Problem-solving • by any other name...

Knowing when to let go is often a dilemma for teachers using open-ended tasks. **TOM LOWRIE** and **TRACEY SMITH** explore this issue by describing the engagement of children in an authentic task and examining the teacher's role more closely.

When we think of problem solving in classrooms, almost all of us would have a slightly different image of what would be occurring. The idea of using the process of solving problems to enhance learning has been with us for many years. In more recent times, the term problem solving has been transformed into other terms that more explicitly represent the type of problems or approach we use in classrooms. For example open-ended tasks or 'good' questions (Sullivan & Lilburn, 1997), rich tasks (Education Queensland, 2001) and problem-centred learning (Wood & Sellers, 1999) all describe ways of coming to know mathematics through an inquiry or investigative approach. In particular, these terms highlight ways of interacting with students that are learner-centred rather than teacher-centred, and open enough to allow for a variety of solution pathways.

Authentic pedagogy

One common element that all these approaches share is that they allow for *authentic* teaching and learning that can lead to *authentic* assessment opportunities. This common element can be thought of as *authentic pedagogy*. The word 'authentic' suggests meaningful and worthwhile learning experiences that are characterised by: an inquiry approach that leads to the formulation of new ideas; developing a connected picture of knowledge; and learning opportunities that are seen as valuable outside the classroom (Burke, 1999).

This characterisation supports the ideas outlined by Sullivan & Lilburn (1997) when they suggest that 'good' questions or open-ended tasks require more than the recall of facts and allow for many pathways towards one or many solutions. Most importantly, the open-ended nature of tasks allows for unexpected solutions and for both students and teachers to learn from the process of doing the task.

This article explores one open-ended task given to a Grade 6 class that illustrates what authentic pedagogy might look like. The task contextualised a number of mathematical processes within a meaningful and worthwhile task that also provided an opportunity to assess students' progress. The significant factor for educators is the variety of methods used by students to solve the task. It highlights how one task that is open-ended can accommodate the differing ability levels of students in one 'straight' Grade 6 classroom.

Providing a context for authentic learning

The grounds outside the Grade 6 classroom were devoid of grass. Many attempts had been made to return this area to its former glory (including some quite expensive proposals), but alas, no improvements were evident. The school's *Parents and Friends* committee decided that turf (grass strips) should be laid and that an underground sprinkler system should also be installed. As part of one of their mathematics projects, the teacher decided to introduce the problem to the class. It was agreed that the class should design an appropriate underground watering system for the school.

Developing measurement and spatial concepts

This task was designed so that students could appreciate that measurement concepts have a practicality function that is often linked to realistic problem-solving situations (NCTM, 2000). Providing opportunities for children to engage in investigations that are personally meaningful (authentic) supports a learning environment that encourages children to reason mathematically and still have ownership of the problem. As they progress through the grades, students should be increasingly challenged to apply appropriate techniques to determine measurements, including the use of various types of informal and formal instruments. The very nature of the present investigation challenged the Grade 6 children to use techniques that complemented their understanding of measurement concepts associated with length, area and angle measures. Thus, individuals used varying degrees of informal and formal measurement techniques to generate a solution.

Approaches adapted in the investigation

The children worked on the activity in class for approximately three weeks. In that time the students needed to consider the cost of various sprinklers, work out their respective water coverage, calculate how many fittings they would need, decide on the hose diameter and ascertain whether the school's water pressure could cope with the design. Additionally, the students had to consider a range of 'what-if' situations including the need for young children to be able to turn the system on, and implications for damage.

Since the task was quite open-ended in nature, the students appreciated that a number of different solutions were plausible. Some children worked in small groups, whereas others preferred to work by themselves. Motivation remained high throughout the activity. It was anticipated that the children would need to consider a number of measurement concepts associated with length, perimeter and area. In addition, spatial concepts related to position and 2D-3D representations would also be evoked. The classroom teacher suggested that a two-dimensional plan of the site (preferably to scale) would be appropriate and that the cost of construction should not exceed $3000. Although the children were challenged to find out as much about watering systems as possible, other resources, including information about the water coverage of particular systems, were made available to the class.

