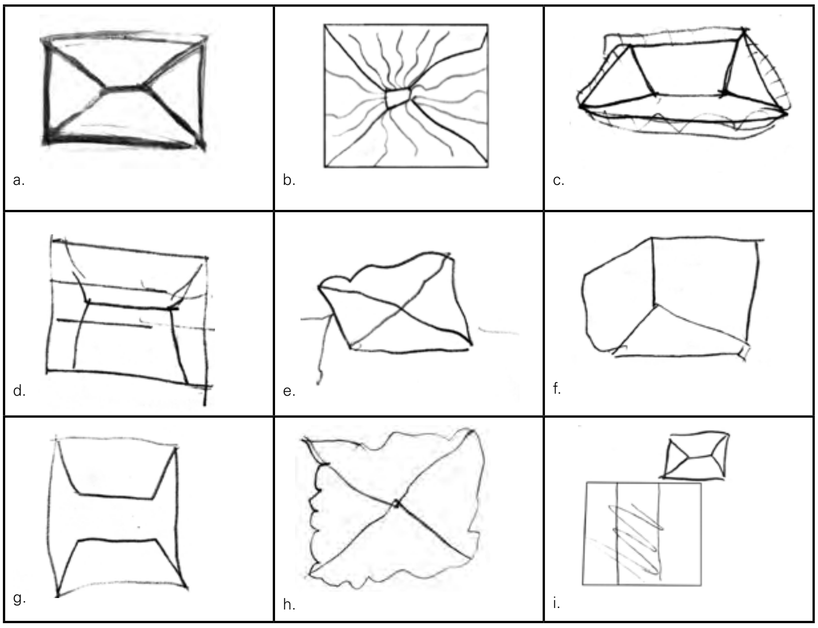


Figure 2. Top view of a roof with a short central ridge.

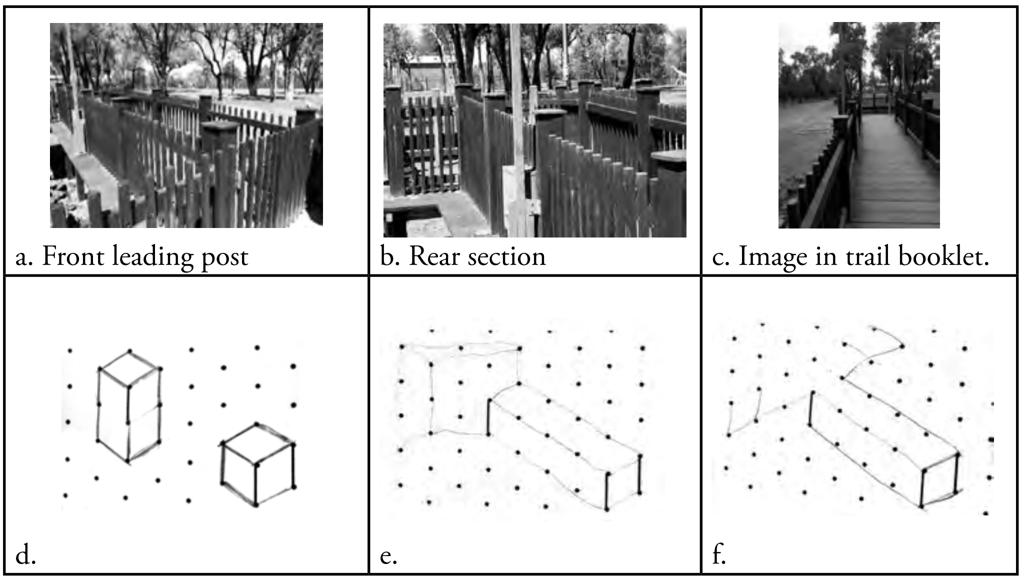
visuospatially. Figure 2a was interesting as he drew artistically starting on the left with the vertical and two oblique lines, then he drew the horizontals at the back, front and for the central ridge and finished with the right vertical and two oblique lines. He had a complete image in his mind that he was drawing. Later, he and another student noted that they had sheds on their farms that were a bit like the shelter and this helped them.

Younger students were more likely to reproduce a pyramid top view (Figure 2e and h), an experience they had in class but older students provided a ridge of some kind as seen in Figure 2a-d, g, and i. Figure 2c also shows the addition of the fence which was sloping outwards around the platform.

Oblique lines in isometric drawings of composite 3d objects

One of the tasks on the mathematics trail was to draw the feeding platform for the giraffes on isometric dot paper as shown in Figure 3a and b. Prior to the mathematics trail, students had drawn 3D objects made from small cubes onto isometric paper starting with turning the shape and looking at the leading edge (not a face) and drawing the edge vertically between dots. Students were encouraged to use one post as the leading edge (front post view in Figure 3a, already drawn on the dots (see red markings on Figure 3d).

A few younger students drew only vertical and/or horizontal lines with no clear connection to the fence. Interestingly four students sitting in the shade on the hot day confirmed that they drew from the picture



in their workbook (see Figure 3c). They did not apply how they had used the isometric paper in class. Hence this picture was a distraction to looking at the corner post of the real object as the front vertical line of the isometric drawing. Two of these students drew orthogonal drawings but two sitting together drew neat isometric drawings of a basic cube and rectangular prism (Figure 3d). They were asked to name the two objects forming the platform before they started. Several times we heard the word cuboid or cube referring to the viewing platform. In fact, the rear shape was another rectangular prism (cuboid) but students thought the question required two different 3D shape names.

A third of the students drew the main rectangular platform having been encouraged to look at the corner post as the front vertical on the isometric paper (Figures 3d–h). There were numerous variations on completing the drawings. Most students had difficulty with completing the rear section. Some managed to get the isometric slope correct after being prompted to recall drawing a similar shape made from blocks.

Discussion

The mathematics trail was an example of how linking mathematics to the students' real world encouraged noticing and deeper analysis because the students were asked to draw what they saw. This required them to notice features of objects. In drawing they were tuned in and reasoning visuospatially. They went through a quick inquiry; finding out: what do I notice, what is significant about what I'm looking at, where will I start, and how will I draw that? Students were also sorting out or self-checking: have I drawn what I see, how may I make it look more like that, what are some features that link to shapes and drawings that I've done before?

Students were going further by improving their drawings, comparing with friends, analysing their drawings in terms of the parts they saw or in terms of their knowledge of a particular shape. However, an emphasis on names (asking for them first in the workbook rather than drawing first) created a problem. In a teaching situation, starting with making and drawing shapes they see before giving them a name reduces the danger of fixating on school mathematics and drawing named objects. The mathematics trail provided a diversity of composite objects and complex imagery, some without a learned name.

Students made connections to what they had learnt in school in terms of the drawing of 3D objects. They used the prior learning of inquiry to get started with the challenge of drawing large objects instead of small objects they could manipulate. They took action by completing the booklet or checking with a friend. Significantly in reflecting, students felt they had ownership over their work and had achieved something mathematical. The difficulties experienced by students suggest greater time is needed for students to develop the art of noticing and to transfer their imagery of 3D objects into 2D representations. Teachers need to encourage noticing features rather than showing or telling mathematics. A trail outside the classroom with large 3D objects will strengthen their noticing and reasoning through drawing and discussing.

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