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Designing a Framework for Problem Posing: young children generating open-ended tasks

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ABSTRACT This article describes the type of problems young children (aged six) generated in problem-posing contexts. With support, the children began to generate increasingly sophisticated problems that were open-ended in nature. The problem-solving situations provided opportunities for the children to pose problems they enjoyed solving and promoted both a more complex and motivating learning environment. The results indicate that the problem-posing actions of students can be nurtured by teachers' actions.

Introduction

Young children should be challenged to solve mathematics problems that are open-ended in nature. These problems may have a number of solution paths, provide opportunities for different forms of representation and usually have more than one appropriate solution. Quality open-ended problems allow students of different abilities to solve the same problem with varying degrees of sophistication and efficiency. Although these problems provide much more scope for students to be mathematically empowered than 'traditional' routine problems, most problems remain teacher or textbook generated. As Silver (1994) argued, 'students are rarely, if ever, given the opportunity to pose in some public way their own mathematics problems' (p. 19). One way to provide young children with the opportunity to engage in more diverse and flexible thinking is to encourage them to pose their own problems.

Problem posing is an important companion to problem solving and lies at the heart of mathematical activity (Kilpatrick, 1987). Problem posing has been used to refer both to the generation of new problems and to the reformulation of given problems (Silver, 1994). In the first instance, 'the goal is not the solution of a given problem but the creation of a new problem from a situation or experience' (Silver et al, 1996, p. 294). Importantly, the problem poser does not need to be able to solve the problem in order for positive educational outcomes to occur because problem posing can occur *before* as well as *during* and *after* problem solving (Silver, 1995).

Benefits of Problem Posing

From a teaching perspective, problem-posing activities reveal much about the understandings, skills and attitudes the problem poser brings to a given situation and thus become a powerful assessment tool (English, 1997a; Lowrie, 1999). Not surprisingly, reports such as those produced by the National Council of Teachers of Mathematics (NCTM, 1989, 1991, 2000) have called for an increased emphasis on problem-posing activities in the mathematics classroom.

Problem posing and problem solving are closely related. As Silver (1995) suggested, problem posing could occur *prior* to problem solving when problems were being generated from a particular situation or *after* solving a problem when experiences from the problem-solving context are modified or applied to new situations. In addition, problem posing could occur *during* problem solving when the individual intentionally changes their goals while in the process of solving the problem. Such metacognitive processes underlie mathematical power and autonomy. Some educators (e.g. Kulm, 1994; Leung, 1996; Lowrie, 1999) have argued that teachers are able to gain insights into the type of explicit knowledge and skills students possess from the actual problems they pose. Thus, problem posing becomes a useful assessment tool.

Since an ability to pose problems is linked to metacognitive thought, it is not surprising that 'more able' students are more successful in generating problems (Ellerton, 1986; English, 1997b). Although some studies have investigated the extent to which young children generate problems (Silver et al, 1995; Lowrie, 1999; Lowrie & Whitland, 2000), most studies have focused on students in upper primary or secondary school settings. One of the main challenges in engaging young children in problem generation is dispelling the belief that problem solving must involve computational complexity. Since most problem-solving experiences presented to students in classroom contexts do not have connections to real-world experiences, it is understandable that students tend to pose problems that mirror school experiences. Unless children are encouraged to talk about problem solving (Lowrie, 1999) and share ideas during mathematical activities (English, 1997a), they will tend to pose traditional word problems that simply are variations of those found in textbooks.

It seems to be the case that young children are more likely to be able to pose 'appropriate' problems when they have a meaningful context in which to situate the problem. Providing opportunities for children to scaffold their ideas and understandings is one way of ensuring that the problem-posing contexts are meaningful. Writing problems for friends to solve (Ellerton, 1986; Stoyanova, 1998; Lowrie, 2000) is another way of contextualising problems.

The very fact that a student must consider the mathematical ability of another person when engaged in free problem-solving situations requires reflection and careful planning. In order to complete the task successfully, the problem poser might not only focus on the underlying structures of the problem but also the extent to which the problem solver will be able to interpret the components of the problem.

