This investigation identified the interactions and discourse employed by students (11-12 years old) when challenged to solve a realistic mathematics problem in a collaborative group situation. The students were asked to use genuine artefacts (including brochures, menus, bus timetables and photographs) to complete an open-ended task in small groups. Although most students were able to establish their own sense of authenticity by aligning the problem to their personal experiences and understandings, it was also the case that the majority found it difficult to establish meaningful, realistic understandings in the group situation. The students were unable to regulate the collective ideas of the group because too much emphasis was placed on personalising the task.


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“If this was real”: Tensions between using genuine artefacts and collaborative learning in mathematics tasks

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Abstract: This investigation identified the interactions and discourse employed by students (11-12 years old) when challenged to solve a realistic mathematics problem in a collaborative group situation. The students were asked to use genuine artefacts (including brochures, menus, bus timetables and photographs) to complete an open-ended task in small groups. Although most students were able to establish their own sense of authenticity by aligning the problem to their personal experiences and understandings, it was also the case that the majority of the students found it difficult to establish meaningful, realistic understandings in the group situation. The students were unable to regulate the collective ideas of the group because too much emphasis was placed on personalising the task.

Keywords: problem solving; collaborative group work; genuine artefacts
“If this was real”: Tensions between using genuine artefacts and collaborative learning in mathematics tasks

Introduction
One way to afford students the opportunity to engage in more diverse and flexible thinking is to encourage them to participate in problem-solving situations that are collaborative in nature. Such situations provide opportunities for students to talk about their ideas, to consider the appropriateness of their solutions and the solutions of their peers, to discuss different representations of the same problem, and to consider the extent to which solutions can be applied to different contexts (Lowrie 2004; Schorr and Amit 2005). Small-group learning situations are recognised as an approach that promotes learning and socialisation (Cohen 1994). In most instances, collaborative learning involves students working together to accomplish shared goals, and through these shared goals students tend to develop a sense of ‘group’ as they recognise the need to help and support each other’s learning (Slavin 1996).

Another approach is to create learning contexts that require students to make connections between their social and personal lives and the learning they undertake in school. Wenger (1998), for example, argued that learning is enhanced when students develop a “thirst for learning of the kind that engages one’s identity on a meaningful trajectory and affords some ownership of meaning” (270). In order to establish such ownership, teachers often attempt to design problem-solving contexts that allow students to access and utilise personal experiences to solve tasks. However, there is certainly a conflict between a teacher’s capacity to construct tasks which are realistic and personalised and the extent to which a problem solver views any task as having a real-life dimension (Cooper and Dunne, 2000).

Some studies have considered the nature of collaborative learning in realistic problem-solving situations. Elbers and Streefland (2000), for example, found that
critical decision making was often influenced by the knowledge students’ possessed, and in more realistic situations, this knowledge was quite different than the type of knowledge utilised in more traditional whole-class settings. Baird and Pedigo (2003) maintained that realistic tasks were a catalyst for engaging and stimulating conversations within collaborative-group environments. Furthermore, the tasks utilised in these problem-solving situations need to be appropriate for this kind of learning dynamic. For example, if a task is open-ended, group members tend to show high levels of cooperation as they discuss how they will proceed as a group, and as they share ideas and information. Cohen (1994) maintained that such tasks ensured that no individual can complete the task alone because it involves sharing resources (i.e., intellectual and material). However, the literature base is certainly not extensive and as Kramarski, Mevarech and Arami (2002) highlighted, more research needs to be conducted on the quality and nature of group interactions when students are engaged in problem solving—particularly when students are required to engage in complex, realistic tasks.

**Using genuine artefacts to enhance problem solving**

A body of research has demonstrated that problem solving can be enhanced when students are afforded the opportunity to personalise the context (Clancy and Lowrie 2002; De Corte, Verschaffel and Greer 2000; Lowrie 2005). De Corte, Verschaffel and Greer (2000) maintained that in order for students to make meaningful connections to personal experience they need to be immersed in innovative learning environments that are radically different to traditional classroom practices. They proposed that tasks should be diverse, complex and have several solution pathways for such connections to be made. Tasks that allow students to personalise the problem scene or utilise genuine artefacts provide opportunities for students to develop skills
in knowing when and how to use mathematical knowledge for representing and solving problems in both practical and realistic situations (Lowrie 2004). Lesh and Harel (2003), for example, maintained that the kind of problem-solving situations that should be emphasised in school are simulations of real-life experiences where mathematical thinking is useful in the everyday lives of the student or their family and friends. As Boaler (1993) noted:

The reasons offered for learning in context seem to fall into two broad categories, one concerning motivation and interest of students through an enriched and vivid curriculum, the other concerning the enhanced transfer of learning through a demonstration of links between school mathematics and real world problems. (14)

The challenge for teachers is to establish a learning environment that encourages students to personalise learning in ways that allow individuals to extend, adapt, revise and adopt mathematical ideas to a context that they can place themselves within. Bonotto (2002) proposed that classroom-based activities that aim to create connections between reality and mathematics should be founded on the use of cultural (genuine) artefacts.

In the present study, these ‘genuine artefacts’ included brochures, menus, bus timetables and photographs. Such artefacts were relevant to the children because they allowed them to make connections to real-world experiences, offered significant reference to concrete situations, allowed them to keep their reasoning processes meaningful (Kaput 1994; van Oers 1996), and enhanced their capacity to think metacognitively (Lowrie and Clancy 2003). Bennett, Harper and Hedberg (2002) commented that the quality and nature of authenticity of an artefact will depend on (a) the level of sensory fidelity in task representation so that practical skills may be developed; (b) the extent to which critical thinking or problem solving can be enhanced; and (c) the potential for social interaction and engagement. In the present study, these three elements were considered to ensure that the richness and
genuineness of the artefact, and its realistic nature, could be mapped against any changes in the classroom culture. Nevertheless, Cooper and Dunne (2000) warned that different types of realistic problems are viewed differently by various social groups and that some students have difficulty “negotiating the boundary between esoteric mathematical knowledge and their everyday knowledge.” (43)

This study sought to gain an awareness of the different interactions that emerge as students collaboratively solve problems that we stimulate by a genuine artefact. The author provided opportunities for students to use personal experiences (or the experiences of peers) to manipulate a task in ways that encouraged students to both personalise and take ownership of the task through the use of a genuine artefact. In light of the work conducted by Cooper and Dunne (2000), there was no assumption that the tasks were authentic but rather more likely to be realistic (and thus engaging) through the use of a tool (i.e., the artefact). The proposition is that the genuine artefact helps establish a more realistic context that could be achieved by problem-solving activities typically presented to children in classrooms.

**Research goals**

Within this contextual framework of using artefacts to generate problem-solving experiences in collaborative situations, this investigation sought to:

1. Identify the influence genuine artefacts had on sense making in group problem solving.
2. Map the group dynamics and interactions that occurred as students engaged with peers in their group and made sense of the problem.
3. Classify interactions among students within groups and determine the extent to which these interactions were consistent (or distinct) across groups.
Methodology
A multimethod sequential research design was utilised for this project, with an initial qualitative study followed by a second, and related, qualitative study (Morse 2003). Both components of the project considered the use of genuine artefacts to promote problem solving. The study which is the focus of this paper investigates a collaborative learning situation, and is an extension of the original study which investigated an individual learning situation (see Lowrie 2004).

Participants
The present investigation involved 28 Grade 6 children (aged 11 or 12) from a small primary school in a large regional city in New South Wales (Australia). As was the case with most single-grade classes in this state, the participants varied in their knowledge of mathematics and literacy concepts and understandings. Participants reflected the cultural and ethnic composition of the local community. Only one student spoke English as a second language and there was one Indigenous Australian student in the class.

Participants self-selected the composition of their groups based on peer interactions and the extent to which they thought they could work with the other members of the group. In a few situations the teacher’s knowledge of the class dynamics was utilised to ensure that the participants would be able to work collaboratively. This process was brief and resulted in seven groups with two of these groups having a gender balance.

The collaborative problem-solving task
The task was given to students and also read aloud, with the children given opportunities to ask questions if they did not understand any of the intended outcomes.

INSERT FIGURE 1 ABOUT HERE
The task was contextualised in the following manner:

Your challenge is to plan a trip to a theme park with other members of your group. The information and maps provided will help you to plan activities and the structure of your day. The pamphlets show the map of the entire theme park and the location of all the rides and attractions. Other information includes accommodation options, transport timetables, and menus for food outlets. You are required to plan the day’s events with appropriate details and budgetary considerations. In planning the day you should use the map as your main reference point. You will need to justify your solution.

Support was provided during problem solving if requested to ensure that the children fully understood the context of the problem.