Following are descriptions of three approaches employed by small groups of children to complete the task. In each case, the group was able to propose a solution that was both realistic (in the sense that the design would actually work) and appropriate (in the sense that it fulfilled the design requirements of the task). The fundamental difference between the approaches was the extent to which the children relied on informal measurement understandings and strategies to complete the task.

Nathan's response

Nathan's story represents a response from a group of students who decided they would use plastic fittings and sprinkler heads that could be purchased from any department or hardware store. Generally, the water coverage area of such equipment is not as large as commercial systems — but they are considerably cheaper. This group found the following information vital to their overall plan:

- full circle sprinkler: water coverage 3 metres
- half circle sprinkler: water coverage 3 metres.

This information was supplied in packaging and related to the water coverage of the respective sprinklers. Nathan described how his group approached the task.

> Before we tried to draw our system on paper we decided to see what it looked like in real life. We got a stack of [traffic cones] and spread them out over the grassed area. We also asked our teacher if we could borrow all the skipping ropes from the sports room. We tied two short ropes together and put blue chalk marks on the rope at 1, 1.5, and 3 metres. We measured 3 metres in from one corner and put a [traffic cone] down. We then measured out 3 metres for the diameter of the full circle and put other ropes down to form a large circle. Sally then paced out how many steps it was from one end of the circle to another, making sure she went through the centre. We put another hat down in a straight line from the 1st one. We kept doing this, making sure we had some overlap between each circle area. We run [sic] out of skipping ropes — but it didn't matter because we picked them up and moved them. As long as the witches' hats stayed where they were we could work out how many sprinklers we would need. (Nathan, Grade 6)

This group of children used a form of trial and error to work out their pattern. They processed the relevant information on-site, actually using a true 1:1 scale; that is, doing all their measurements in the designated area. Although this approach was relatively effective for this task, it would have been quite laborious if the area had been larger. They were not able to draw a scaled diagram to complete the task without first doing it in a more informal manner. In this instance, insisting that these children should design a scaled drawing would not have worked: conceptually they were not ready for this so they produced their scaled drawing after developing their concrete representation.

In this approach, the concrete nature of the solution path ensured that the children did not lose sight of the main goal of the problem. It may be the case that they were not able to make the transition from a 3D to a 2D context without first completing the task. Their 'step by step' approach allowed them to test and modify changes using a combination of formal and informal measurement techniques.

Shannon's response

Shannon's group represented their proposal through a scaled diagram (see Fig. 1). This group actually shaded the proposed area of water each sprinkler would cover. Importantly, they have also indicated the lengths of hose required between each sprinkler and calculated the total amount of piping they would need for the project.

Unlike the first group, these children did most of their estimations and calculations on paper away from the site. One of the members of this group, Shannon, described the manner in which they represented the task.

> The first thing we did was use a trundle-wheel to measure the area of the playground we wanted to lay turf on. It was