For young children to successfully generate problems that encourage the investigation of problems in realistic and personally satisfying ways, it is essential that they understand important elements of the problem-solving process (including the way mathematical ideas are related within the context). The children also need to engage in a variety of problem-solving activities in open-ended contexts so that their perception of what constitutes a problem is not restricted to traditional computational problems. Furthermore, they should be encouraged to talk about the type of activities that interest them. This will allow teachers to gain a better idea of their needs and preferences so that problems can be modified in accord with an individual's mathematical capacity and interests.

The present study adds to the research literature on problem posing in a number of ways. First, it investigates the problem-posing capabilities of children who are only six years of age –most studies in the past focused on children aged 10 or older. Second, the children involved in the investigation recently had been involved in a teaching–learning programme that encouraged problem posing. This programme provided a worthwhile starting point for problem-posing investigations.

The Study: initiating the problem-posing framework

If students are asked to generate problems without being offered a supportive environment in which to construct the tasks, they will generally not make the connection between real-life situations and mathematical ideas (Ellerton, 1986; Silver, 1994). This is particularly the case with young children since they do not possess an extensive repertoire of processes or skills to utilise in problem formulation.

In this investigation, a class of 25 Grade 1 students (aged 5-6) worked with undergraduate students [1] studying primary teaching for one hour per week for five weeks. In most instances, a strong rapport was developed over this period of time – with the teachers developing an understanding of the children's mathematical ability and preference for investigating particular types of problems in addition to a range of affective factors, including confidence and task persistence.

The teaching–learning sessions were constructed on the philosophy that the teacher would serve both as a role model for designing the problems and as an expert who would provide appropriate 'mathematics knowledge' support. Three initiatives were proposed for these sessions. The children would be:

- exposed to a variety of open-ended investigations;
- encouraged to talk about the type of activities that interested them; and
- challenged to consider relationships between mathematical ideas, offer alternate solutions to problems and identify components of a given problem that may be difficult to solve.

The teachers posed questions that encouraged the children to (a) describe the types of understandings and strategies they would need to consider in order to solve the problem, and (b) ascertain why they liked solving the particular problems they had posed. Although the specific questions were influenced by the nature of the problem posed, the questions followed a similar structure. The interview protocol included the following questions:

- What do you have to think about to solve this problem?
- What things will you need to help you finish the problem?
- What is going to be the hardest thing to do?
- Will we need to use number (mathematics)?
- When do you think you will need help from me?
- Where should we begin?
- Could we solve the problem another way?

It could be argued that these guided questions not only allowed the children to consider the knowledge and strategies required to solve the problem but also allowed them to consider the extent to which the problem evoked interest.

The teachers also attempted to gain insights into the children's overall perception of problem solving. In particular, the extent to which the pupil enjoyed solving the problem, their likes and dislikes with regard to the structure of this type of problem, and how they may be able to make their problem more interesting were considered. The format of this session allowed the children to consider relationships between problem posing and problem solving, as discussion moved between the problem they had posed and possible solutions to the problem.

Data Collection and Analysis

Two forms of data were collected: (1) data associated with the young students' generation of open-ended tasks; and (2) information concerned with the scaffolded learning environment. Data were collected as the students constructed problems and as they described the way in which these problems were developed. In addition, student–teacher interactions were observed, while the reflections of the teachers were also sought. All sessions were audiotaped, with the author taking notes and posing questions to both teachers and children on a regular basis.

The analyses of these data are presented in the following sections. The types of problems generated by the children in the sessions are presented and described in relation to the scaffolded environment. Information about the types of problems constructed by the children after participating in the five-week programme is then presented.

Results

Over 50% of the children (13 of the 25 students) began to generate open-ended problems while participating in the activities presented in the problem-posing environment. Examples of the types of problems posed by the children during these sessions follow. Descriptions of how the children thought they would solve the problems are also included.