Design principles
A number of design principles were considered to evaluate the extent to which the task was relevant and connected to the students’ mathematical knowledge and capacity to solve the task in a collaborative manner. A rubric was established to consider these elements in relation to the students’ sense making within the problem solving task. The rubric became a mapping tool which facilitated recording the observable behaviours of the participants throughout the activity. The rubric was used to determine whether or not the task was viewed as being authentic and conducive to collaborative problem solving. Fundamentally, the design principles were assessed under categories associated with genuineness, mathematics sense making and collaborative learning behaviours (see Table 1).

All groups had at least one member who had been to either this theme park or a theme park that was similar in nature and scope. Moreover, this explicit knowledge of the context allowed students to have a realistic connection to not only the purpose of the problem but the challenges that would be faced. Thus, even though the participants were too young to be placed in a situation that would demand such processing and strategy use, they were aware of the possibilities and options that could be used.
Data analysis

Creating a framework for the interpretation and analysis of data
Peter-Koop (2002) utilised four basic interaction sequences that can be used to monitor participants’ 'lines of thought' as they engage in group activities. These classifications of group interaction were adapted from the work of Jones and Gerard (1967) and include pseudo, asymmetrical, reactive and mutual contingencies. The first class of interaction (cited in Peter-Koop 2002)—'pseudocontingency'—describes a limited case of social interaction. In general, group members follow their own pre-established plans in these situations. The second category—'asymmetrical contingency'—classifies situations where Person A tends to access strategies and ideas whereas Person B’s responses are determined predominately by social stimuli produced by person A. In Peter-Koop’s (2002) study, this category tended was frequently dominated by high achievers. With respect to ‘reactive contingency’, neither A nor B tend to follow self-produced stimuli, rather individuals tend to react in ways that are dependent on proceeding social stimuli. The fourth category, ‘mutual contingency’, describes situations where sense making and conversations are mutually driven. In such instances, according to Jones and Gerard (1967), ideas are continually recast in light of the responses and actions of other participants in the group. In Peter-Koop’s (2002) investigation, such episodes are infrequent and tend to occur when high achievers are working together.

Analysis of group dynamics and targeted group interviews

Group interactions
Data were collected as the students engaged in the task. Classroom interactions were audio-taped and detailed observations were taken by two researchers who divided their time among the groups. Anecdotal notes were taken with a range of behaviours and interactions observed—in particular the ‘lines of thought’ used by individuals to
construct meaning within the respective groups. Importantly, the researchers were conscious of not interrupting the students’ thought patterns or approaches to solving the task.

**Targeted group interactions**
After completing each activity, target groups were chosen for further interview on the basis of their metacognitive sophistication as ascertained from the group interactions and observations from the previous study (see Lowrie 2004). The participants were interviewed (in their small focus groups) for approximately 40 minutes to encourage them to reflect upon the way they represented the problem and the strategies they used to complete the task. During these interviews, questions were posed along three themes, namely the (a) influence of the genuineness of the artefact; (b) extent to which each group contextualised the task in a realistic manner; and (c) shared strategies and processes used by the group to make sense of the problem.

**Interpretation of data**
Portions of the audio tapes were transcribed for analysis based on the conceptual elements of the framework. In the first instance, these data were presented in a pictorial form and generically matched to determine whether the ‘lines of thought’ were associated with either ‘contingency’ or ‘interweaving’ processes. A finer-grained analysis of conversational turns was then conducted to determine framework categorisation within these general processes. For example, could the conversational interactions be aligned to either a ‘reactive contingency’ or an ‘asymmetrical contingency’ after an initial contingency categorisation?
Results and discussion

The classroom dynamic
Realistic scenarios or simulations of real-life experiences were often formulated in the early stages of problem representation. Most of the participants intuitively placed themselves within the setting to contextualise the problem—and in the early stages of the activity these personalised intentions caused students to become distracted. They began to discuss approaches that were not necessary for the task to be solved. Once each group’s focus reverted to the task at hand, it was evident that the mathematical knowledge required to complete the task was fundamentally shaped by these initial personalised decisions.

Approximately 20% of the children had actually visited the theme park referenced in their problem—with almost 85% of the participants previously attending some form of amusement park. Consequently, their mathematics sense making was formulated from a desire to create a realistic scenario. Furthermore, their problem-solving approaches were strongly influenced by previous ‘theme park experiences’ with out-of-school knowledge influential from both sense making and process perspectives. The genuine artefacts (brochures and menus) required to complete the problem within this context also encouraged the children to make connections to real-life experiences.

