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Abstract: The article focuses on the teaching method Mathematics Register Acquisition (MRA) model used by teachers in educating Māori student with mathematics in Māori language and its four strategies including noticing and intake in New Zealand. Noticing involved by making students aware of the new and old layers of mathematical writings and the teacher sees to it that the new writing is visible. The intake stage is where students are doing almost all of the cognitive work.

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Learning how to represent mathematics on paper

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If you walk into any primary school, the walls are usually covered in students' work, showing what they have done in a range of subjects. These exhibits not only have an authentic audience but also provide students with alternative ways to present their ideas. The exception in these classroom displays is mathematics. Yet displaying children's mathematical writing is just as valid as displaying their writing in other subjects. Maybe part of the reason is that mathematics is equated with numbers and who wants to just see numbers displayed, no matter how eloquently they explain a problem?

We have been working with a school in New Zealand that teaches mathematics, in the Māori language. In the last two years, the focus has been on improving the quantity and the quality of writing in mathematics. For us writing meant using words, diagrams, symbols and graphs, either individually or in combination. Collaboratively, we explored what was already being done and how we could build on the existing strengths. We believed that writing can be referred to many times and this supports students' reflection on their learning (Southwell, 1993). Writing explanations and justifications support students to think mathematically and this can begin in the early years.

In working with the teachers, we classified the strategies for working with the students around the Mathematics Register Acquisition model (Meaney, Fairhall & Trinick, 2007). The four strategies of the model were:

1. Noticing:
   Students are made aware of new aspects of the mathematics register, whether these are new layers of meaning for already known ideas or for completely new ideas. The teacher does almost all of the cognitive work by ensuring that the new writing is needed and seen frequently. If students do writing the focus is on the physical aspects of the writing, such as producing a round circle.
2. Intake:
   By this stage, some of the cognitive load has shifted to the students so that they actively engage in understandings when and how new language is used. Students use the new terms and expressions in restricted ways, with the teacher channelling them into giving appropriate responses.
3. Integration:
   At the integration stage, students have a good understanding of the new aspects of mathematical writing. Mostly the teacher's role is to remind students of what they can do. If the student struggles to operate at this stage, the teacher is quickly able to supply help by returning to the Intake stage.
4. Output:
   In the final stage of the MRA model, students show their fluency in mathematical writing without any support from the teacher. The teacher's role is simply to provide opportunities for students to make use of the fluency that they had acquired.

The following sections outline some of the strategies seen at the different stages.

Noticing

At the noticing stage, teachers used a wide range of strategies to focus students attention of the written representations of mathematical ideas. Quite often it was the teacher rather than the students who did the actual writing as they modelled what it looked like and how to produce it.

Modelling

Doerr and Chandler (in press) stated that "students needed to have models of good writing before they could be expected to write such responses independently". Our research showed several different types of modelling done by teachers. These were: the writing of words, symbols of diagrams as a part of a focussed discussion; the modelling by the teacher of the genre that students would use in an activity; and the modelling of writing that students would copy into their books.

Figure 2 shows the teacher drawing a square as she talked. She emphasised the features of a square through words, symbols or diagrams. The word for square in Māori is *tapawhā rite*, that literally means four equal sides. Following is an extract from a classroom video where the square is drawn on the board.

I kī koe i mua he tapawha rite. He aha te tikanga o tērā? He pai nga ingoa Maori nō te mea ka whakāmarama i te āhua i roto i te ingoa, nē? He tapawhā rite. He aha te tikanga o te rite (draws shape on board)? He ōrite te aha? He rite nga taha. Mehemea ka whakamahia au taku ruri… he rite ia taha? Nā reira he tapawhā…. He tapawhā… he tapawhā rite, na te mea he ōrite nga taha

You said it was a square. What does that mean? Maori names are good because the shape is explained in the name, isn't it? A square. What is the meaning of equal (the same)? [draws shape on board] What is the same, the sides are the same. If I use my ruler are the sides the same? Therefore it's a four side… four side….. It's a square, because the sides are the same.

