It is the paper published as:

Author: B. Perry
Title: Australian teachers' views of effective mathematics teaching and learning.
Journal: ZDM **ISSN:** 1863-9690 1863-9704
Year: 2007
Volume: 39
Issue: 4
Pages: 271-286

Abstract: Thirteen Australian teachers who had been nominated by their professional mathematics teachers' associations as excellent teachers of elementary school mathematics were interviewed on their beliefs about mathematics, mathematics learning and mathematics teaching. In particular, they were asked to discuss the characteristics of effective teachers of mathematics and excellent mathematics lessons. In spite of their differences in location, experience and teacher education, the teachers displayed a lot of consistency in their responses and in their lists of characteristics. While this group of teachers cannot be claimed to be representative of Australian teachers, they have provided a snapshot of what is regarded as effectiveness in mathematics education in Australian elementary schools

Author Address: bperry@csu.edu.au

Australian Teachers' Views of Effective Mathematics Teaching and Learning

Bob Perry
Charles Sturt University

Thirteen Australian teachers who had been nominated by their professional mathematics teachers' associations as excellent teachers of elementary school mathematics were interviewed on their beliefs about mathematics, mathematics learning and mathematics teaching. In particular, they were asked to discuss the characteristics of effective teachers of mathematics and excellent mathematics lessons. In spite of their differences in location, experience and teacher education, the teachers displayed a lot of consistency in their responses and in their lists of characteristics. While this group of teachers cannot be claimed to be representative of Australian teachers, they have provided a snapshot of what is regarded as effectiveness in mathematics education in Australian elementary schools.

Introduction

Australian school education is the constitutional responsibility of the six states and two territories that make up the Commonwealth of Australia. While the Australian government seeks to influence what happens in Australian schools through various funding arrangements, especially in the areas of professional development for teachers, starting age for children and attempts to develop national assessment and curriculum approaches, the current situation is that school education varies quite dramatically from one state / territory to another. For example, while some states / territories have 7 years of compulsory primary (elementary) schooling, some have 8 years and, at least until 2005, one had 6 years. The ages at which children are legally eligible to commence primary school – and, hence, the ages at which children reach each of the structural educational milestones in their schooling – differ from one state / territory to another, as does the mode of entry into school (Dockett & Perry, 2007). There are currently moves afoot from the Australian government to standardise school commencing ages and to further rationalise assessment and curricula.

The study reported in this paper was conducted with a group of teachers who were currently teaching in three states – New South Wales (NSW), Victoria and Tasmania - and the Australian Capital Territory (ACT). Some of these teachers had taught in more than one constituency in Australia and two had taught overseas.

The school mathematics curricula in these parts of Australia had all undergone revision in the four years before the study was conducted and, at least in the primary school, were all characterised by what might be called a socio-constructivist approach to learning. In all of the study sites, strong systemic numeracy programs such as *Count Me In Too* and the *Early Numeracy Project* were well established (Bobis et al., 2005).

Teacher education for elementary teachers in Australia is conducted by universities and can be in a number of different forms. Currently, there are two major structures for initial elementary teacher education – an integrated 4-year Bachelor of Education degree or a 1- to 2-year Bachelor of Teaching or Master of Teaching degree that generally follows on from a 3-year undergraduate degree such as a Bachelor of Arts or Bachelor of Science. The

4-year degree may or may not contain specific content studies in mathematics but will contain mathematics pedagogy studies. Depending on what they have completed in their undergraduate degree, preservice teachers undertaking the BTeach or MTeach degrees have from 3 years of mathematics content study through to none by the time they start teaching. It is unusual for Australian elementary teachers to have undertaken formal postgraduate studies in education, although many will have completed extensive inservice programs.

Method

Using the questions outlined by Cai (2007, this volume), each of the 13 Australian teachers was interviewed via telephone by the researcher. Such interviews took between 35 and 70 minutes and were tape recorded and later transcribed. Under each of the major categories:

- Teachers' Views about Mathematics;
- Teachers' Views about Mathematics Learning; and
- Teachers' Views about the Teacher and Teaching

data were analysed according to the emergent themes presented by the teachers.

The participants

The Australian teachers for this study were selected after a request was made to the Australian Association of Mathematics Teachers (AAMT, the national professional association) and the Mathematics Association of New South Wales (MANSW, the state affiliate of AAMT in NSW) for these organizations to nominate elementary school teachers who were 'excellent teachers of mathematics'. The organizations were asked to use their own criteria for doing this as there was at the time no generally recognised way of celebrating excellence in mathematics teaching. (It is worth noting that AAMT has since established a process for such recognition in the context of its establishment of professional standards of excellence (Australian Association of Mathematics Teachers, 2006).) The organizations made the nominations and checked with the nominees to be sure that they were willing to be involved in the study. Only after this had been done were the nominees contacted by the researcher.

Thirteen teachers were chosen through these approaches and all agreed to participate in the project. All of the teachers were quite experienced, with between 11 and more than 32 years teaching. There were nine females and four males in the sample. Other details of the participants are given in Table 1.

Table 1. *Details of Australian participants*

Code	*Sex*	*Degrees / Certification*	*Years of teaching*	*Other relevant data*
AUT1	M	DipTeach	30+	• Fellow of College of Teachers, London • Fellow of Royal Geographic Society • Excellence in Teaching award from Australian College of Education • Former Head of Mathematics / Deputy Headmaster of prestigious independent school

AUT2	F	BTeach	11	• School art coordinator • *Count Me In Too* training
AUT3	F	DipTeach	20	• Professional development in gifted and talented education and systemic mathematics program
AUT4	F	BEd GradCert (Productive Pedagogy)	20	• Extensive *Count Me In Too* training • Former mathematics consultant • Taught in England and Ecuador
AUT5	F	BEd	25	• Training and experience in special education • Former mathematics consultant
AUT6	F	BEd	32	• Conference presenter
AUT7	M	BEd	30	• Extensive *Count Me In Too* training • Assistant Principal
AUT8	F	DipTeach	20+	• National Literacy and Numeracy Award • Former mathematics consultant • Extensive *Count Me In Too* training
AUT9	M	BEd	25+	• Council Member, Mathematical Association of Victoria • School Numeracy Coordinator
AUT10	F	BEd	28+	• Conference presenter • School Numeracy Coordinator
AUT11	F	BEd, GradCert (Teaching)	18	• School Numeracy Coordinator
AUT12	M	BEd, MEd	19	• Taught in Canada for 8 years
AUT13	F	BA, GradDip (Primary)	20	• Taught Spanish in primary schools for 13 years

Given the criteria of 'excellence' used in the recruitment of the teachers in this study, no claim can be or is made concerning the generalizability of the results obtained in this study to all Australian elementary school teachers.