> *What would a seesaw look like if you sat on one end I sat on the other end?*
>
> *I know you (the teacher) weigh more than me so I will be up and you will be down on the seesaw but I need to know how much higher I will be ... I will have to draw a picture to get the answer. (James, Grade 1)*
>
> *How long would it take for me to tie your shoes up?*
>
> *You should close you eyes and pretend that you are doing up your shoe lace and count slowly as you go ... You also have to remember that you have two shoes not one. (Emily, Grade 1)*

It is not surprising that the children were able to pose open-ended problems when you consider the fact that they were exposed to a range of motivating open-ended investigations during the five-week teaching investigation. More unusual was the fact that the other children were reluctant to generate problems that had more than one answer, even though they had been encouraged to consider their personal thinking processes in detail whilst solving a number of open-ended investigations during the one-on-one sessions. As two of the teachers explained:

> *Sally did not have the confidence to put her ideas together either on paper or verbally. Even when I offered to write it down for her she was reluctant to formulate any problem. After some time she simply replicated a problem she had completed in class. (Claire, Teacher)*
>
> *John had a clever way of constructing problems. He would think of an answer and then work backwards to write his problem. What he could not do was think of a number of solutions then design a problem to satisfy the criteria. (Jane, Teacher)*

These children were unable to propose problems that had more than one answer or could be solved using a variety of approaches. Thus, their problems were restricted to traditional problems commonly presented in classroom mathematics lessons. Although these children had only been at school for 18 months, it could be argued that their beliefs about what constituted a mathematics problem were already well defined. An alternate explanation would be that the task of proposing open-ended tasks was too difficult for them.

Nurturing Problem Posing

The problem-posing sessions provided opportunities for the teachers to develop the children's awareness of how problems are structured. The interview protocol outlined in the problem-posing framework section of the article formed the basis for the investigations presented to the children. Most of the teachers opted to present children with tasks along an open-ended continuum as a way of exposing them to different types of problems. As Sara (James's teacher) maintained:

> *It took a while for my student to realise that a problem could have more than one answer. For quite some time he would say that he had finished and would not consider alternatives. Once we began to solve problems that could be solved in several ways he started to get the idea. We would then talk about why each answer was still correct as a form of justification. This seemed to really help him understand how problems were structured. He would find different solutions, then prove to me why they were correct. (Sara, Teacher)*

These sessions not only provided opportunities for the explicit teaching of problem design but also exposed the children to challenging open-ended investigations. As the children's confidence grew, the 'content' of the problems became more aligned to their personal interests. Moreover, the problems became increasingly sophisticated. Although this was partly due to the fact that many of the children developed an instant rapport with their teacher and enjoyed the scaffolded support provided for them, it was also a response to the freedom they received to shape their own learning contexts. Jim (Emily's teacher) recognised that:

> *Emily began to create problems that were essentially experimental. How long to tie your shoes or to walk a certain distance? She was competitive and enjoyed creating challenges for us. Even though these problems seemed trivial, there was quite a lot of mathematics in solving the problems ... the trick for me was to know when to let her go and when to offer support. As she learnt about the process, her problems became more mathematical. (Jim, Teacher)*

Although the children who were able to generate open-ended problems were not necessarily the most capable students in the class, those students who had sound mathematics skills were more likely to be able to create open-ended tasks. Nevertheless, a number of other factors contributed to the students' problem-posing success, including their desire to take risks, perceptions of what constituted a mathematics problem, and willingness to work cooperatively with their teacher.

Open-ended Problem Posing

Following the completion of the five-week problem-posing programme, the children were challenged to pose a problem *that a friend could solve* (Ellerton, 1986, Lowrie, 1999, 2000). This activity required the children to generate a problem that would be both appropriate and enjoyable for a friend in their class to solve. Instead of modifying a problem that had already been constructed, the children were asked to develop their own problem. Moreover, they were encouraged to generate a problem that could have more than one correct answer. Although this would generally be a difficult task for young children, the concentrated focus on engaging students in activities that were open-ended in nature provided them with a framework that increased the likelihood that they would be able to pose appropriate tasks for others to solve.

Fifteen of the children (60%) were able to generate an open-ended problem. Although this proportion is not much higher than the figure reported during the five-week investigation, it is important to note that the students did not have the support of a teacher to scaffold their understandings in this context. Therefore, it could be argued that the five-week programme had a positive effect on the children's mathematical power – that is, their ability to make decisions about the type of mathematical investigations they enjoy solving and an awareness of how mathematics is related to their lived experiences.

Types of Open-ended Problems Posed

A majority of the problems posed were associated with number sense concepts (60%), with fewer associated with measurement (26%) or spatial sense (14%), despite the fact that each area had been given equal attention during the five-week programme. These problems are open-ended in nature because each problem has more than one possible answer and more than one solution path. The following are examples of the problem in each of the defined categories.