Erica: The price (admission costs) is valid until 30th November. That means the prices will change next week (the interviews were conducted on the 21st November)... over Christmas the prices could change.
Sue: Over Christmas they always go up.
Sally: Because more people will be going there and they will want more money, and then they can build more rides.
Sue: They have built more rides since I was there.
April: Yeh and like through Christmas time, it is sort of like holidays...so they’d want to be going on holidays.
Erica: I reckon they’ll probably go down when school starts again...maybe by 20%.

Importantly, the participants, both individually and collectively, sourced a great deal of visual, spatial and graphical information from the artefacts which enhanced the
scope of the activity. Moreover, the students accessed important personal knowledge, such as prices going up over Christmas, despite the fact that there was no evidence of this on the artefacts supplied. The problematising that occurred not only showed that the students have multiple perspectives and offer multiple pathways or solutions, it also provided the children with the opportunity to create connections between reality and mathematics.

**The influence of the genuine artefacts in sense making**
The genuine artefacts (brochures and menus) required to complete the problem within this context, encouraged the participants to make connections to real-life experiences. The students certainly appreciated having the actual brochure rather than some form of replicated artefact. One of the participants, Matthew, provided some insights into the use of the artefacts which were reinforced by many of the participants. He commented that it would be difficult to navigate his way through the problem and ‘visualise’ the environment without such detail.

Int: What are some of the strategies you need to know when you are going to a theme park?
Matt: [I] look at the signs and follow the paths and things like that…You can watch where people are going and you can work out that that must be a good ride, and there are a few other strategies like, if you go up a path, you have to try and stay on a budget like when you go to a ride…when it’s a fake map, you can’t really feel it being there and it’s hard to visualise how that is actually happening [and] it makes the activity twice as hard to do.

Int: What things would be disappointing for you if they weren’t there?
Matt: Well I think it would be less fun to do because all the pictures, and you can see the people enjoying themselves there. And you can actually see the rides going. Like that’s like a type of action sort of a thing there where it is going around…if it wasn’t there…it’s a bit hard to understand.

The genuine artefacts not only allowed the students to make sense of a range of spatial concepts, but also established a strong motivational intention for the open-ended task. Moreover, group accountability (Slavin 1999) was heightened as students within the group developed a sense of task credibility.
This section summarises the way in which each group worked together to represent and interpret the task. In particular, the data were examined in relation to the tensions that existed between the participants establishing a ‘personally’ authentic solution whilst working in a ‘cooperative’ manner. It is important to note that when these students completed similar activities (on an individual basis) most strategies and ideas were formulated from a desire to create a realistic setting. Furthermore, their problem-solving approaches were strongly influenced by previous ‘theme park experiences’, and as such, their out-of-school knowledge was influential—from both sense making and process perspectives. It was also the case that most of the students were able to generate quite sophisticated solutions to these problems.

By contrast, the quality and depth of the solutions in the group situations ranged considerably in both the sophistication of the generated solution and the processes undertaken to develop a realistic group response (see Table 2).

**INSERT TABLE 2 ABOUT HERE**

Only two of the seven groups were able to generate a solution that could be justified by more than one member of the group. Although two other groups (Groups 3 and 6) displayed strong sense making during the process, they were unable to organise their individual ideas and strategies in a manner that satisfied the conditions of the open-ended task. As a consequence, the meaningfulness of the solution tended to be restricted to individuals applying personal knowledge and experiences to the problem context. By contrast, few individuals were willing or able to modify or reconstruct the problem in order to satisfy the group element of the task.

**Group interactions in collaborative situations**

There was considerable variation in the commitment the participants displayed in generating solutions that were collaborative in nature. As is the case with many
collaborative interactions, the high-achieving students tended to dominate much of the
discourse (Esmonde 2009; Peter-Koop 2002). In the following example (see Figure 2), where each comment is strongly influenced by the proceeding social stimuli (Jones and Gerard 1967), one of the group members, Tony, becomes isolated from the
discussion. In subsequent dialogue he remains attentive, however essentially silent, as
the other two members of the group make decisions about menu selection and the
approach to take in completing the task. Some of Tony’s personal preferences and
ideas were challenged and he did not have the confidence to justify his convictions.
Group interactions and the interrogations of his ideas overwhelmed his careful
thinking—to the extent where his ideas lost impact.

