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Title: Investigating and visualising in space and geometry
Journal: Australian Primary Mathematics Classroom **ISSN:** 1326-0286
Year: 2003
Volume: 8
Issue: 2
Pages: 14-19

Abstract: For teachers to assist students to think mathematically in space and geometry, they themselves need to experience and understand how students visualise and how their imagery and concepts can be developed. Theoretical ideas gleaned from the literature and from researching classroom experiences is incorporated into a number of activities. These activities should be completed by teachers to understand the theoretical ideas and then to modify the activities for their own classrooms.

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Investigating and Visualising in Space and Geometry

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For teachers to assist students to think mathematically in space and geometry, they themselves need to experience and understand how students visualise and how their imagery and concepts can be developed. Theoretical ideas gleaned from the literature and from researching classroom experiences is incorporated into a number of activities. These activities should be completed by teachers to understand the theoretical ideas and then to modify the activities for their own classrooms.

Theoretical Ideas

Going Beyond The Names for Shapes

Teachers are often concerned that many students entering high school are only able to name a few two-dimensional shapes in geometry. They have little knowledge of the properties of shapes or how these shapes are related. The names of some common shapes may be familiar to them as isolated pieces of knowledge without any sense of relationship. Many have misconceptions that they parrot. For example, "a trapezium has only one pair of parallel sides." In fact, a better definition is that "a trapezium has a pair of parallel sides." It may have two pairs and in this case we denote it as a special trapezium with the name of parallelogram. Similarly many students do not realise that a square is a special kind of rectangle. It has all the properties of a square but it also has the extra property that adjacent sides are equal and hence that all sides are equal. Why has this lack of dynamic integrated knowledge happened and what can teachers do about it? One reason is the need for experiences that help students to build their visual imagery along with some related facts.

Seeing Parts

Students need to *explore* the shapes. They need to be able to "see" parts of shapes if they are going to be able to explore them and investigate properties. We call this *disembedding*. It is the kind of skill that is used when children are given puzzle books and they have to find the sheep hidden in the picture of trees and bushes. It has been discussed by psychologists for many years.

Making Shapes

Letting students make and draw shapes is a key to building imagery and linking this to concepts. They can cut them out, make them on geoboards, make them with sticks, use computer drawing packages, make them with other shapes like those found in tangram and pattern block sets. Making shapes encourages ownership and hence a willingness to talk about their shapes. Making these shapes might require students to try and use properties and they might result in different versions of the shape given a particular name. This can lead to rich discussion, classifying, and investigating. The shapes that they represent could be two or three dimensional. The property use can be as simple as having a straight side that joins to another straight side rather than curved sides or lines that do not join. It could be as sophisticated as getting the angles of a square exactly at right angles and sides equal and parallel.

Developing Tactics

Investigating shapes may require students to develop some tactics. For example, folding is an important tactic for checking equal sides or symmetry. Creasing hard makes edges when 2D nets become 3D shapes. Overlaying pieces to check the size of angles is another tactic that can lead to a great deal of sense about angle size. Figuring new arrangements of shapes using the tactic of flipping as well as sliding and turning is an important skill in making and exploring tessellating patterns, solving jigsaw type puzzles, and creating composite shapes. Systematically exploring alternatives like where to place squares in an arrangement with certain criteria is another tactic. This can be an efficient strategy in finding all 12 pentomino shapes in which 5 squares are joined together with sides joining exactly. Students may carry out their investigations and e unaware they are using these tactics but an encouragement to express their key tactics for solving the problems can develop this meta-thinking.

Developing from Pictorial Imagery to Dynamic and Pattern Imagery

When students first develop images of concepts they are fairly static. They can also suggest to students certain features that the student initially but incorrectly associates with a concept. For example, a student might have a fixed image of a rectangle with a horizontal base that is roughly two squares. Long thin rectangles, obliquely placed rectangles and squares would be discounted by this limited image. Imagery needs to become dynamic and more diverse. Shapes made from elastic, drawn on stretchy material, or in a computer drawing package can be stretched to give a good sense of the diversity of shapes that are all called by the same name. Stretching shapes also helps students to see how shapes are related by the number and type of parts. By forming a rectangle and pulling a side so it remains parallel and the angles at ninety degrees, a full range of rectangles including the square can be made. Forming a rhombus and drawing in the diagonals, a full range of rhombus can be made including the square and students will notice that the diagonals always remain at right-angles. This can also be done with geosticks and elastic for the diagonals. These explorations develop *dynamic imagery*. However, discussion about the changes and the properties that vary or stay the same will help to develop their conceptual understandings.