Results

Teachers' Views about Mathematics

What is mathematics?

The responses to the question "What is mathematics?" showed a diversity of opinions among the teachers.

Mathematics is useful

All of the teachers interviewed felt that mathematics had many utilitarian aspects. While AUT1 agreed, suggesting that it is "one of those very essential subjects that allows us to function in the world", he also felt that "mathematics is a precise science and a subject that contains vast amounts of fascinating and interesting material that can be integrated with other subjects". AUT2 also saw mathematics as "essential for life in all

areas" but did not agree with the traditional notion of mathematics being a fixed body of knowledge. She said "I think it's not something that is static. I think it's an ever changing thing that is being constantly developed". The usefulness of mathematics was reiterated by AUT9 who saw it "very much connected with life experience and it very much influences your daily living". However, when pressed about the perceived usefulness of all the mathematics children might meet in primary school, he did concede that "you don't meet fractions coming down the street these days" (Australia has been a metricated country since the early 1970s). While agreeing that mathematics had its utilitarian aspects, AUT10 reiterated the 'science' aspects with "it is a system of ordering ideas, using numbers, measurement, space; a science which uses numbers and symbols and includes arithmetic, algebra, geometry, calculus and trigonometry".

Mathematics and numeracy

Some of the teachers wanted to make a distinction between 'mathematics' and 'numeracy', a concept and language that has been used extensively in Australian schools since the 1990s to indicate the ability to use mathematics in everyday life. The Australian Association of Mathematics Teachers (1997, p. 15) defines 'numeracy' in the following way: "To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community life".

For example, AUT5 suggested:

> To be numerate is to be able to use the mathematical ideas that we have effectively and to make sense of the world. It involves drawing on lots and lots of different things: deciding how to use mathematics; what to use; when to choose different skills and evaluating, critically evaluating what you're using at the time.

In contrast, AUT7 took a quite different view of numeracy as "just the study of number while mathematics covers the application of number in all other areas" while AUT8 did not differentiate between the two feeling that, in her school and with her children particularly, "mathematics needs to be like numeracy".

Mathematics is a set of ideas and tools that help us to solve everyday problems formally and informally

A number of the participants linked mathematics to the solution of 'everyday' problems, suggesting a particular way in which mathematics could be useful. AUT8 saw mathematics in much the same way as she saw literacy "as a way to see the world. I think that maths is basically the same thing, it's a way of getting the students to explore their world, to solve problems and see things". This notion was reinforced by AUT7 who suggested that "it's probably a study in the use of numbers to make our lives easier and it also allows us to solve problems in an efficient manner".

Mathematics is a language to explain phenomena that happen around you

One of the teachers (AUT3) was very clear that mathematics is a language that can be used to help you understand your environment:

> We talk about mathematics as just another language that we are all able to speak to varying degrees. It's just a way of describing things that happen around them, be it numerically or in terms of space or shape. It's just understanding that part of the world around them. So to me, mathematics is basically learning a language to explain phenomena that happen to you.

While the notion of mathematics as language was raised by two other teachers, neither saw it as a central facet of mathematics learning in primary schools.

Mathematics is a number of strands such as those reflected in the syllabus

Many of the participants seemed to understand mathematics as what was taught in primary schools. For example, AUT13 suggested that "it's like the study of numbers and shapes and their relationships, and of trying to apply them and not just looking at mathematics as a separate subject". While AUT11 also saw these strands, she was convinced that mathematics was more of a framework than just a collection of strands:

> Mathematics is the framework of processes and skills that allows us to manipulate, calculate and explain number problems and amounts in our world. For me, mathematics is the framework and the framework has five quite distinct application areas.

One of the key messages from many of the teachers was the importance of patterns and relationships in mathematics. For example, as well as emphasizing the syllabus strands, AUT5 supposed that "it's all the different substrands, isn't it? If you're working mathematically it's your numbers, it's your patterns; mathematics encompasses all the things we do". This approach was reiterated by AUT4: "If you're doing all of those things in relation to patterns and number patterns, then I think you are doing mathematics".

Summary

The overwhelming response to the question "What is mathematics?" was that mathematics is something that allows people to function in and understand their worlds. As might be expected in a country where the 'mathematics versus numeracy' debate has been ongoing for years, there was some discussion around this dichotomy. However, there was a general tone coming through the teachers' responses that they thought that mathematics was what they taught in their mathematics lessons. Few of the teachers interviewed had studied mathematics at a tertiary level – other than in their teacher education courses where there study would have dominated by pedagogical rather than content issues. Hence, it is not surprising that their view of mathematics might not expand beyond their own experiences.

Truth and mathematics

While a few of the teachers interviewed thought that there were some 'truths' in mathematics, most of them were a little more unsure about the value of taking a hard and fast stand on this question. There was also some confusion in the answers to this question as to whether it was meant to deal only with primary school mathematics or whether it was a question about more general areas of mathematics. The teachers were encouraged to choose whichever aspect they wished and to explain their answers from this perspective.

A lot of things in mathematics must simply be accepted as 'true'

At the primary school level, in particular, some teachers felt that there were some 'truths' that needed to be accepted. For example, AUT3 suggested:

> Some things have to be accepted as true because they are. That doesn't mean that there isn't an explanation for them. I think it's important that we make children understand that there's always an explanation for things. We might not understand it at certain times, but there's always going to be some sort of explanation.

Sometimes, it was felt that the explanations of certain mathematical ideas may be beyond primary-aged children, so that it was a waste of time trying to explain them:

> There are a lot of factual things that need to be recalled. (AUT2)

> At primary level, I think that's partly true. There are some things that I used to teach in mathematics at primary level where I wasn't sure why we needed to do it but we did it. I would not have been able to explain adequately why it worked and I did ask students to accept that it was true and to perhaps explore the reasons for it at a later stage when they were able to grasp the explanation more easily. (AUT1)

Everything in mathematics can be questioned

The most common philosophy in mathematics syllabuses in Australia adheres strongly to the socio-cultural approaches of Vygotsky (1978). Many teachers mouth this philosophy without really understanding its importance. Teachers in this project were able to articulate their beliefs around 'truth in mathematics' quite strongly.