Figure 2: Writing as part of the discussion in the lesson

Teachers also modelled how they expected students to record information. Figure 3 shows a teacher setting out the results from using a spinner and this highlighted the features of a table. When students produced their own tables, they would be operating at the Integration stage.

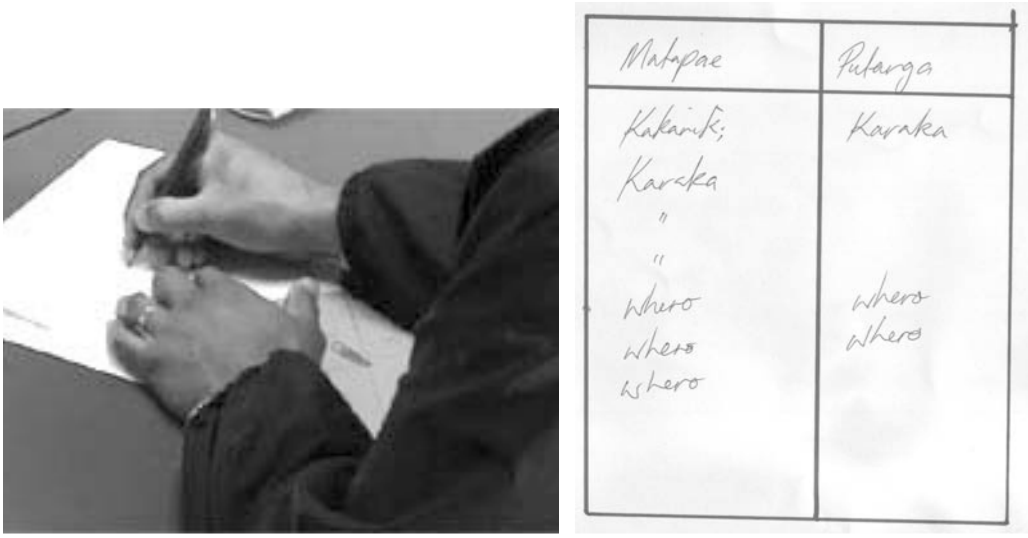


Figure 3: Teacher recording results

Another example of modelling was when the students copied the teacher's work from the board. They could be used as examples at later times. Figure 4 shows a ruler being drawn by a teacher and the copy in a student's book.

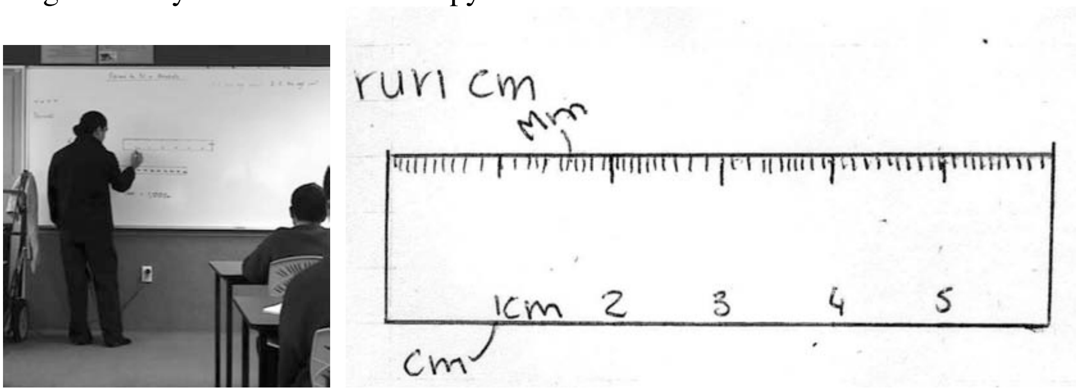


Figure 4: Teacher writing on the board which is then copied into students' books.

Providing examples of new writing

Teachers often began new topics by emphasising new material. In the following extract, the teacher has *āhuahanga* (geometry) written on cardboard. She then had the children in her Kindergarten class read it with her. She finished by drawing different shapes and having the children name them.