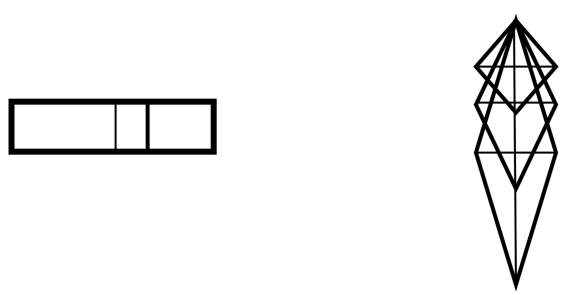


Figure 1. Static drawings of dynamic imagery of the stretched rectangle and rhombus

The transformations of flip, slide and turn are predicted by dynamic imagery supporting the analytical skills needed to solve the problem.

Viewing from another perspective and re-orienting shapes also requires dynamic imagery.

Another extension of imagery beyond the simple, static picture of a shapes results in developing a pattern of shapes like those found in tile patterns, or grids of squares that are commonly found in traditional arts. The squares can form a grid for solving problems like where does the last square go to create symmetry (see below) or make a new pentomino shapes.

Developing Strategies

Investigating is the way for students to first develop a primitive knowing (Pirie & Kieren, 1991) or *emerging* strategies. Students then begin to explore their physical environment and materials that they are given using *perceptual strategies* to solve problems, make images and have an image on which they can notice properties and develop their *imagery strategies* and concepts. First these images are static but then they can be developed – these become *dynamic and pattern*ed. As concepts and imagery develop, more *efficient strategies* will be used to solve spatial problems.

Reflections on investigations

When you do the following activities, ask yourself or your group the following questions:

What tactics did you use to investigate?

What imagery strategies were you developing?

How did language, listening and looking at others, and manipulating materials help your learning?

What new conceptual knowledge have you clarified or gained?

How many nets of a rectangular prism can you make?

Without looking at your boxes or blocks, try to imagine a toothpaste box and draw its net.

What things did you consider?

Now take your box and free-hand sketch the net. Imagine folding it up. Will it work? Why?

Now make an accurate drawing by toppling the box or cutting along a few edges and flattening it out ignoring any flaps. Draw around it, cut it out and fold into the box.

To make an edge, what do you have to do?

Now draw a number of other possible nets of the same rectangular prism. If you are not sure of any particular one, draw more accurately, cut out and try.

What systematic ways can you work out the different nets?

Would it make a difference if you used a special rectangular prism like the cube?

Unusual Tangram

Take the common seven-piece tangram and explore the different shapes to find out which sides and angles are the same.

By overlaying the angles of the pieces, decide how big each angle is (there are three different kinds).

Make some straight angles with the pieces.

Now take the unusual tangram. Take pieces marked 1 and 2 and make a simple four-sided figure.

Include 3 and make a simple four-sided figure.

Include 4 and 5, make a simple four-sided figure.

Include six and make a simple four-sided figure.

You might need to think about what you found out with the common seven-piece tangra m.

Try another way.

You might need to think about what you found out with the common seven-piece tangram.

Try another way.

Could you be more systematic in solving this problem? What could you consider? What do

Halving a Rectangle

Fold your rectangle into two equal parts.

Can you do it another way?

How many different ways can you fold it in two?

Share your answers with other investigators.

Can each investigator prove the answer.

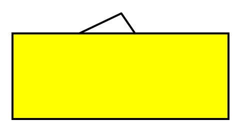
Can you work out if there is a pattern to explain how many answers there are?

What is the difference between halving and lines of symmetry for the rectangle?

What would happen with other shapes like an equilateral triangle or a circle or a square?

What Shape is It?

A shape is partially hidden under a cardboard cover.



Trace and say what you think the shape is.

Why did you select that shape?

Could it be another kind of shape? Why?

Could it be another shape of the same kind? Why?

What kind of imagery are you using to explain your answer?

Seeing Shapes Within Shape Designs

Try the various match stick activities on the cards.

Which shapes were hard to make?

Which ones were easy to see shapes within? Which were not?

Did you get better even though the cards were getting harder?

Make up your own match stick puzzle.