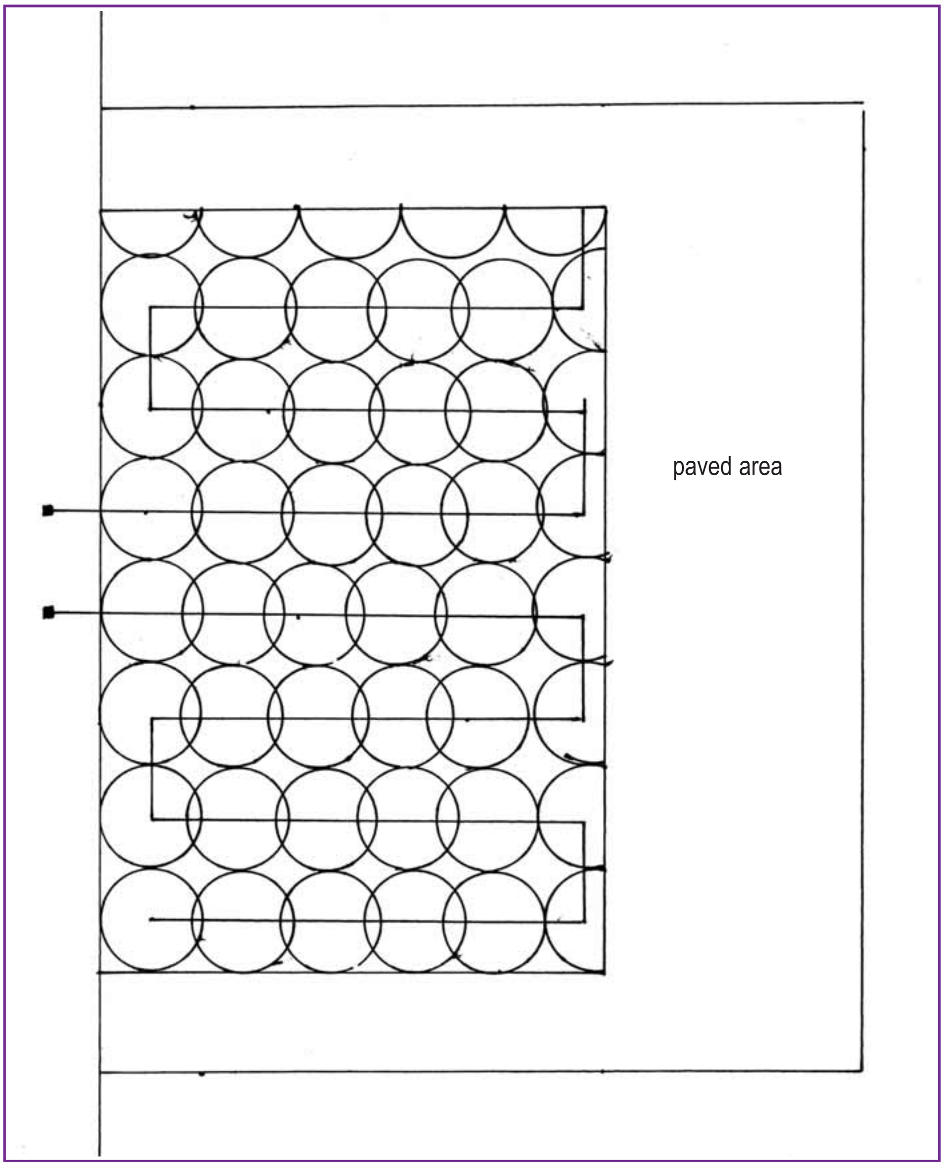

Figure 1. Originally drawn with a 1 cm : 2 m scale with a five-cent coin used to represent the water coverage of 3 metres.

> almost 30 metres long and 20 metres wide. We then had to decide what type of sprinklers we would use. We felt that the little ones would be best and they covered an area of 3 metres across. Our scale was one metre to two metres because we could use a five-cent piece [a coin] to show the water coverage… We had a lot of sprinklers but it was a large area we had to cover. (Shannon, Grade 6)

The groups' selection of a scale for their plan was quite interesting. The decision was based on the ease with which they could actually represent their work. Generally, they felt that using a compass set to draw circles was problematic (possibly based on their experience with using such tools) and consequently decisions about the selection of an appropriate scale for their representation were associated with the ease with which they could draw circles. In this case the students' notion that the scale was 'one metre to two metres' should have been expressed as 1 cm : 2 m. Consequently, they understand that the five-cent coin (1.5 cm diameter) represented the three metre coverage of the water. Although the way they represented their scale was incorrect, conceptually they were able to demonstrate an awareness of how to use scale. The selection of appropriate units and scale are important skills that children need to develop as they progress through the middle years of school (NCTM, 2000) and the accuracy achieved with this approach was quite acceptable.