> I think children need to experience and find the truth for themselves. They need to really get into it, explore, experiment and struggle with mathematics until, hopefully, they get to an understanding, get to the truth behind it and see the real mathematics behind the situation. (AUT8)

> Even Einstein's being questioned at the moment so was that true? Is it true? We don't know, you cannot just accept that. (AUT7)

> I don't believe that in maths you can just accept that things are true, you need to challenge them. (AUT12)

> I think that everything in mathematics can be questioned. There is more value in actually questioning as much as you can in mathematics because that's how you're actually going to develop a true understanding of how things work. (AUT4)

In mathematics, things are true because we can show that they are true

Three of the teachers responded to the question about truth in mathematics by referring to what they saw as one of the underlying processes in mathematics – proof. Their argument was that if something can be proven mathematically then it is true mathematically. Even at the primary school level, they saw this to be a central tenet of mathematical thinking.

> We used to think that in high school. They would say 'just do it'. I don't anymore but rather look at everything that we do and say 'OK, it is true because we can prove it through this process'. 'Do you believe that this is right?' 'Yes, because I am able to prove it by doing X, Y and Z.' … I believe that there is an explanation and everything can be proven. (AUT5)

> Yes, you can arrive at the truth, but it's how we choose to arrive at the truth [that is important]. We strive for truth arrived at with purpose, therefore with meaning. Accepting truth at face value with no explanation closes the door on making connections and meaningful learning. (AUT11)

Summary

The teachers interviewed in this study struggled to answer this question. Many of them simply attached it to their practice in school mathematics and claimed that there were clearly aspects of that mathematics that were true. Examples included basic addition and multiplication facts. On the other hand, some of them relied on being able to prove things to show that there were truths in mathematics. Others, still, interpreted the question to mean whether or not mathematical results and knowledge could be questioned. Among these teachers, such questioning was an accepted part of their concept of mathematics.

Abstraction and mathematics

Teachers were asked about abstraction in mathematics and mathematics learning. Their responses reflected both the level at which they were teaching as well as a broader understanding of mathematics per se. Some of the responses also reflected the perceived need to make all mathematics learning relevant to the children's current environments and interests.

Abstractions can be related to real life

While the over-riding practice in elementary school mathematics teaching in Australia remains tied to concrete demonstrations of key mathematical ideas, the introduction of approaches based on 'number sense' and more emphasis on mathematical thinking (McIntosh & Dole, 2000; McIntosh, Reys, & Reys, 1997) have caused teachers to rethink their approaches. Nonetheless, the importance of everyday examples, often related to concrete materials (manipulatives), is evident in many comments.

> Maybe higher maths is abstract but the maths that we deal with every day, for just about everything I teach, I think I could work in concrete materials first. (AUT10)

> Yes and no. There's stuff that you can see, there's stuff that you can't and what you have to be able to do is think both abstractly but also see what is staring you in the face as well. (AUT3)

> Are we stuck on maths being abstract because we believe that's how we were taught? Do we say it is abstract because we cannot see its relevance in everyday life? I think maths is real, it is part of everyday life. Are we saying maths is abstract because we are not recording all that we are doing mathematically? (AUT11)

> The abstract will come after they have had all the concrete and the practice and the manipulation of patterns and numbers and shapes etc. (AUT5)

> To me, maths is abstract but I think it is also very logical and sequential. … I think they have to be shown it originally, they've got to explore it and do it themselves and have me model not just what is happening but the language that goes with it. Then the abstraction will come down the track. So I think all this practical hands-on stuff that we do with our students leads to abstraction eventually. (AUT8)

> To me, mathematics is both. It is abstract to some extent but that is because it's about people developing and creating new ideas – I think that is the creativity of mathematics. (AUT12)

Abstraction, understanding and application

A few of the teachers felt that there were strong links among children's abilities to think abstractly, to understand what they were doing, and to apply their knowledge. For example, AUT2 suggested that mathematics "is an abstract thing, but children need to construct it". However, as a teacher of gifted and talented children, she went on to espouse a belief that "bright children don't need to construct anything, they just do it abstractly in the first place". Other teachers in the sample made other interesting observations.

> I think it is not just in mathematics. I think in general you're trying to help children think abstractly. That's just part of the learning process right through. (AUT13)

> It is probably an abstraction if they may know or have an idea of what it is but not know how to apply it. (AUT7)

Teacher AUT1 saw abstraction as an important part of mathematics learning: "I think it is good training for children to learn abstract concepts. It broadens their thinking and that's one of the good things about mathematics. It gets people to think outside the square". On

the other hand, AUT7 considered abstraction as a beginning (lack of understanding) rather than an end. He linked understanding not to abstraction but to mental 'concreteness'. While this notion represents an outlier in the overall data collection, it is still worth considering.

> In most instances the introduction of new concepts probably is abstract in a way, in that an understanding has not been developed. But your understanding of the abstract thing is when it is used practically. So, the way I look at it is that I just start with something for which there is little meaning or understanding so it is abstract to the child. Once they see a little more information or how it can be used or where it happens in real life, then it goes from being this abstract thing that they don't really understand into something that is concrete in their mind.

Summary

The teachers in this study often made a link between abstraction in mathematics and lack of concreteness or relevance in the children's current lives. While this link has some foundation in current syllabus documents, it is not the complete story. Many of the teachers saw mathematics as becoming progressively more abstract as children moved through their schooling but most saw that their role was to encourage abstract mathematical thinking in their students as soon as they could. However, many suggested that this could not occur until after concrete, relevant experiences had established 'understanding' which was seen as a prerequisite to abstract thinking.

Teachers' Views about mathematical learning

Learning mathematics with understanding

Understanding was generally seen to be an important aim in the mathematics education of children. However, there were some variations in what the teachers considered 'understanding' to be and how it might be achieved.

Understanding is being able to apply what has been 'learned'

Of the 13 teachers interviewed, 8 were quite clear that being able to understand a mathematical concept was equivalent to being able to apply the concept in novel situations. All of these teachers felt that children could be assisted in their understanding by giving them the opportunity to meet novel problems in situations different from those encountered before. The following comments typify the beliefs of these teachers.

> You have understood if you can apply that understanding to something else. If you have learned something in isolation and you cannot transfer that knowledge to another situation, then you probably haven't truly understood. (AUT10)

> [Understanding] is being able to relate it to the known world or to previous experience or to future problems; being able to see where it will apply. (AUT3)

> If you really understand a concept then you are able to apply it in many different situations rather than just applying a skill or a piece of knowledge in one situation repeatedly. (AUT4)

> [Understanding] is being able to use what you have learned appropriately in another context. (AUT12)

> I relate it to like knowing another language, where you can actually use it. (AUT13)

> You understand what you are doing if you have the confidence to carry out the process, without hesitation, in a wide variety of situations. (AUT1)

Understanding is being able to communicate knowledge

The importance of language in mathematics learning is well documented (Cobb, Yackel, & McLain, 2000; Ellerton, Clarkson, & Clements, 2000; National Council of Teachers of Mathematics, 2000). Three teachers related their definitions of understanding to the children being able to use language – in its broadest terms – to communicate what they have learned.