More Thoughts on the Activities

Each of these activities are designed so that the investigator, yourself or students, can get started. The investigator achieves an early sense of accomplishment and becomes engaged in moving on. However, the activities become more involving and challenging. Investigators take risks and *try and modify* ideas. Here the challenge is engaging many stored memory thoughts. They will think about properties to try to come up with another idea or more efficient idea. They will try manipulating images in their minds. They will use tactics to help with manipulations and experiments so they can see different possibilities. They will evaluate their trials and solutions. The evaluation might just be that what they see did not turn out as expected but the evaluation might involve thinking about the properties and whether it is a possible solution.

Often it is what is noticed, what comes to your attention that will help in the solution of a problem. This selective attention can be directed by seeing, verbal interactions, and stored memory, especially rich visual imagery. Changes in the position of the materials often leads to new insights.

Eventually these activities try to draw the investigator into a more abstract thought process. This is obvious in finding patterns for the painted cube. Other thoughts try to involve the investigator in relating the new experience with older experiences and developing conceptual knowledge.

If you investigated with other people or with other groups in the room, you might have noticed how interactions with each other influenced you. Another group may have given a little exclamation of satisfaction in finding a solution, they might have done something which you could see sparking off a new idea in your mind. They might have given a verbal cue. If you discussed your thinking at times, group interaction will extend your conceptual thinking and tactical ideas. Expressions of feelings about the activities and how they might have influenced your work might be discussed.

These outcomes might be expressed in terms of *responsiveness*. That is the investigator participates at both the emotional and thought level. By initially responding, changes are made in equipment, comments from others in your group and these influence you to have further thoughts that impinge on your responses. By being engaged, you are being responsive. This is one kind of productive pedagogy.

The following diagram was developed by me (cf. Owens & Clements, 1998) some years ago to explain what happens during problem solving within the context of a classroom. It is a helpful summary of the above discussion.