Interestingly, this group calculated the number of sprinklers and the length of pipe they would need to purchase for the project from the scaled plan they had designed — they did not physically visit the surroundings once they had determined the area of the site. This explains why their water coverage did not cover the 30 metre by 20 metre grassed area. Instead, their watering system only covered an area with dimensions of approximately 24 metres by 16 metres. As Figure 1 shows, the paved area surrounding the sprinkler area needed to be included in the plan in order to cover the intended area (30 m × 20 m). For teachers, this reinforces the importance of children moving between the three-dimensional and two-dimensional contexts. It also reiterates the important role that justifying solutions plays in working mathematically.

Rebecca's response

By contrast, Rebecca explained that her group regularly moved between the physical environment and their 2D representation as they engaged in the activity. As Rebecca indicated, the children in this group used the actual environment to monitor the authenticity of their 2D design.

> After we had drawn our design it looked like we had too many sprinklers for the amount of space we needed. We all went down to the playground and decided to check our plan. Jo had the largest steps so he paced out where seven metres would be in order to cover the diameter and continued doing this across the length of the playground. The total was close to what we had drawn on our plan so we knew we were right. (Rebecca, Grade 6)

These children were able to verify their results by representing the problem in a concrete manner. Furthermore, they demonstrated an ability to interpret their results across different forms of representation. Such problem-solving and reasoning skills should be encouraged in the mathematics classroom (NCTM, 2000).

This group decided to use a one centimetre to one metre scale for their 2D design (see Fig. 2) with Rebecca suggesting that they could 'use up all our page because the paper was almost 30 centimetres long and the playground was 30 metres long.' Rebecca's group decided on an appropriate scale once they had considered the dimensions of the physical area and the length of the A4 paper. Shannon suggested that her group selected their scale after considering the ease with which they could represent the water coverage of the sprinklers, hence the selection of a five-cent coin. In both