> Understanding is that they're able to show that they know how to get an answer. (AUT5)

> Understanding is achieved when they are able to explain the 'why', the 'how' and the 'do' in a situation using mathematical language to support their explanation. It's using prior knowledge and skills in these situations. It's making connections from past and present learning that support this explanation of 'I got here because …'. (AUT11)

> The greatest litmus test of understanding is when a person can communicate [the idea] to another in a meaningful way. (AUT9)

Understanding is excitement

One of the teachers (AUT8) had quite a different conceptualization of 'understanding'. While still involving the need to be able to communicate new knowledge, her emphasis was more on affective issues.

> I think students reveal understanding when you see this little light bulb go off and they get excited about what they have just discovered or done or finished. You know that they have understood it because they get excited about it and they like to share it with other people.

Summary

'Understanding' is clearly a critical component of the learning of mathematics. Teacher AUT2 suggested that "complete understanding is vital". Teachers' explanations of how understanding might be demonstrated varied but all had a component of children demonstrating their understanding through either direct application of the new knowledge to a novel task or explanation of their knowledge through the use of mathematical language.

Concrete experiences

The use of concrete materials (manipulatives) in mathematics learning, especially in the elementary school, is almost universal in Australia. However, there has been a gradual move away from the belief that concrete materials will 'automatically' result in appropriate mathematical learning, particularly under the influence of a drive for more abstract mathematical thinking. In this context, the teachers' comments around the role of concrete experiences were pertinent.

Concrete materials are necessary for mathematics learning to happen

All of the participants believed that there was a place for the use of concrete materials in elementary school mathematics learning. Eight of the teachers were clear that they thought it necessary to have materials available at all times so that either they could demonstrate a mathematical idea with them or children could use them to assist in their understanding.

> I use a lot of concrete manipulatives in my room. Concrete helps my students to understand, to visualise things and to actually see what is happening in front of them. I think with my students, particularly my Aboriginal students, these concrete, hands-on type activities really work effectively. (AUT8)

> Virtually everyday we have concrete experiences in maths. It's definitely necessary; I think it is necessary forever. However, concrete doesn't mean MAB [Base 10] blocks. It means manipulating things like puzzle tiles, paint etc. (AUT3)

> I am really into concrete, being able to manipulate it so that they have a better understanding of it. In every part of the syllabus, everything I do, the kids are manipulating or playing – we call it play – with concrete materials so that they are understanding what's happening. (AUT5)

> Concrete materials enhance the learning of connections by accessing more than one sensory form of memory; for example, touching it, talking about it, listening to others shares ideas. (AUT11)

Concrete materials are not a panacea

Four of the teachers expressed some misgivings about what they saw as an over-emphasis on the use of concrete materials in mathematics learning. While they all believed that there was a place for them, some thought that there was over-reliance, particularly on structured concrete materials such as Base 10 blocks.

> I think using concrete materials is really important as well. However, I think we need to use them not just as 'here, go and do this activity with these concrete materials' but use them in a way that actually helps them build up visualization. … I think above all we need to help children see the connections between the mathematics that they are learning and real life so if you can teach as much of it as you can through real life problems and scenarios then hopefully they'll make those connections themselves. (AUT4)

> There can be an overuse of concrete materials to the detriment of the child where, without concrete materials, they are lost. Understanding also involves being able to move from concrete materials to just being able to process without all those things involved. … If concrete materials are used in the upper primary school, they should be 'more mature' and we should not underestimate the ability of the children to deal with this sophistication. (AUT1)

> I don't think concrete materials are always necessary and not always possible but, wherever possible, the majority of children benefit from their use. (AUT2)

> Well I don't think it's always the case because there are some times a child will suddenly see the application for what you are doing without needing to manipulate or move materials. However, for a lot of children, they do need to have that actual touching or moving or manipulating. (AUT7)

Summary

The use of concrete experiences in mathematics learning is accepted by all of the teachers interviewed. However, some of them were reluctant to classify all concrete materials or all experiences with structured materials as necessary. While many concrete experiences offer great opportunities for mathematics learning, the individual characteristics of the learner need to be taken into account before offering particular materials. In many ways, teachers saw that the best solution to this dilemma was to have the material available and to allow students to choose whatever they saw as useful in their learning. However, even this flexibility has some associated dangers in terms of reliance on materials or reluctance for some children to be seen using materials when their peers are not.

Some of the teachers were clear that they thought that concrete materials were particularly relevant to certain groups of children and not for others. For example, Aboriginal children were felt to need more experience with concrete materials, perhaps because of a stereotypical belief that these children have much more kinaesthetic learning

styles than non-Aboriginal children (Malin, 1998). In contrast to this, one teacher clearly felt that her gifted and talented students had no need for concrete materials because they were already capable of structured logical thinking. There is a need to remember that even though children may be part of one or more groups, they are, above all, individuals with individual needs. None of the teachers interviewed in this study would argue with this but sometimes even they forget it in the classroom milieu.

Memorization

If memorization is taken to mean 'recall of pertinent information' then all of the Australian teachers interviewed would advocate for memorization as an important component in children's mathematical learning. However, if memorization is associated with rote learning and drill such as was common in Australian schools 30 or 40 years ago, then only a few of the teachers would agree that memorization has a place in mathematics classrooms. This dichotomy of understanding is reflected in the comments of the teachers.

There is a place for memorization

> Memory is very important. They have to start off with a core amount of information. They need to be able to recall it all. They do need to be able to memorise it. (AUT2)

> I still think memorization is really important. I guess there's a lot of people with the view that, for example, with times tables they should forget the times tables and just spend all your time trying to build up understanding. I agree and I disagree with that. I think that understanding has to come before rote learning but if a child gets to a point where they actually fully understand the concept then they would be just that more efficient if they actually then could learn those times tables off by heart and have those number facts at their fingertips. (AUT4)

> I think there's a place for memorization. I'm glad to see that the new syllabus puts some emphasis back on learning times tables. I think that is very important. Along with that comes understanding. (AUT1)

> In a way it is essential. I don't mean just the old tables and the drill but … you've got memorization in terms of techniques that can help you find an answer. There are some things that children just remember so that they can get on with other things. I call them 'essential facts'. (AUT7)

> Facts and skills help with connections. Good connections lead to good, quick recall. Memorization is in a sense a by-product of repeated connections. Certainly the memorization of facts is advantageous, especially with computations. Memorization of facts is convenient. (AUT11)

> When they've got the understanding, then the memorization comes in. Then, if for some reason they forget, they've got those strategies rather than just rote learning. (AUT10)

There is very little rote learning or memorization in my class

Only two of the teachers clearly advocated for little memorization in their mathematics classes.