INSERT FIGURE 2 ABOUT HERE

In this example, the reactive contingency of the discourse dampened Tony’s
enthusiasm and he became increasingly disempowered by the activity. The small-
group environment did not provide him with the opportunity to contribute his ideas or
‘line of thought’. This was disappointing for Tony since he felt that he had “become
really good with budgets” and was “helping his mum with their budget at home”.
Although the group dynamics remained positive, Tony (and to a lesser extent Chris)
was reluctant to challenge Jacob’s ideas and approach to the task. Consequently, Tony
did not embrace the task with the same enthusiasm or commitment he displayed in
individual activities. Essentially, he wanted the opportunity to personalise the context
and to construct meaning at a less frantic pace. In this group context, this did not
eventuate.

The influence of a high-achieving student is clearly represented in situations
that produce asymmetrical contingencies. In such circumstances, most engagement is
influenced and even determined by the ideas of one person (in this case, Matt),
whereas other responses are generally influenced by the ideas and strategies that have been formulated by this person. Although Daniel and Luke provided strong support within the group, much of the discussion was orchestrated by Matt (see Figure 3).

**INSERT FIGURE 3 ABOUT HERE**

This open-ended task had captured Matt’s imagination, and he was skilled at partitioning the activity into smaller manageable tasks—with the intention of ensuring that everyone in the group had meaningful involvement. Impressively, he considered the views and ideas of the other group members in a respectful and empowering manner. Nevertheless, he remained a powerful presence in the group context and orchestrated most meaningful approaches to the problem.

By contrast, the members of another group (Group 2) were able to build upon one another’s ideas and respond with questions or comments that either recast the previous comment or provided more depth to the group’s conversations. This ‘interweaving of ideas’ (see Figure 4) helped the group to navigate their way though the open-ended task and produced solutions that were collective responses. This group was able to make assumptions about aspects of the problem (for example, the time it would take to go from one ride to another or waiting times on specific rides) that were informed by personal experiences at the respective theme parks. This allowed the group to establish a sense of balance between making reasonable and realistic estimates of schedules and costs while keeping the investigation manageable.

**INSERT FIGURE 4 ABOUT HERE**

Interestingly, these participants relied upon the expertise and ideas of one another (Slavin 1996), and could not realistically work alone (Johnson and Johnson 1999) on the task. As Dekker, Elshout-Mohr and Wood (2006) suggested, these students continually restored the group ‘balance’—allowing individuals to show and explain
one another’s work and ideas in an environment where learning outcomes and time spent on the activity were relevant to their group’s learning outcomes.

**Personalising problems in group situations**
Generally, the participants found it quite frustrating to consider the task from a realistic perspective in the collaborative situation. In most instances, a considerable amount of time was taken negotiating meaning and contextualising the problem within a framework that was acceptable to all members of the group. The more personalised the participants wanted the problem to be, the more likely they were to complete aspects of the problem on an individual basis. Consequently, most groups found it difficult to restore the balance that is continually disturbed in group situations (Dekker, Elshout-Mohr and Wood 2006). Interestingly, all but two of the groups attempted to personalise the scenario from an idiosyncratic viewpoint in order to establish a sense of authenticity. In other words, for the problem to be realistic most participants felt that it was important to produce a solution they considered to be appropriate. As Tony (Group 1) maintained, “two of us had one idea but the others said no…we ended up doing different things”. The time and effort it took to get to this point was considerable. This group became somewhat fragmented and consequently decided to produce their own solutions. Unfortunately, the task was too demanding for this option to be worthwhile or successful. Not surprisingly, there were other ‘tensions’ that encouraged the students to work by themselves. Nathan (Group 2) argued that “if this was real we would all be over 18 and allowed to do what we want…so my friends are not going to help me say what I can eat”. It is important to note that this response came from one of the most cohesive groups in the investigation.
A substantial amount of time was spent debating such issues. When these individual approaches and ideas were combined to form a group response the solutions were, in the main, haphazard. Moreover, the solutions did not possess the level of depth and sophistication provided by the same students on similar tasks when working individually (Lowrie 2004). There was also less attention to detail—and as a result of the failure to recognise the importance of a group approach—few students took the responsibility to validate findings. The participants did not establish ownership of the activity and tended to become fixated on tasks within the central activity. The students focused on aspects of the problem that were appealing and ignored other elements of the task—generally the components that required computations or mathematical justification. In the case of the mixed-gender group (Group 4), the authenticity of the task, and the fact that they decided to personalise the task, hindered the potential for sophisticated sense making.