T3:	Ko tō mātou mahi i tēnei rā, kua timata he kaupapa hou – ko te āhuahanga. Koutou katoa…
Katoa:	Āhuahanga
T3:	Āhuahanga
Katoa:	Āhuahanga
T3:	Āe. Anei te kupu. Kōrero mai.
Katoa:	Āhuahanga
T3:	Āhuahanga. Titiro. Ko te kupu āhuahanga e pā ana ki ēnei mea (kei te tuhi i runga i te papatuhituhi)
Katoa:	Tapatoru - Tapaono
T3:	Ko ēnei ngā aha?
Tamariki:	Porohita – Taimana

T3:	Our work this day is something new- it is geometry. All of you
All students:	Geometry
T3:	Geometry
All students:	Geometry
T3:	Yes, here is the word, say it.
All students:	Geometry
T3:	Geometry. Look. The geometry word that relates to these things (draws on board)
All students:	Triangle- hexagon
T3:	What are these?
Students:	Circles- diamonds

Teachers also highlighted new words that would be used in the lesson by displaying them on the walls as can be seen in figure 5.

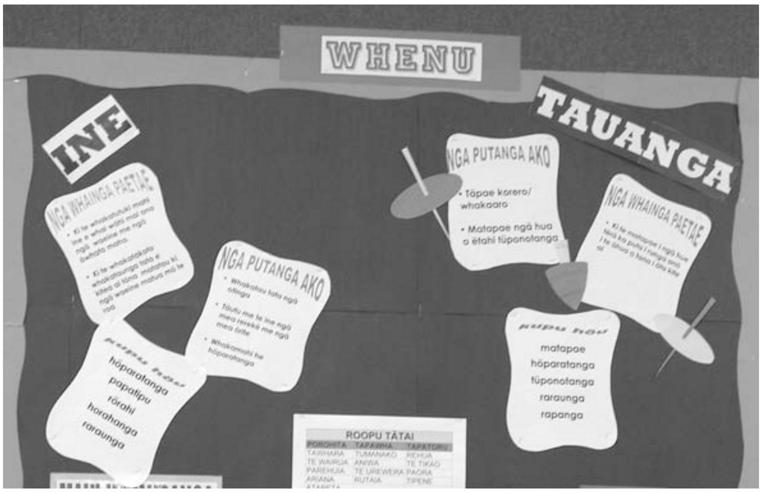


Figure 5: Wall showing *ine* (measurement) and *tauanga* (probability) words

Kinaesthetic activities

As an introduction to the diagrams or symbols needed for writing, some teachers involved the students in physical activities to highlight features. Figure 6 shows a teacher with her students "drawing" triangles with their bodies.


Figure 6: Making shapes with the body.

Previously, the students manipulated concrete examples of the different shapes. Using their bodies to make the shapes is one level of abstraction away from this manipulation. Drawing on paper would be the next stage.

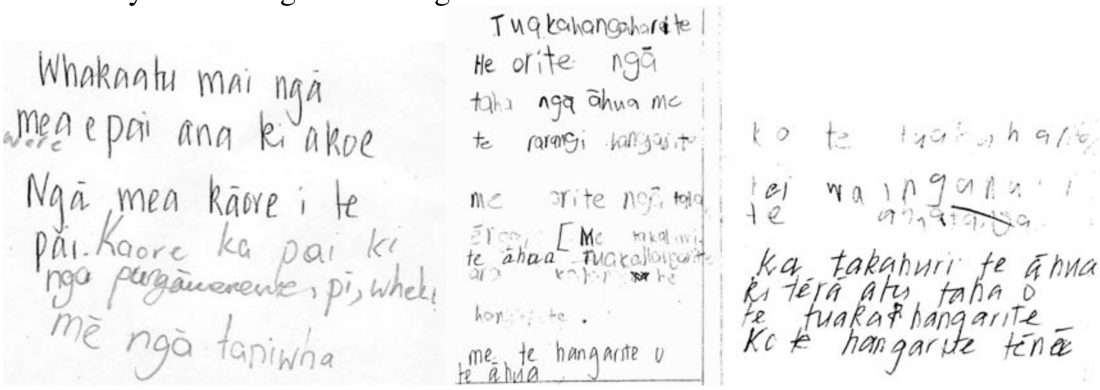
Intake

At this stage, students are the ones who are doing the writing but the activities often provided clues that supported students to produce appropriate mathematical representations. Consequently the teachers often had students complete worksheets that both modelled the writing but also provided opportunities for students to do some independent writing. The other common strategy was to work with students' own writing.