> If they have a full understanding of what they're doing in mathematics, they don't need as much memorization. (AUT5)

> I have very little rote learning or memorization in my class. I have never ever sung, said or chanted times tables for instance. I have one child in my class who is in Year 5 and still needs to do his 10 combinations. For these, I do a lot of visual things such as ten frames and card games and other things he can remember. It's not rote learning, it's not memorization. He is doing it through seeing and visualizing it. (AUT8)

Summary

Memorization as a notion is tainted with links to the rote learning of times tables through meaningless drill and practice. Such links were generally rejected by the Australian teachers interviewed. However, for the most part there was agreement that it is useful to the children and the teachers if the children can recall some 'facts', including their addition and multiplication facts.

The perennial question about whether understanding comes before memorization or vice versa was raised again by the teachers and there were examples of both positions among those interviewed. Consistent with their earlier explanations, however, the majority of the teachers felt that understanding should precede rote learning or memorization.

Practice

Closely linked to memorization, at least in terms of the 'instant' recall of basic numerical facts, is practice, often in the guise of 'drill and practice'. Not surprisingly, given the variety of responses to the notion of memorization, there is a wide variety of responses to the notion of practice.

Practice is 'extensive experience'

For those teachers who saw value in memorization after understanding, practice could be seen as a way of gaining more meaningful experience in thinking about and using a mathematical concept rather than an approach involving rote learning.

> Practice is the gradual build up of memory. Practice is extensive experience. (AUT9)

> If you don't practice, you forget. They just need to reinforce their knowledge. Just once or twice isn't enough. We have to keep going back over and over and over. (AUT10)

> I don't think that it necessarily hurts. … We do a lot of practice of number facts but we do it in games. (AUT3)

> I think practice is very important, but, again, practice in the everyday world. (AUT1)

> Practice is just giving you experience to use [what you have learned]. I think the more experience you have of using different parts of mathematics [the better].

There can be too much repetition

As further emphasis on the difference between drill and practice, some teachers made it clear that they thought children were often subjected to too much meaningless repetition under the guise of practice.

> I think practice is important but I guess I do think that too much time is spent on practice and writing out times tables or writing out the answers as quickly as you can and that sort of thing. I do think there is too much time spent on it but it is a valuable thing to do. (AUT4)

> We find as adults, if you don't get a chance to practice some of the things that we are going through, they don't stick but you don't need to do a hundred exercises of the same. (AUT13)

> I think you only need to go so far as realizing when most people are confident with the idea and not flogging it to death. (AUT1)

> Well I don't do much. You would never walk into my room and see sums on the board. I very rarely do that sort of thing. I don't have textbooks in my room either. We do some practice but it's got to be really meaningful and they've got to know why they're having a practice at it. I try to make most of the practice fun and engaging. (AUT8)

Summary

Most of the teachers interviewed believed that there was a place for practice once the children had gained understanding of the concepts being considered. However, the practice needed to be based on relevant problems and did not necessarily need to be extensive. Teacher AUT12 proposed a useful distinction between drill and practice.

> To me, practice is something that happens when the children have a grasp of understanding and they know the real reason for doing the mathematics. The practice is reinforcing that understanding to make it easier for them. Drill is giving the children a page of 50 questions and saying you have half an hour to do this whether they really understand it or not. They can get 50 questions right but they may not be able to ever use that in another context.

Most of the Australian teachers interviewed were happy to have practice in their children's mathematics learning but were not inclined towards drill.

Teachers' Views about the Teacher and Teaching

Characteristics of an effective teacher of mathematics

All of the Australian teachers interviewed in this study had strong opinions about the characteristics of effective teachers of mathematics. Many of these characteristics reflected their own practice and the practice of key mathematics educators with whom they had worked and/or by whom they had been influenced. As well as recognizing the impact of effective teachers on children in their classes, many of the characteristics involved the influence of the teachers on people other than children, particularly other teachers and parents.

Effective teachers have passion, enthusiasm and love of mathematics learning

Most of the teachers interviewed exuded a passion for mathematics - as well as for their teaching in general and their teaching of mathematics in particular - that was infectious.

> You have to have a love of the subject. You've got to have a burning desire to investigate and be inquisitive. (AUT9)

> Someone who displays an interest or a love of learning themselves. I think that a lot of it comes back to motivation. The teacher needs to be motivated in order to motivate the children. (AUT2)

> You need to be enthusiastic and passionate and make it interesting for the children. (AUT10)

> I really value maths and mathematics teaching now and I think the students do too. I'm really enthusiastic. I get excited in my classroom and the children can see that you really are interested, that you really love this thing, and it carries over to them. … I'm getting towards the end of my teaching career but I'm loving it at the moment. … My students are really into maths, they love to do maths. It's important not only for teachers to be enthused but the students need to enjoy being enthused by mathematics. (AUT8)

> An effective teacher has a passion for the subject. They really enjoy mathematics, keep themselves abreast of mathematics learning and read about mathematics themselves. They communicate that passion and interest to the children and they excite the children about the potential of mathematics. (AUT1)

Effective teachers have knowledge of the subject and syllabus

In Australia, most elementary teachers are generalists, teaching all of the subjects in the overall curriculum. They tend not to specialise in mathematics or any other subject, even if they have particular interests or skills in one of them. Nonetheless, the need for effective mathematics teachers to have a strong knowledge and understanding of both the subject itself and the syllabuses being used was consistently proffered by the participants of this study.

> I believe that the teacher has to have a good, strong knowledge of mathematics. (AUT9)

> They have to have an understanding of the syllabus to start with and what they should be teaching. (AUT5)

> An effective teacher plans with a clear scope and sequence and accesses the guidelines articulated by the [education] department. (AUT11)

> I think effective teachers need to have an understanding themselves. They have to have a knowledge of the content. (AUT7)

> You should have a good understanding of what you are trying to teach. (AUT13)

Effective teachers show the relevance of mathematics in today's society

As has been reported earlier in this paper, making mathematics relevant to children is accepted as important by all the teachers interviewed in this study. However, one comment seems to sum up this disposition.

> I think an effective teacher would be someone who is able to take the curriculum and make it alive for kids. Someone who can make it interesting and who can show the relevance of mathematics in today's society. (AUT12)

Teacher personality

One teacher felt that humour was an important trait for effective teachers to have and that effective teachers needed to be approachable. At the same time, however, effective teachers needs to establish their authority in the classroom.

> I think humour helps. The personality of the teacher is important. You need to be approachable but you also need to have some authority so that you can get the point across. (AUT13)

Effective teachers know their children

The teachers in this study indicated that knowing the children they teach was very important. This knowledge assisted them in making their mathematics lessons more relevant because they could be attached to the children's interests and environments.