This group was interested in recording what they had intended to eat and were satisfied that their solution was both appropriate and realistic. From this point on, however, this particular group became increasingly interested in talking about the food they would choose to eat and the shopping they would do. They generalised (not always with the reasoning presented in Figure 5) their spending, and found it difficult to justify solutions because few of their ‘estimations’ were challenged or reflected upon.

On the other hand, Group 7 was able to consider the authenticity of the task as a collective—thus appreciating that in realistic situations individuals would have to negotiate and compromise on a number of issues. In a similar set of interactions to those highlighted in Figure 4, the group’s ideas were continually recast in light of
other participant’s responses (see Figure 6). Unlike the interactions of Group 4, this group was able to contain their sense making within the boundaries of the open-ended task. This type of interaction was uncommon across the groups.

INSERT FIGURE 6 ABOUT HERE

Yackel, Cobb and Wood (1993, 11) maintained that “the interactions that occur when children work together in small groups are intimately related to children’s mathematical conceptions as well as the social norms that have been negotiated in the classroom”. This group had a common purpose—both from a mathematics and social perspective. The group was ‘at ease’ with the manner in which individuals within the group wanted to mathematically represent the task—possibly because there was no dominant high achiever. The social milieu of the group was amenable and importantly, the group was able to present an authentic response that was collective in nature.

**Conclusion**
The social conversations that emerged as a result of the activity brought a diverse range of numeracy understandings to the problem scenario—many from specific out-of-school experiences. The genuine artefacts provided an opportunity for all of the participants to engage in a problem scenario that was contextually meaningful. Although most students were able to establish their own sense of authenticity by aligning the problem to their personal experiences and understandings, it was also the case that the majority of the students found it difficult to establish meaningful, real-world understandings in the group situation. Such ‘high-level’ collaborative tasks required the students to interact about the process and discuss planning, decision making and the division of labour. Generally, this was done quite well. Difficulties arose when the students had to consider the dimensions of group engagement in
relation to the substantive content of the problem. The tension between embracing a ‘realistic’ experience and generating a ‘collective’ mathematical solution was too difficult for most groups. Moreover, the open-ended nature of the task may have heightened the tensions between notions of realistic experience and the necessary collective agreement. However, the realistic intent of the task could not be made authentic if the students did not have scope to engage with the material in personalised ways.

In group situations where interaction is vital to productivity, it could be expected that children would engage in more productive discourse as they worked to resolve the problem activity. Instead, many of the students decided to solve components of the problem on their own—maintaining high levels of individual authenticity. However, from a group perspective, solutions remained chaotic and lacked the detail that was required to justify solutions. In her study, Peter-Koop (2002) found that haphazard solutions did not have a negative impact on the quality of the solutions, nevertheless group work that was dominated by reactive dialogue became time consuming. In this study, all forms of interaction and dialogue tended to be time consuming, and for the most part, these interactions did not produce ‘productive’ mathematical outcomes. In other words, the students were unable to regulate their own collaborative learning (Dekker, Elshout-Mohr and Wood 2006).

It is important to note that the students cooperated with one another, displayed respect for all members of the group, and valued one another’s opinions. Most of the groups, however, had one or two enthusiastic members (often high achievers) who dominated the discussions. The interactions, therefore, tended to be either asymmetrical contingencies or reactive contingencies—and as a consequence, the interweaving of ideas was not common.
Although the collaborative group activity fostered cultural, social, and cognitive phenomena that cannot be separated (Schoenfeld 1989), the mathematical sense making was quite limited, and was in stark contrast to the reasoning that was developed by most of these students when given the opportunity to solve problems individually (Lowrie 2004). The most productive groups tended to engage in conversations that were mutually interactive and contained ‘feedback loops’ that allowed participants to confirm or reinforce ideas. With such engagement, the students were more likely to present solutions that had a consistent group focus. Framing such tasks within a learning context that promotes cooperation and a collective response highlights the problematic nature of combining two productive forms of student engagement.