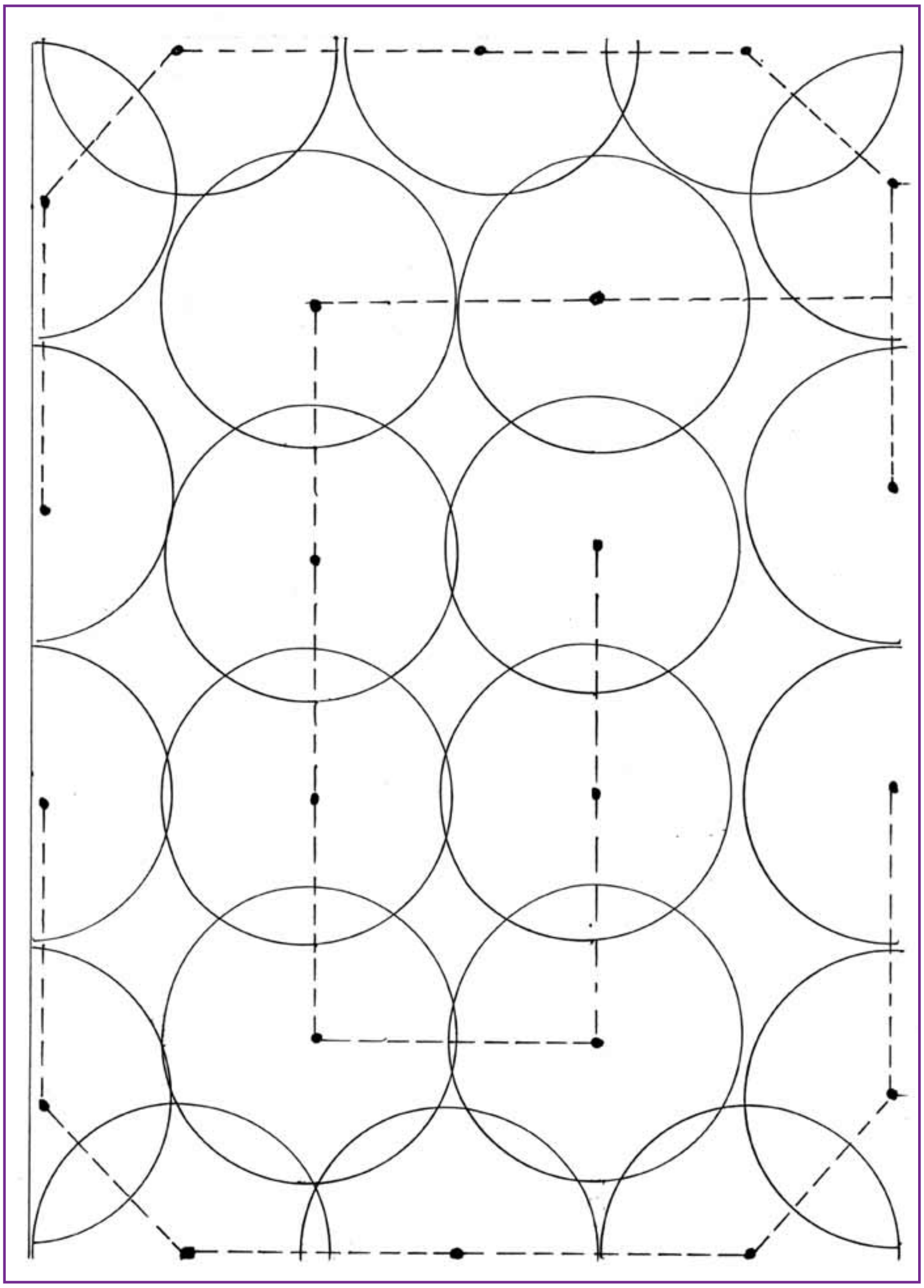

Figure 2: A 1cm to 1 metre scale with a 7 metre water coverage for each sprinkler

instances, the actual scale was selected after considering a number of other factors. We would suggest that both groups had begun to develop relatively sophisticated understandings of scale.

Implications for the classroom

The authentic nature of this teaching and learning activity not only provided an opportunity for the children to be inquisitive and reflective, it also allowed them to be 'empowered' in the sense that they had the flexibility to direct and monitor their learning. The three responses in this investigation described different ways of knowing and justifying their approaches in relation to the skills and understandings they possessed.

The open-ended nature of the task allowed access to learning for all ability levels. Children who had a limited knowledge of scale, or who found it difficult to move between the 2D and 3D contexts, were able to use varying degrees of informal techniques and measurement understandings in order to make connections to more formal procedures or analytic reasoning. Although the three responses outlined quite different approaches to completing the task they were able to make a convincing case for their respective approaches. The opportunity to reflect on their approaches, and to be exposed to the methods and approaches of other groups, made the learning experience even more valuable.

Knowing when to let go

One of the difficulties teachers face when they decide to implement open-ended tasks is knowing what role they should take in the process. How much assistance is too much? The teacher in this story exemplified the role of a facilitator, who provided guided assistance to students (Tharp & Gallimore, 1988). The focus for the teacher was on listening and responding rather than telling and directing. For each learner or group of learners, the amount of 'scaffolding' varied considerably. In other words, using open-ended, authentic tasks does not, and should not require teachers to surrender their place in a learning community. Rather, the teacher's role becomes one of a guide who provides a level of assistance that is negotiated as learning progresses.

The use of open-ended tasks supports the notion of authentic pedagogy, which relies on establishing a caring, educative relationship with learners. The example in this article described the use of an authentic, open-ended task that promoted risk taking and fostered ownership of learning. By doing this, students became more empowered to direct their own learning futures. The teacher's role has never been more important!

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Tracey Smith and Tom Lowrie teach at Charles Sturt University, Wagga Wagga.

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