> You have to know your students pretty well. (AUT10)

> You need that rapport with the children too; that's personalities, understanding, knowing a little bit about them, but the most important thing is where they are, where you want to get them and lots of different ways of doing it. … I like to think that my classroom is a sanctuary [for the children]. It should be a comfortable environment. (AUT7)

> Once there is a level of empathy with the student so that you know this person reasonably well, at least in terms of their interests, you can start to get somewhere. (AUT9)

> I think that comes from knowing your students. I really find that maths can knock down a lot of barriers. (AUT8)

Effective teachers are co-learners with the children

Two of the teachers expressed the view that effective teachers needed also to be co-learners with their students, so that even if they did not fully understand the mathematics being taught, they could model the learning that was necessary.

> I think they should always strive to know more mathematics but perhaps if they view themselves as co-learners, if they do lack confidence then as a co-learner, they get all of that knowledge and skill that they need by actually exploring it with their students. (AUT4)

> They need also to be a learner. They need to be willing to learn from the kids because it is amazing the amount of information that they have and you don't realise. (AUT3)

Effective teachers mentor other teachers and engage in their own professional development

While only two teachers explicitly mentioned the mentoring of other teachers as an indication of effectiveness, most of the teachers were or had been involved in mentoring roles in mathematics education.

> I think effective teachers mentor other teachers. They accept that they need professional development, it's ongoing and that there are always better ways to do something. I think effective teachers share good practice, are willing to use a variety of resources and accept that they can be wrong too. (AUT11)

> I talk to a lot of other teachers, I'm always on the go. All the teachers ask me to do things and ask me to come into their classroom. (AUT8)

Effective teachers are facilitators as well as explicit modellers

Among the teachers interviewed for this study, there seemed to be a balance between the facilitation of children's learning and instruction.

> They should have good role modelling for the kids, they can show kids how to work out a problem, giving them some scaffolding of what to do with certain problems, how do you go about solving a problem, what steps you need, what formulas you would need to use. So they do all that modelling, they teach the kids the modelling, they get the kids to work together. (AUT12)

> An effective teacher gets other sources – in this case, students – to give you some ideas and that, hopefully, might trigger somebody else who is sitting over there in the corner who doesn't understand what you, the teacher, may have been doing and suddenly goes 'Ah, click. Yes, that way makes sense to me". (AUT7)

> They've got to view themselves as two things – as a facilitator, so that they set up scenarios where the students actually can create their own understandings through discussion and working with each other but also they need to be really strong at explicitly modelling things as well so that the student might come up with a strategy and then the teacher can go on to explicitly model that strategy and perhaps get another student to have a go at that strategy. (AUT4)

Effective teachers have high expectations for their children's learning

All of the teachers interviewed in this study had high expectations of themselves and their own teaching. Some expressed the need for them to have high expectations of their children as well.

> An effective teacher communicates very clearly to the students what the goal for achievement is in that lesson. (AUT1)

> I have such high expectations of my students with maths. We come from a supposedly low socio-economic area where the children come to school with very little maths but I have high expectations and I challenge my kids. I think they're achievable, sometimes they're very difficult but I give them enough confidence for them to have a bit of a go at it. I think if you expect highly of your students they will come up to that, especially if you can make them achievable and you support them. (AUT8)

Effective teachers involve parents

The two teachers who had mentioned mentoring other teachers also mentioned how they try to involve parents in their children's mathematics learning. In Australia, parental involvement in literacy classes is quite common, especially in the first years of school. In mathematics classes, parental involvement is not as common.

> I provide lots of activities that can be shared with parents or the school community. I use interactive displays here and I have found these to be really effective. (AUT11)

> I always have a lot of parents in my room, which is good. I try to get them involved in it because I think then that my students can see that the parents are valuing what's happening in school and the school is valuing what the parents bring. (AUT8)

Summary

There are many characteristics of an effective mathematics teacher arising from the above results. Key among these seem to be knowledge of the subject and a passion and enthusiasm for both the subject and its teaching. The Australian teachers interviewed also felt the need for effective teachers to know their children well and to make sure that their lessons were fun and relevant, both for the children and for the teachers. Some of the teachers interviewed felt that effectiveness might have dimensions beyond the classrooms and beyond their immediate students in these classrooms. So, mentoring other teachers and working with parents were highlighted by a small number. Teacher AUT8 summarised her feeling about effective (good) teaching in the following way.

> I think good teaching, as in anything, is an attitude thing. If teachers enjoy it and get into it the students will respond and reflect your enthusiasm and attitude. I think that is the whole part and parcel of it. If you're really enthusiastic and have this positive attitude to maths and maths teaching I think it'll roll over into the students.

Characteristics of an effective mathematics lesson

Effective mathematics teachers engage their students in effective mathematics lessons. All of the Australian teachers interviewed in this study were deemed to be effective mathematics teachers. While all of them expressed humility in their interviews, they all agreed that most of their mathematics lessons seemed to be effective. Hence, they felt that they were in a position to provide some advice on the characteristics of an effective mathematics lesson. Some of this has already been canvassed during the teachers' characterizations of effective mathematics teachers but other comments are also revealing.

Children and teacher respect each others' views

Many of the teachers interviewed described their typical lesson structures, including some time in which children had the opportunity to express their own opinions about the tasks being undertaken to the rest of the class and the teacher. This is one way in which children can be shown respect for their views. Teachers were also adamant that when they had something to say to children, they expected that the children would listen. However, this was not seen as a behavioural issue but, rather, as an issue involving the children's

respect for the teacher. Both directions of respect need to be earned and neither can be taken for granted.

> You try to make sure that children have the opportunity to question, to discuss, to answer and that there's an atmosphere where the children and teacher respect each other's views and that those are listened to. (AUT1)

Active engagement, often through curiosity

The Australian teachers interviewed in this study all reported that effective mathematics lessons needed to engage children actively. The influence of the productive pedagogies approach to teaching and learning (Gore, Griffiths, & Ladwig, 2002) which has been systemically adopted in a number of states of Australia can be seen in this language. Nonetheless, the comments from the teachers in this study show clearly that their practice goes well beyond rhetoric.

> As far as possible, you vary the activities throughout the lesson and move from some practice to some explanation to some discussion to some fun activity and you keep the children's interest at a very high level throughout the lesson. There's no part of a lesson which either the teacher or the children consider to be totally and utterly boring. (AUT1)

> [An effective mathematics lesson is] one where all the conversation is about the maths, the students are engaged and they are participating and there is not too much teacher talk. (AUT13)

> I think curiosity is a big thing with kids … and active student involvement. (AUT2)

> You have to have engagement by means of curiosity. (AUT9)

There is a clear goal to be reached that is known by the children

Another aspect of the productive pedagogy approach is that the students should be clear about what it is that they are supposed to be learning in each lesson. The interviewed teachers tended to take this for granted and did not speak about it in detail. However, a couple of them had strong points to make.