**Implications**
The implications of this investigation should be treated with caution given the fact that the study was drawn from a single classroom. Nevertheless, it was evident that genuine artefacts promoted personalised learning contexts, and thus allowed students to access relevant personal experiences to make sense of relatively complex problem-solving activities. These artefacts establish a sense of problem solving ‘integrity’ that not only heightens student motivation but also helps to establish meaningful engagement between peers—through both one-to-one and group interactions. However, as the students accessed and used personal knowledge to solve problems, they were less likely to monitor and manage collaborative group goals. A group’s capacity to develop and regulate their collaborative learning is reduced if the complex personalised nature of problem solving is simultaneously demanded (Dekker, Elshout Mohr and Wood 2006).
From a classroom perspective, this study has shown that teachers should be mindful of creating realistic scenarios that unintentionally interfere with the already demanding responsibilities group members have in balancing one another’s ideas and approaches to solving open-ended tasks. Nevertheless, those groups able to manage the sophisticated interplay between realistic solutions and group goal setting tended to manage group dynamics by weaving one another’s interactions into the conversations by revisiting ideas rather than being overly influenced by dominant individual’s ideas and approaches. It is certainly the case that mutual recognition of ideas and solutions, where views are revisited rather than acted upon, can help facilitate the difficult goal of creating realistic problem-solving situations for small groups to engage with. A fundamental tension exists between the use of realistic tasks that are quite open-ended and the collaborative-group dynamic. This is particularly the case when individuals (understandably) want to personalise the problem context. This tension warrants further research. Such avenues include the potential importance of creating tasks which have more defined solution pathways; the extent to which the members with a group dynamic can be encouraged to refocus their ideas when presented with dilemmas that were ‘unrealistic’ or unfounded in relation to their experiences; and the role of the teacher in disrupting reactive contingencies in order to refocus the group.
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References


<table>
<thead>
<tr>
<th>Genuineness of task</th>
<th>Mathematics sense making</th>
<th>Problem solving behaviours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attendance at a theme park</td>
<td>Able to interpret coordinates</td>
<td>Solved the problem as a group</td>
</tr>
<tr>
<td>Recognised artefact as a realistic tool</td>
<td>Capacity to use and access</td>
<td>Considered one another’s personalised experiences within the problem scene</td>
</tr>
<tr>
<td>Verbalised experiences about theme park attendance</td>
<td>Experience with map reading and timetables</td>
<td>Students’ ideas were generally taken seriously</td>
</tr>
<tr>
<td>Contextualised themselves as adults</td>
<td>Recognition of the open-ended nature of the problem</td>
<td>Students were satisfied with their response despite not having one correct answer</td>
</tr>
<tr>
<td>Experience with menus, accommodation and money handling</td>
<td>Capacity to transfer knowledge from similar activities</td>
<td></td>
</tr>
<tr>
<td>Utilised personal knowledge and experiences to complete task</td>
<td>Using number, measurement and spatial sense</td>
<td></td>
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</tbody>
</table>
Table 2
A summary of group interaction in relation to outcomes and dynamics

<table>
<thead>
<tr>
<th>Group profile and number</th>
<th>Quality of solution</th>
<th>Authenticity</th>
<th>Group processing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixed Achievement Boys (1)</td>
<td>Solution fragmented</td>
<td>Individually authentic but no group structure</td>
<td>Generally worked individually</td>
</tr>
<tr>
<td>Mixed Achievement Boys (2)</td>
<td>Justifiable solution reached</td>
<td>High group authenticity</td>
<td>Strong group interaction</td>
</tr>
<tr>
<td>Mixed Achievement Boys (3)</td>
<td>Well structured however central question not addressed</td>
<td>Not realistic</td>
<td>Dominated by one high achiever</td>
</tr>
<tr>
<td>Mixed Achievement and Gender (4)</td>
<td>Estimations realistic however computations inaccurate</td>
<td>Some group authenticity but generally individual</td>
<td>Some productive interaction of ideas</td>
</tr>
<tr>
<td>Mixed Achievement and Gender (5)</td>
<td>Task not completed</td>
<td>Individually authentic but no group structure</td>
<td>Generally worked individually</td>
</tr>
<tr>
<td>Mixed Achievement Girls (6)</td>
<td>Task not completed</td>
<td>Individually authentic</td>
<td>Generally worked</td>
</tr>
<tr>
<td>Mixed Achievement Girls (7)</td>
<td>Individual aspects strong</td>
<td>Justifiable solution reached</td>
<td>Strong group authenticity</td>
</tr>
</tbody>
</table>
**Figure Captions**

*Figure 1.* Using cultural artefacts to complete the authentic task.