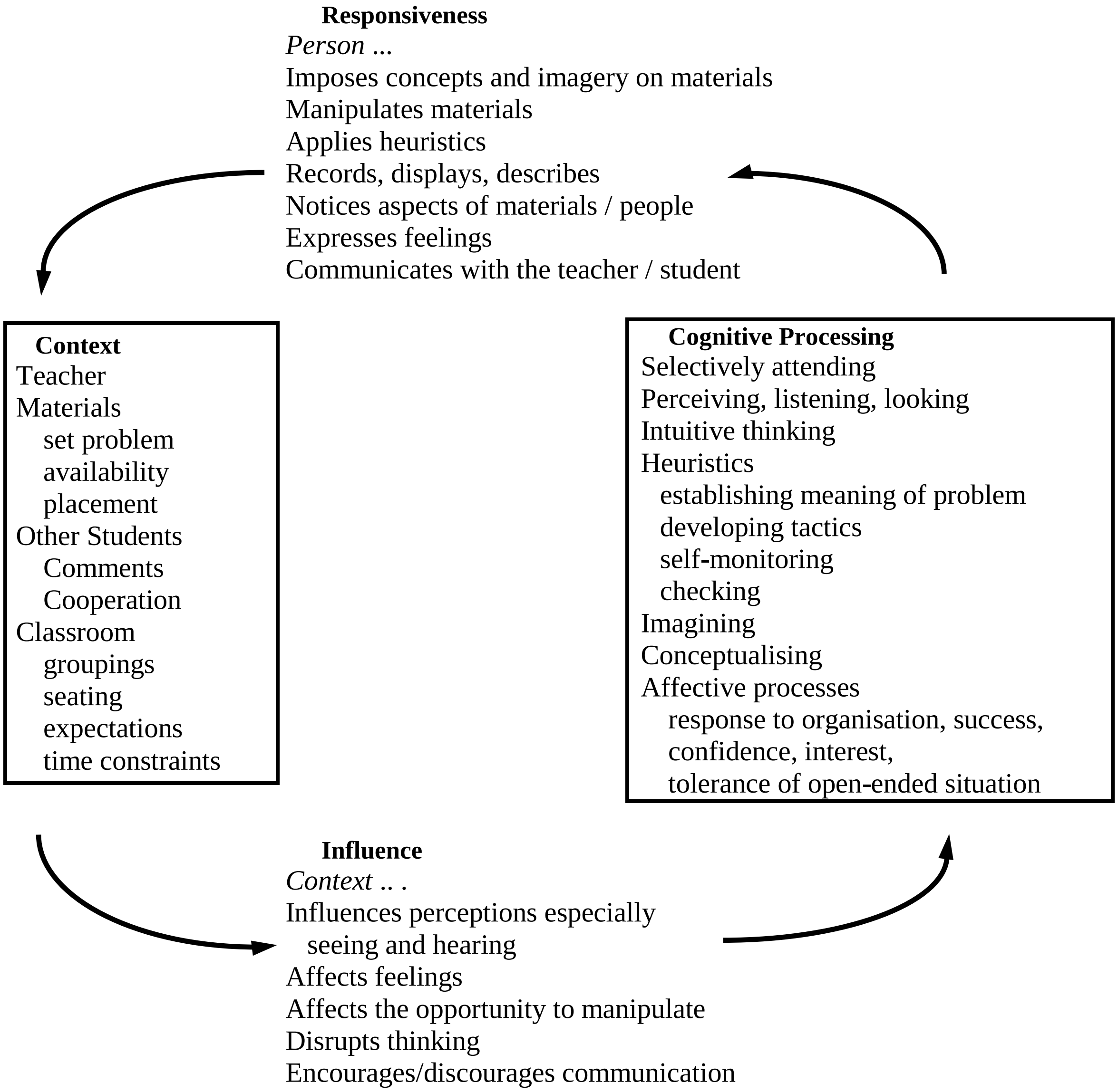


Figure 2. Aspects of problem solving

A final comment could be made about the way that you had to rearrange the pieces in the tangram activity in order to fit all six pieces into a four sided figure. Metaphorically, this is like constructivism. When we rearrange our ideas based on new experiences, we are constructing our knowledge. Concrete investigations often precipitate mental investigations and the visual imagination that is involved in these activities.

References

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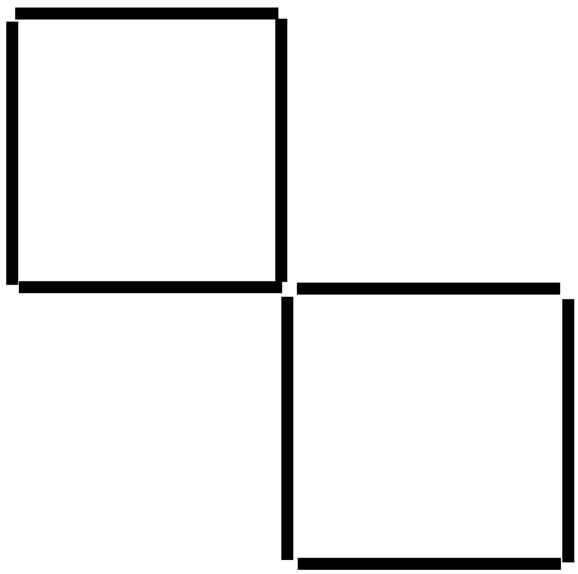
Pirie, S., & Kieren, T. (1991). Folding back: Dynamics in the growth of mathematical understanding. In F. Furinghetti (Ed.), *Proceedings of 15th conference of International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 169-176). Italy: Program Committee for PME15.

Card 1 - Designs with Sticks

What you will need:
12 sticks



What to Do:
Make the design.
Add sticks. For each puzzle, go back to the original design.

a. Add 2 sticks, to make 3 squares.

b. Add 4 sticks, to make 3 squares.

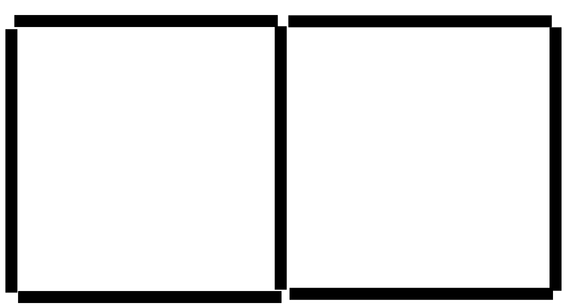
c. Add 4 sticks, to make 4 squares.

Card 2 - Designs with Sticks

What you will need:
12 sticks

What to Do:

Make the design.



a. Add 5 sticks to make 4 squares in a square.

   Leave the sticks for the next part.

b. Remove 2 sticks to leave 3 squares.

   Go back to the big square.

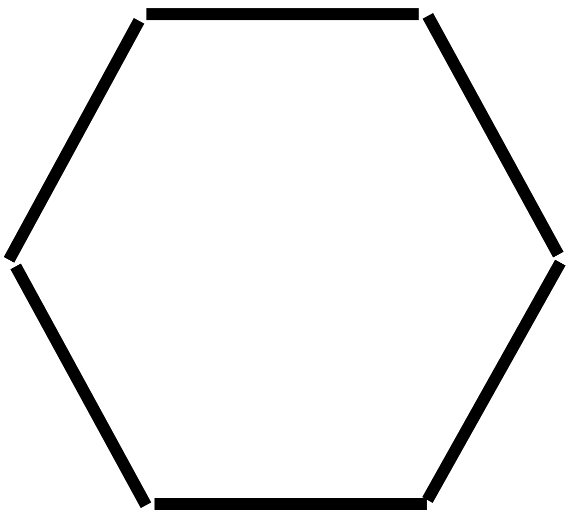
c. Remove 4 sticks to leave 2 squares.