Using students' own words as a starting point for writing

Scaffolding students' writing by providing some support was very common. One teacher transcribed some students' contributions if they were slow writers. At the beginning of the next lesson, she asked them whether they still agreed with the ideas and if they wanted to add anything to them. She found that doing the writing for these students meant that their ideas were valued by others.

Another teacher scaffolded students' writing. In the first example in figure 7, she began the writing and then the student completed it with a sentence. In the second example, the teacher corrected the student's narrative and in the third example she had the student interpret what he had wanted to write and then rewrote it for him. The range of strategies suggests that the teacher was actively monitoring students' work whilst they were doing this writing.



Show the things that you like, the things you do not like. Spiders are not nice, bees, octopus and monsters.

Mirror line
The sides, the shape and the mirror line are the same. Let the sides be the same but turn the shape..
The shape is symmetrical

The mirror line is between the shape. If you turn the shape to the other side of the mirror line that is symmetrical

Figure 7: Working with student's own writing

Integration

The teacher's role at this stage is to provide opportunities for students to use the newly acquired ability to represent mathematical ideas on paper but in a way where she can put them back on track if they start making mistakes. Students have the major responsibility for using the new skills whilst the teacher's one is to keep a watching brief.

Correcting students' writing

A very common strategy at this stage is for teachers to collect in students' work and check it for accuracy. In figure 8, an earlier version of the sentence can be seen faintly underneath the final sentence. This is most obvious in the writing of *kahuri*.

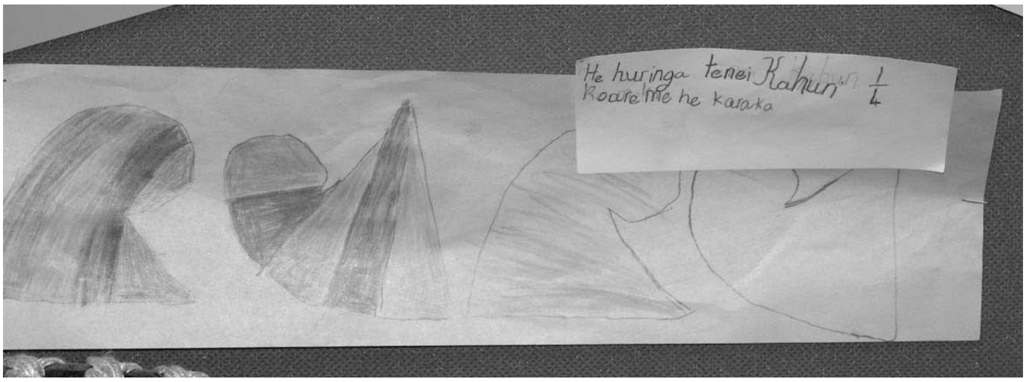


Figure 8: Public display about rotation

Writing using computers

Technology can be employed at any stage of the MRA model to support students' representations of mathematical ideas. This is because it can reduce the tediousness of physically representing some mathematical ideas, such as tessellating patterns and drawing graphs, engage more with these topics. Brown, Jones, Taylor and Hirst (2004) found that students were more able to engage with a problem about the diagonal properties of quadrilaterals using Geometers Sketch Pad than using a pencil and paper technique. For reluctant writers of mathematical paragraphs it may also provide an incentive to produce longer work of a higher quality. In our research, one teacher had students use MSWord drawing functions to produce tessellating patterns. Figure 9 shows the development of a translated pattern. The software allows the quick development of a complicated pattern that would have taken many hours to draw. The immediacy of working with a program such as MSWord provides instant feedback to the students. The ability to make judgements about the appropriateness of what they are doing remains with the student.

