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Abstract: The construction of streamlines is one of the most common methods for visualising fluid motion. Streamlines can be computed from the intersection of two nonparallel stream surfaces, which are iso-surfaces of dual stream functions. Stream surfaces are also useful to isolate part of the flow domain for detailed study. This paper introduces a technique for calculating dual stream functions for momentum fields that are defined analytically and depend on only two variables. For axi-symmetric flows, one of the dual stream functions is the well-known Stokes stream function. The analysis reduces the problem from the solution of partial differential equations to the solution of two ordinary differential equations. Example applications include the Moffat [17] vortex bubble, for which new solutions are presented.

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Creating Streamribbons Based on Mass Conservative Streamlines

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Abstract—Streamribbon is used to visualize the rotation of the fluid flow. The rotation of flow is useful in fluid mechanics, engineering and geophysics. This paper introduces the construction technique of streamribbon using the streamline which is generated based on the law of mass conservation. The accuracy of constructed streamribbons is shown through two examples.

Keywords—Mass conservation, streamline, streamtube, streamribbon.

I. INTRODUCTION

STREAMLINE, streamtube and streamribbon are three of the most fundamental features for visualizing steady flow fields. Streamlines are paths of massless particles that are released in a steady flow. Streamtube and streamribbons show the expansion and rotation of the flow.

The path swept by a deformable line segment is a streamribbon. Thus a streamribbon can reveal the translation, the angular rotation, and the rate of shear deformation of the flow. In [1] streamribbons are used in identifying causal relationships between fluid motion and local flow variables such as temperature, static pressure, turbulent kinetic energy, and vorticity.

This paper follows the streamribbon construction in [2, 3] where one streamline and constant-length vectors normal to the local velocity are used to form a streamribbon. In this paper, we use the streamlines constructed in [4]. The process of creating a streamribbon is: (1) generating a streamline; (2) generating a ribbon by passing through all constant-length vectors along the streamline.

The accuracy of created streamribbons is dependent on both the accuracies of the streamline and the constant-length vectors. Li [4] introduced an adaptive streamline tracking method for three-dimensional CFD velocity fields based on the law of mass conservation. This method suits to the CFD velocity fields that more data is available. The advantages of the method introduced in [4] include that the accuracy of the tracked streamlines for a given mesh can be controlled by the threshold number T, i.e., the larger the threshold number T, the more accurate the tracked streamlines are, and the tracking process is stopped when not enough data can be provided. The overall accuracy of tracked streamline depends on the initial mesh and the threshold number T. When CFD velocity fields are given as numerical solutions of mathematical models, more data of the velocity fields are possible to calculate.

The streamtube construction followed the methods in [2, 3] has been introduced in [6] where the streamline for constructing streamtube is generated by the approach in [4]. Such a streamtube is created by connecting the circular cross flow sections along a streamline. The radius of a cross flow section is determined by the local cross flow expansion rate.

The streamribbon construction method in this paper is based on the adaptive streamline tracking method in [4] and the formula for the rotation angle given in [2]. The construction is for incompressible flows but the method can also be used to compressible steady flows by replacing the CFD velocity fields with CFD momentum fields.

II. STREAMLINE CONSTRUCTION

Assume that a CFD velocity field is given in a hexahedral mesh and the further data of the velocity field is available. We subdivide a hexahedron into five tetrahedra (refer Fig. 2 in [4]) when we apply the conditions given in [4] to the hexahedron.

Assume that $V_l = AX + B$ is the linear interpolation of a given CFD velocity field at the four vertices of a tetrahedron, where

$$V_l = A \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} + B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix},$$

are constant matrix and vertical vector respectively. $V_l$ is unique if the four vertices of the tetrahedron are not on a plane [5]. Assume that $f$ is a scalar function and $fV_l$ satisfies the law of mass conservation (continuity equation) on the tetrahedron. We can find the expressions of $f$ for different Jacobian forms of matrix $A$. More data will be required only when $fV_l$ does not satisfy the law of mass conservation in a hexahedron. We subdivide the hexahedron where $fV_l$ does not satisfy the law of mass conservation into eight smaller hexahedra. In [4, 5], we find eight conditions of $f$ for which $fV_l$ does not satisfy the law of mass conservation, and then use these conditions for mesh

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refinement. The expressions of \( f \) for the eight cases are listed in Table 1 in [4].

\[\partial \frac{\partial}{\partial z} = \omega \cdot s\]  \( \tag{1} \)

where \( \theta \) is the rotation angle and

\[\omega = \text{curl} \left( \vec{u} \right), \ s = \frac{\vec{u}}{\|\vec{u}\|}.\]

Since

\[\text{curl} \left( \vec{u} \right) = \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)\]

for \( \vec{u} = \langle u, v, w \rangle \) in Cartesian coordinates, we evaluate the above curl using the values of velocity fields at two neighboring nodes on the streamline and the difference method for partial derivatives.