> I guess coming from a productive pedagogy kind of viewpoint now I would say that an ideal mathematics lesson would ensure that first and foremost the students would know what the aim of the lesson was, what the whole purpose of it was. So that would be made explicitly clear right from the beginning. … What needs to be added here is that it needs to be made explicit to the students what is expected of them so that when they go off to do an activity they know what their final product needs to have. (AUT4)

> I have clear goals to be reached, they know where the journey is going, it's very clear and they have to be very focussed. (AUT11)

A lot of meaningful questioning

In an approach which relies on the teacher facilitating children's learning of mathematics rather than telling them the mathematics they need to learn, there is a strong reliance on appropriate questioning by both teachers and children. This reliance was in evidence in some of the teachers' comments.

> The language of mathematics is important, how to talk with children, how to question them and how to get them to talk about mathematics in a certain way, to get them to contribute to the maths that is happening in the classroom. (AUT8)

> Excellent mathematics lessons have a lot of questioning children, questioning what they are doing and why they are doing things. (AUT5)

Lessons are planned but flexible enough to change

All of the teachers interviewed believed that they needed to plan their lessons, although given that they are such an experienced group of teachers, such planning is not necessarily recorded in a written form. However, the teachers were also well aware that planned lessons can be disastrous if they do not have sufficient built-in flexibility to allow change in unexpected circumstances. Planning is not only about the mathematics to be learned but also about the learners, their needs and their dispositions.

> Some lessons go over really well and some don't and you never know which ones they will be. (AUT10)

> Most of my lessons are planned. … For example, you would have to have the resources you needed there and if there is a child who needs concrete material then you have to have it available. There has to be an ability to change. Some of the best lessons are the ones where you know where you're going but you can be taken off on a tangent because it is really helping the kids understand what is going on. Then you bring it back. (AUT7)

> You need to have plenty of material available for those who finish tasks early so that they're not sitting around twiddling their thumbs and can move to other things to do. (AUT1)

> I never teach a lesson the same way twice because I think 'That didn't work very well, I'll have to change this". I'm constantly changing things and I think "That was a disaster". It may not have been a disaster but I think "That was terrible, I should have done this". Next time I teach something that is similar I think "No, I'm not doing that again" and I totally change what I do. (AUT5)

> When new ideas have been discovered, when perhaps what I had planned is not what we've done at all, which is what happened to us this week because somebody came up with something and we've gone off on a tangent and discovered something totally new, then that is an excellent lesson. (AUT3)

There needs to be routine and structure

Many teachers who were interviewed in this study emphasised the need for routine and suggested that children generally flourished when there were set routines. That doesn't mean that routines can never be broken but rather that the stability provided by routines facilitates children's mathematics learning.

> I do maths straight after recess every day so that they know the routines. We have maths every day for at least an hour. (AUT8)

> There is the basic structure [of a lesson]. There's that tuning in and engagement, sometimes by means of curiosity. There could be modelling of the strategy and then from there it could be trialling that strategy. So, there are those structures you have in place. (AUT9)

> Then, at the end of the lesson, I always bring the whole class back together and we always say "OK, what did we do today?"; "What can you tell me that you have learned today?"; "What did you do that was different to this person?". The kids, usually, come up with some great things about what they did each time. (AUT5)

Children can be organised into groups

While the teachers interviewed did not necessarily see small group work as either necessary for, or a hindrance to, excellent mathematics lessons and most of them would

use such groups in their teaching, three of them made specific mention of the ways in which they used groups.

> They are in small groups of six, so I am running five groups, five rotational groups which supports me as a teacher because I am not planning every single day. (AUT11)

> At times you need to have groupings. So you might have a core lesson and some work that the children who have obtained or understand that knowledge can go on with. Then you can spend some intense time with the ones that don't. (AUT7)

> You need a variety of things so that you can extend the ones that pick it up really quickly but you can still give time to the ones that need more time to absorb what you are teaching. Peer tutoring works at times but you cannot expect the brighter ones [to do this all the time]; to keep them interested you need to keep them extending themselves. (AUT13)

Acceptance that the learning is a journey

Two of the teachers emphasised the importance of children knowing that their learning takes a series of steps and that getting to an answer is not necessarily the end of the investigation.

> You will have kids who are creative and come up with some good ideas of how to solve a problem but it may not be the best way of solving it. They may actually come up with better ways than you might but sometimes they need redirection, saying that what you have done is great but there is an easier way of doing it. (AUT12)

> I think that excellent lessons are those that accept that the learning is a journey. In the classroom, the children see that on the journey there are skills that are going to make the journey easier and I am the vehicle to give them those skills. (AUT11)

Start the lesson with something that's really interesting and get the kids really excited about it

Many of the teachers in this study saw an excellent mathematics lesson as one where there was excitement on the part of all concerned, relevance for the students and sufficient time, space and resources to allow for the children to explore the ideas being developed for themselves. Some teachers even suggested that if lessons began well enough, in an interesting way, then the children would be motivated to continue their learning.

> When it is time for maths groups it should be "Yes! Off we go to maths" and they should be coming into the classroom excited. For all sorts of reasons, the mathematics classroom should be a place where they feel really good about themselves, where they're feeling really enthused to be there, not somewhere where they have to sit for whatever length of time, but somewhere where they don't want to leave at the end of it. … Not everybody feels like that all the time but there are times when the recess bell has gone and I am shooing them out the door and they're still not going. (AUT3)

> I like to start the lesson off with something that makes the children think. It doesn't have to be anything to do with the particular topic that you're learning but it just means that you are trying to get the answer to something. (AUT13)

> I think the project base is often a really good way of getting kids hooked in, where you develop a project around them trying to solve a problem. At the same time, they decide that they really need a lot of support. They say "How do we find the area of a circle?", for instance. Then you can come back and say "Let's look at this. How could we solve the problem?" (AUT12)

> I heard somebody once describe a day as a little bit like the news. You have your "Here's the headlines – bang". Then you have your news and then you have a wrap-up at the end. That's

> probably what a maths lesson needs to have too. "Today we are going to – boom". Then you do what happens and you also say "This is what we are going to produce at the end". Then you get to the end and you might do it in terms of "Here's a really good thing that so and so did" or "What did you think about this?" or "Where could we have gone here?". So, you have your introduction, your main body, your conclusion and some sort of evaluation or feedback to the kids about how they went; how the lesson went or where you are going next. (AUT7)

Summary

All of the teachers interviewed in this study had been chosen as effective mathematics teachers. Hence, it is reasonable to assume that they had implemented many effective or even excellent mathematics lessons. All had strong opinions on the characteristics of such lessons. Key among these was the need for lessons to be exciting, active and relevant to the children. As well, teachers saw a need for there to be mutual respect among teachers and children, particularly in terms of listening to what each other has to say. There was quite a lot of emphasis on the structures and routines in mathematics lessons with the teachers feeling that such routines in particular were needed by children. The influence of the productive pedagogies approaches that are currently being introduced at system level in many states of Australia was seen in the teachers' comments about explicit teaching, children's awareness about what they are expected to do and learn and the importance of meaningful questioning.