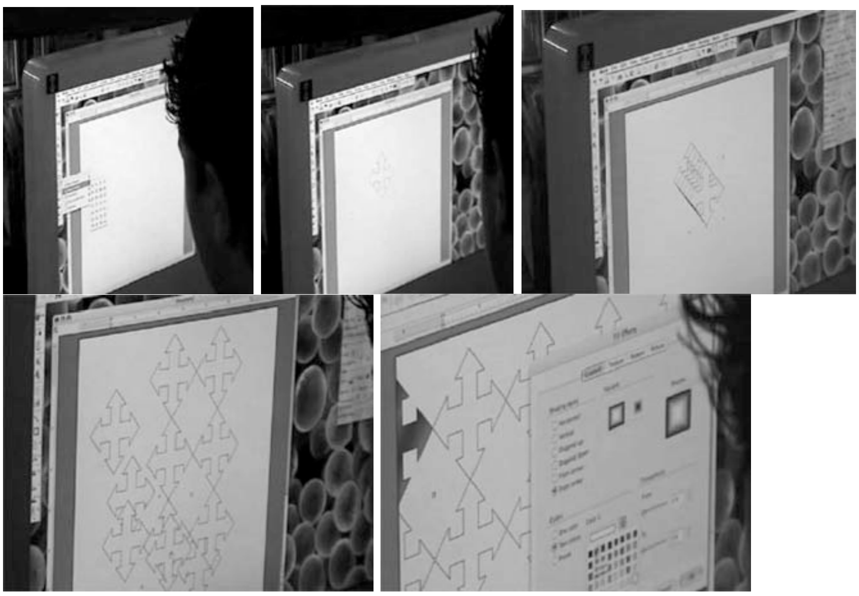


Figure 9: Stages in developing a tessellating pattern using translation

The first picture shows the student choosing a shape. The next activity is to draw the original shape, copy it and then paste several examples onto the screen. The student slides (translates) the copies around the page to form a pattern. This was then coloured in.

Writing in public places

One of the junior classes used large pieces of chalk to draw 2-D shapes on the concrete. This can be seen in figure 10. There was a strong link to oral language where the students' recording was one part of developing the students' understanding of shapes. The teacher gave a description of the shape, students had to draw it and jump into it when they had finished. Some students then took on the task of describing the shapes.

Figure 10: Drawing shapes on concrete

When students did mathematical writing on school playgrounds or on whiteboards, they displayed their fluency, but not in the same way as the static posters. Public writing was done quickly and was only available for immediate scrutiny as it would be removed at the end of the lesson if not sooner. The nature of this writing meant that it was very easy for teachers or other students to highlight difficulties in understanding the meaning that the writer was trying to display. Consequently, the students themselves would clarify the meaning that they were trying to give.

In asking students to display their knowledge, it is assumed that they have the skills to do so and that the classroom environment was supportive. In this environment, students can be reminded of what they already know and if they cannot resolve difficulties, then the teacher can intervene.

Output

Students have competence in being able to represent the mathematical ideas so the teacher's role is to provide opportunities for the students to show this competence, preferably in meaningful situations.

Figure 14 shows a student completing a tally to record their results from using a spinner as part of a probability task. This student had no difficulty with this part but their description of what they had done needed some improvement.