> *Number Sense - Sally had $5 to buy lollies at the shop. What did she buy and how much money would she have left? (Claire, Grade 1)*

Claire indicated that the person solving the problem would need to 'make a list of all the things she would have to buy ... add them up and make sure it did not cost too much ... then see how much money you would have left'. It was evident that Claire had a sound understanding of the processes required to solve the task. When asked what aspect of the problem was the most difficult, she suggested, 'you have to make sure you can add up all the lollies properly and get it close to five dollars'. Claire's reasoning may have been associated with the notion that the closer the problem solver was able to get to five dollars, the better the solution would be.

Measurement Sense - How many steps will you take if you walk real fast in 1 minute? (John, Grade 1)

In commenting on how a person might solve his problem, John indicated, 'you would need to have a watch [because] it would be too hard to count the time and steps at the same time'. John was unable to propose another way a person could solve the problem but appreciated that you could have several answers, 'depending on how fast you walked'.

Spatial Sense - Draw a nice pattern with two triangles, two rectangles and a square. Then you have to colour each thing a different colour. (Jayne, Grade 1)

Jayne suggested that one way to solve this problem was to 'make sure that you have the square in the middle, then the rectangles with the triangles on the outside'. Although this problem offers a range of possibilities – for example, whether or not the problem solver would use three colours or five colours – Jayne was unable to offer other suggestions, despite the fact that she generated an open-ended problem.

Problem posing is a powerful form of learning because the problem poser is generally able to make reasonable predictions about the kind of knowledge and strategies that would be required to solve the task, even though they may not yet be able to solve the problem themselves (Silver, 1994; Lowrie, 1998). It does, however, become increasingly difficult for young children to comment on the structure of problems when they are open-ended. This is due to the fact that these problems do not have straightforward solutions and consequently young children find it difficult to be able to focus their attention on solution paths.

Conclusions

All of the children who participated in this study were able to pose problems and consider the type of mathematics content that would be required to complete the task successfully. More than half of the children were able to pose problems that were open-ended in nature. Although the children were quite young, they were able to identify important components of the problem and suggest which mathematical operations would be required to solve the problem. This is consistent with the findings of recent studies with children in Grade 1 (Lowrie, 2000) and Grade 3 (Lowrie & Whitland, 2000). These studies found that children were able to construct problems and then identify the types of processes that would be required to complete the task successfully.

When first asked to construct a problem, the children were more likely to pose problems that could be classified as word problems. They were, in fact, posing problems that reflected the type of instructional practice presented in the classroom (van den Heuvel-Panhuizen et al, 1995). Once the children started engaging in the problem-posing activities that were more open-ended in nature, approximately half of them were able to generate such problems within the five-week period. Although it could be argued that these children were simply modifying or adapting problems that they had previously solved during the one-on-one sessions with their teacher, the very fact that they were able to alter the structure of the problem had merit and educational value. Moreover, the fact that the children were challenged to consider important elements of problem structure (including the component of the problem that would be most difficult to solve) would have had an influence on an individual's ability to generate an open-ended problem. Thus, the scaffolded framework exposed the children to a range of problems that were quite different from the type of problems commonly found in textbooks and predominantly discussed the classroom.

Since these open-ended problems did not have straightforward solutions, some of the children were unable to focus their attention on solution paths. As Silver (1994, p. 26) commented, 'when one poses a problem, one may not know whether or not the problem will have a simple solution, or a solution at all'. On the other hand, these more non-traditional problems allowed the children to explore, conjecture and examine in ways not possible in more routine situations. The children's evolving awareness of problem posing was facilitated by the teacher's pedagogical actions in the scaffolded environment.

The movement toward young children generating more sophisticated problems occurred in the structured problem-posing environment that challenged individuals to pose problems they enjoyed solving. Within a short period of time, many of the children began posing problems that were quite different from those they usually solved in the classroom. Although it is not possible to replicate the one-on-one learning environments reported in this study in typical classroom situations, the study shows how quickly young children begin posing problems that are more complex in nature, and challenging to solve, than the type of problems presented in textbooks and typical classroom discussions.

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Note

[1] The undergraduate students are referred to as teachers for the remainder of the article.

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