Conclusion

The key aim of the study reported in this paper was to ascertain what Australian teachers who had been designated as excellent teachers of mathematics believed were the characteristics of effective teachers of mathematics and excellent lessons in mathematics. Not surprisingly, given the variation in experience and teaching contexts of the teachers, there was a wide variety of responses from the teachers. However, some commonalities could be discerned.

It was clear during each of the interviews that all of the teachers in this study displayed a deep passion and enthusiasm for mathematics and mathematics education, so it is not surprising that the presence of such passion and enthusiasm was seen to be one important trait of an effective teacher of mathematics. So, too was knowledge of the subject content and of the syllabus. This needs some interpretation because most of the teachers interviewed were without tertiary mathematics experience, except in terms of what they learned in their mathematics education courses. So, when these teachers talked about 'knowledge of the subject content' they were referring to knowing the mathematics that they needed to teach. This is a very restricted view of mathematics but is typical of Australian elementary school teachers. All of the teachers interviewed were quite conversant with the content of their state or territory's mathematics syllabus and used it extensively in their preparation of mathematics lessons. However, their planning went well beyond the syllabus and was structured very much to meet the needs of their particular group of children.

One of the key characteristics of excellent mathematics lessons reported by the teachers in this study was the need for there to be mutual respect among the children and teacher. This respect is much more forthcoming if people know each other, know what each other is interested in and if the activities designed for classroom experiences are relevant to these interests. This means that teachers need to know their children beyond the

confines of the classroom and the school and use that knowledge to help plan and implement relevant mathematics learning experiences.

One interesting feature of the teachers' responses about effectiveness as a mathematics teacher was the emphasis by some on aspects of a teacher's profession beyond teaching children. Clearly, effectiveness was seen to have a leadership component, especially in terms of the mentoring of other teachers.

Mathematics lessons described by the teachers in this study always revolved around well planned activities designed to meet the needs of the students in the class. However, there was more to them than this. The lessons were seen to be fun, full of experimentation, exploration and active engagement. They provided the children with interest and excitement in their learning and resulted in challenge in a supportive environment. The lessons were well planned but a number of the teachers made the point that planning is not enough. There needs to be sufficient flexibility in the planning – and in the teacher – to allow for unforeseen deviations from the plan as something interesting comes up during the lesson. Excellent teachers of mathematics are well prepared in terms of what it is that they wish the children to learn but they also well prepared in terms of letting the children run a little when opportunities arise. The structures and routines need to be in place but the ability to move away from these when it suits is very important.

While there may be a certain Australian stereotype of a laid-back, casual person who is willing to let things happen as they will, this is not reflected in the teachers in this study. They are clearly a dedicated group of professionals who have thought very carefully about their practice as teachers of mathematics and whose children and peers have benefited from this thought. The passion and enthusiasm expressed by these teachers is infectious and it has been a pleasure to share some of this in this paper. No doubt, teacher education and experience can make a great deal of difference to the quality of teachers in classrooms. However, so many of the characteristics of effective teachers of mathematics and excellent lesson in mathematics discerned in this study are personal qualities that are not the focus of teacher education courses. Perhaps the combination of these personal qualities can be developed through the confidence and self-belief that can result from greater knowledge about mathematics and mathematics learning and teaching gained from preservice teacher education, professional development, reading and sharing with colleagues. Whatever is the case, the teachers in this study, chosen for their effectiveness, have provided great insight into how this effectiveness might display itself in the mathematics classroom.

References

Australian Association of Mathematics Teachers (1997). *Numeracy = Everyone's business.* Adelaide: Author.

Australian Association of Mathematics Teachers (2006). *Standards for excellence in teaching mathematics in Australian schools.* Adelaide: Author. Available on-line at: http://www.aamt.edu.au/standards/standxtm.pdf

Bobis, J., Clarke, B., Clarke, D., Thomas. G., Wright, R., Young-Loveridge, J., & Gould, P. (2005). Supporting teachers in the development of young children's mathematical thinking: Three large scale cases. *Mathematics Education Research Journal, 16*(3), 27-57.

Cai, J. (this volume). What is effective mathematics teaching? A study of teachers from Australia, China, Hong Kong-China and the United States. *Zentralblatt für Didaktik der Mathematik* .

Cobb, P., Yackel, E., & McClain, K. (Eds.) (2000). *Symbolizing and communicating in mathematics classrooms.* Mahwah, NJ: Lawrence Erlbaum.

Dockett, S., & Perry, B. (2007). *Starting school: Perceptions, experiences and expectations.* Sydney: University of New South Wales Press.

Ellerton, N., Clarkson, P., & Clements, M. A. (2000). Language factors in mathematics education. In K. Owens & J. Mousley (Eds.), *Research in mathematics education in Australasia 1996-1999* (pp. 29-95). Sydney: MERGA.

Gore, J. M., Griffiths, T., & Ladwig, J. G. (2002). Exploring 'productive pedagogy' as a framework for teacher learning. Paper presented at the *2002 Annual Conference of the Australian Association for Research in Education*, Brisbane. Available on-line at: http://www.aare.edu.au/02pap/gor02267.htm

Malin, M. (1998). They listen and they've got respect: Culture and pedagogy. In G. Partington (Ed.), *Perspectives on Aboriginal and Torres Strait Islander education* (pp. 245-273). Katoomba, NSW: Social Science Press.

McIntosh, A., & Dole, S. (2000). Mental computation, number sense and general mathematics ability: Are they linked? In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000* (Proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia, Fremantle, pp. 401-408). Sydney: MERGA.

McIntosh, A., Reys, B. J., & Reys, R. E. (1997). Mental computation in the middle grades: The importance of thinking strategies. *Mathematics Teaching in the Middle School, 2*(5), 322-327.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Vygotsky, L. S. (1978). *Mind in society*. Cambridge: Harvard University Press.