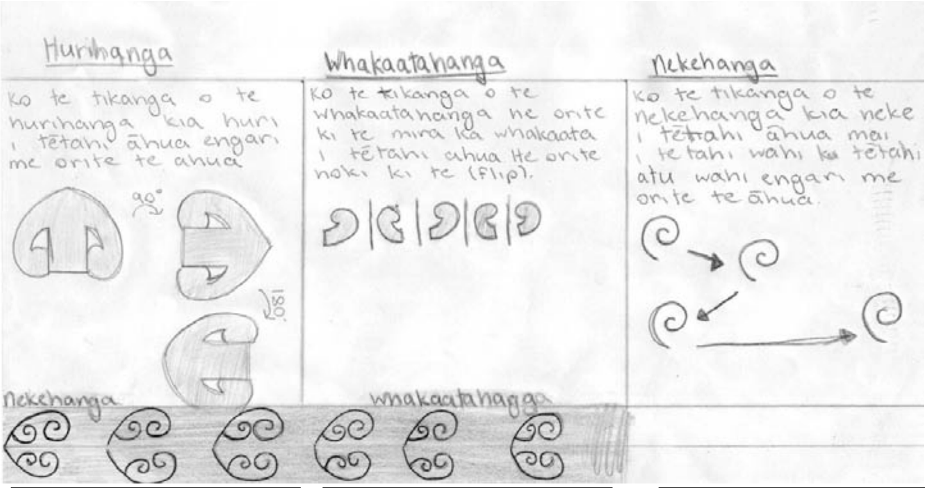
Ta	||||	Hinga
Te	HHH	Toa

1 Mena i tau te pine pepa i runga i te nama iti, i peaianei ki ahau nui ake engari ka tau ki runga i te nama orite te nama iti.

If the paper clip lands on the small number perhaps, to me it's bigger, but if it lands on the same (equal) number, smaller number

Figure 14: Recording the results of a spinner using a tally.

Assessment tasks also tested students' mathematical fluency. Two teachers asked students to write about a topic both at the beginning and at the end of a unit of work. The teacher and the student could see what had been learnt and what improvements had been made in their writing. A teacher in Doerr and Chandler's (in press) research had used a similar approach in that she had given students the same writing prompt at the beginning, middle and end of a unit. Figure 15 provides the two pieces of writing from one student. The prompts were not the same as in the first example the teacher had felt that the students responses had shown them staying with simple shapes that were easy to transform. For the second prompt she asked them specifically to choose traditional Māori shapes and describe how these had been formed. This was a more difficult task but still provided students with a large amount of freedom to choose shapes that they felt comfortable describing. It is possible to see a significant change in the type of transformation that is being discussed. Although the text appears not to be more challenging what has been attempt is more difficult because it is not just a definition of the transformations but an attempt to explain how different patterns were formed using different transformations.



Rotation The meaning of rotation is to rotate a shape but it remains the same shape	Reflection The meaning of reflection is when the same shape is in the mirror. It is the same when you flip it	Translation The meaning of translation is to move a shape from one place to another but the shape is the same

What I see in this kowhaiwhai [pattern] is a reflection between the columns. If you look closely you will see that both sides are the same. The difference is that this kowhaiwhai is reflected

An example

What I can see in this kowhaiwhai is a reflection on both sides of the diagram
An example
Hammerhead shark (the kowhaiwhai pattern)

Figure 15: Transformation assignments

Conclusion

For students to have fluency in mathematical writing so that it can support their thinking, teachers provided opportunities from each of the MRA's stages. In the initial stages, the teacher needs to entice students into using new terms and expressions. However, as they gain proficiency, the teacher's role becomes one of providing opportunities for students to take control not just of their own writing but also of the evaluation of its appropriateness. In this process, opportunities to display their writing both in permanent and temporary form is essential. Mathematics need to be seen as well as heard.

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References

Brown, M., Jones, K., Taylor, R., & Hirst, A., (2004). Developing geometrical reasoning. In I. Putt, R. Faragher & M. McLean (Eds.), *Mathematics education for the third millennium: Towards 2010*. (Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia, Townsville, Vol. 1, pp. 127-134). Sydney: MERGA.

Doerr, H. M., & Chandler-Olcott, K. (2009, forthcoming). Negotiating the literacy demands of standards based curriculum materials: A site for teachers' learning. In J. Remillard, B. Herbel-Eisenmann, & G. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (Studies in Mathematical Thinking and Learning Series, A. Schoenfeld, Ed.) (pp. 282-301). New York: Routledge.

Meaney, T., Fairhall, U., & Trinick, T. (2007). *Te Reo Tātaitai: Developing Rich Mathematical Language in Māori Immersion Classrooms*. Wellington: NZCER.

Roth, W-M. (2001). Gestures: Their role in teaching and learning. *Review of Educational Research*, *73*(3), 365-392.

Southwell, B. (1993). Development and Assessment of student writing. In M. Stephens, A. Waywood, D. Clarke, & J. Izard (Eds.), *Communicating mathematics: Perspectives from classroom practice and current research* (pp. 223-236). Melbourne: The Australian Council for Educational Research Ltd.