*Figure 2.* An example of a reactive contingency that led Group 1 participants to work on their own.

*Figure 3.* An example of an asymmetrical contingency that led Group 3 participants to select appropriate accommodation for the open-ended task.

*Figure 4.* An example of the interweaving of ideas (Group 2) in order to complete a sub-goal of the task.

*Figure 5.* Group 4 interweaving generating an estimation cost through mutual contingency.

*Figure 6.* Group 7 employing mutual contingency discourse with “feedback loops” directed toward the actual open-ended task.
Figure 1. Using cultural artefacts to complete the authentic task.
Tony: I don’t want to eat this stuff…we are on holiday.

It means adding five dollars.

Jacob: It’s included in the price…like it’s free.

Or sixty in total. That is too much. You’ll be OK.

Chris: It gives us more money to spend for tea too.

That’s twenty [dollars] if we all do it.

*Figure 2.* An example of a reactive contingency that led Group 1 participants to work on their own.
<table>
<thead>
<tr>
<th>Matt</th>
<th>Let’s think about where we will stay</th>
<th>How much will it cost?</th>
<th>We will need two rooms. Anything better?</th>
<th>For $50 more. Does your one have a pool Daniel?</th>
<th>Daniel’s seems the best then</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luke</td>
<td></td>
<td></td>
<td></td>
<td>Let me see</td>
<td></td>
</tr>
<tr>
<td>Daniel</td>
<td>This looks good</td>
<td>$200 per night for 2 bedrooms</td>
<td></td>
<td>I think so</td>
<td></td>
</tr>
<tr>
<td>Hayden</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>This is $250 but it has a great pool</td>
</tr>
</tbody>
</table>

*Figure 3*. An example of an asymmetrical contingency that led Group 3 participants to select appropriate accommodation for the open-ended task.
<table>
<thead>
<tr>
<th>Alex</th>
<th>[The Park] opens at 10.00 [am]</th>
<th>We need to finish at Movie World</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chris</td>
<td>Let’s look at the timetable</td>
<td>It won’t matter it is in the morning</td>
</tr>
<tr>
<td>Daniel</td>
<td>It’s in 24 hour time</td>
<td>It’s the same</td>
</tr>
<tr>
<td>Nathan</td>
<td>So when will we have to leave? [the hotel]</td>
<td>Let me see</td>
</tr>
<tr>
<td></td>
<td></td>
<td>We can work back from 10.00 [am]</td>
</tr>
</tbody>
</table>

*Figure 4. An example of the interweaving of ideas (Group 2) in order to complete a sub-goal of the task.*
<table>
<thead>
<tr>
<th>Sarah</th>
<th>What will that give us?</th>
<th>But we need to know what we want to eat.</th>
<th>My toast is only $5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ziggi</td>
<td>How much will we spend on breakfast?</td>
<td>That sounds about right</td>
<td>Yes</td>
</tr>
<tr>
<td>James</td>
<td>It's only an estimate</td>
<td>Let's just make sure it's a good estimate</td>
<td>About $10 each will be OK</td>
</tr>
<tr>
<td>John</td>
<td>At least $10 each</td>
<td>And I want bacon and eggs</td>
<td>Mine is $12</td>
</tr>
</tbody>
</table>

*Figure 5.* Group 4 interweaving generating an estimation cost through mutual contingency.
<table>
<thead>
<tr>
<th>Isy</th>
<th>We should have a separate table for food and drinks</th>
<th>Because we want to make sure that we keep costs down</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jay</td>
<td>How are we going to make a budget?</td>
<td>Good so we can make sure we stick to a budget</td>
</tr>
<tr>
<td>Ellie</td>
<td></td>
<td>We can have separate totals for each day</td>
</tr>
<tr>
<td>Katie</td>
<td>We should use a table that lists our food and the actual costs</td>
<td>And it lets us see if we can save money</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Good idea...like separate bills</td>
</tr>
<tr>
<td></td>
<td></td>
<td>And lets us see if we can share and not have too much over</td>
</tr>
<tr>
<td></td>
<td></td>
<td>We can then see if we have to go without some things on a day too</td>
</tr>
</tbody>
</table>

*Figure 6.* Group 7 employing mutual contingency discourse with “feedback loops” directed toward the actual open-ended task.