   Go back to the big square.

d. Remove 2 sticks to leave 2 squares.

Card 3 - Designs with Sticks

What you will need:
12 sticks



What to do:

Make the design.

a. Add 6 sticks to make 6 triangles.

b. Remove 3 sticks to leave 3 rhombus.
Go back to the 6 triangles.

c. Remove 4 sticks to leave 2 trapezium.
Go back to the 6 triangles.

d. Remove 4 sticks to leave 2 rhombus.
Go back to the 6 triangles.

e. Remove 3 sticks to leave 3 triangles.

Card 4 - Designs with Sticks

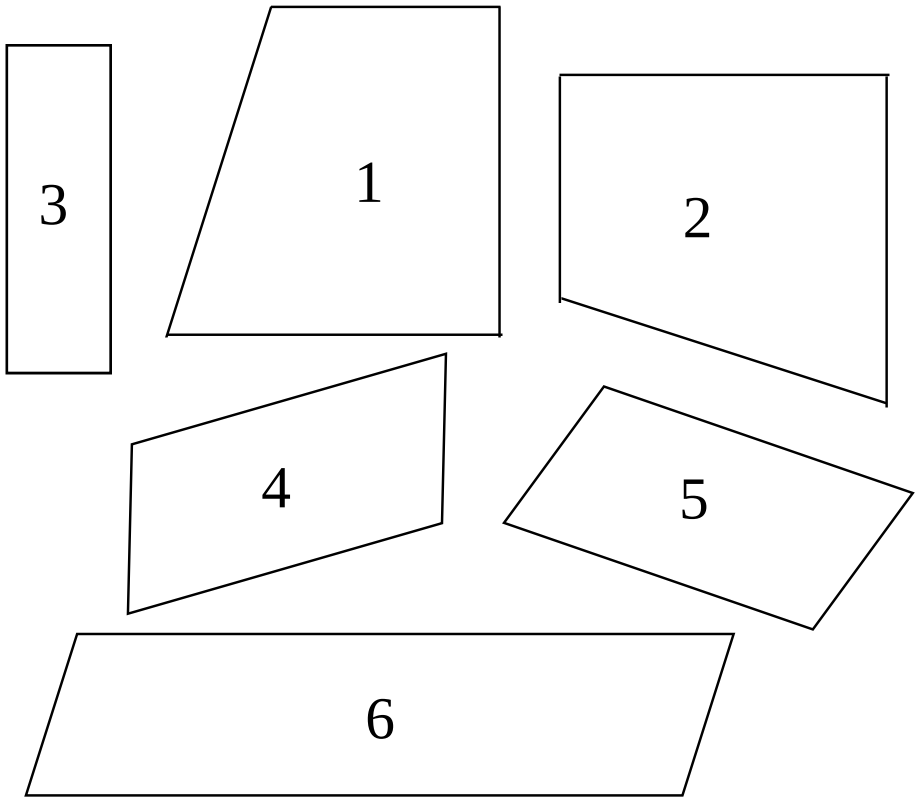
What you will need:
24 sticks

What to Do:

Use all the sticks each time. Make equal sized squares every time.

Take 12 sticks.	Take 24 sticks.
a. Make one square.	a. Make one square.
b. Make 3, then 4 squares.	b. Make 2, then 3 squares.
c. Can you make 2 squares.	c. Make 4, then 9 squares
	d. Try to make 5, 6, 7, and 8 squares. One of these you cannot do.

The Unusual Tangram



Symmetry

Take the tiles and make the following design.

Take an extra tile and work out where it can be placed to make a symmetrical shape.

Draw your new shapes and mark in the lines of symmetry or explain rotational symmetry.

Use the five tiles and make as many symmetrical designs as you can with the sides touching exactly.

Draw each of them and mark in the line(s) of symmetry?

How many asymmetrical shapes can you make? Draw them.

Pop-Up Cards

Fold the paper in half and cut a slit perpendicular to the fold. Fold the card to form a V and then fold against itself. Open and close the paper to check the card pops out.

Try using 2 cuts and making the paper fold out.

Use these to make some interesting pop up cards