We can evaluate \( \theta \) by taking the previous time step \( T_0 \) to the current time step \( T \) after approximating \( \text{curl} \left( \vec{u} \right) \) described above.

\[\int_{T_0}^{T} \frac{d\theta}{dt} = \frac{1}{2} \omega \int_{T_0}^{T} \frac{\vec{u}}{\|\vec{u}\|} \, dt\]

\[\approx \frac{1}{2} \omega \left( \frac{T - T_0}{2} \left( \frac{\vec{u}(T)}{\|\vec{u}(T)\|} + \frac{\vec{u}(T_0)}{\|\vec{u}(T_0)\|} \right) \right)\]

\[= \frac{T - T_0}{4} \omega \left( \frac{\vec{u}(T)}{\|\vec{u}(T)\|} + \frac{\vec{u}(T_0)}{\|\vec{u}(T_0)\|} \right)\]

where \( \vec{u}(T_0) \) is the velocity field at previous time step and \( \vec{u}(T) \) is the velocity field at current time step.

For steady flows, we can write Eq. (2) as

\[\theta(T) = \theta(T_0) + \frac{T - T_0}{4} \omega \left( \frac{\vec{u}(\xi_T)}{\|\vec{u}(\xi_T)\|} + \frac{\vec{u}(\xi_{T_0})}{\|\vec{u}(\xi_{T_0})\|} \right)\]

or

\[\theta(h) = \theta(0) + \frac{h}{4} \omega \left( \frac{\vec{u}(\xi_h)}{\|\vec{u}(\xi_h)\|} + \frac{\vec{u}(\xi_0)}{\|\vec{u}(\xi_0)\|} \right)\]

where \( \xi_0 \) represents the previous node and \( \xi_h \) represents the current node, and \( h \) is the time taken from \( \xi_0 \) to \( \xi_h \).

The normal vector \( \vec{n}_1 \) at the current time step (i.e., at point \( P_1 \)) is located on the plane which passes through the point \( P_1 \)
and normal to the velocity field at the current time step (at point \( P_1 \)). \( \theta(h) \) can be interpreted as the angle between normal vectors \( \vec{n}_0 \) and \( \vec{n}_1 \) when both have initial points at \( P_1 \).

Let \( \vec{n}_1 = \langle x_1, y_1, z_1 \rangle \), \( \| \vec{n}_1 \| = a \);

\( \vec{n}_0 = \langle x_0, y_0, z_0 \rangle \), \( \| \vec{n}_0 \| = a \), where \( a \) is the constant length for all normal vectors on the streamribbon. Now we find \( \vec{n}_1 \) for the current time step when \( \vec{n}_0 \) is known as the normal at the previous time step. Since \( \vec{n}_0 \cdot \vec{n}_1 = \| \vec{n}_0 \| \| \vec{n}_1 \| \cos \theta(h) \), we have

\[
\begin{align*}
x_0 x_1 + y_0 y_1 + z_0 z_1 = & \\
a^2 \cos \left( \frac{h - \omega}{4} \right) \left[ \vec{u}(\xi_h) \cdot \vec{u}(\xi_0) \right] + \vec{u}(\xi_0) = & \\
\end{align*}
\]

and

\[
\vec{n}_0 \cdot \vec{u}(P_1) = x_1 u_1 + y_1 u_2 + z_1 u_3 = 0
\]

Where \( \vec{u}(P_1) = \langle u_1, u_2, u_3 \rangle \). Taking \( z_1 = 1 \) leads to a linear system

\[
\begin{align*}
x_0 x_1 + y_0 y_1 = & a^2 \cos \theta - z_0 \\
x_1 u_1 + y_1 u_2 = & -u_3,
\end{align*}
\]

Solving (3) for \( x_1 \) and \( y_1 \), and then let

\[
\vec{n}_1 = a \left\langle \frac{x_1, y_1, z_1}{\| x_1, y_1, z_1 \|} \right\rangle.
\]

IV. STREAMRIBBON CONSTRUCTION

Theoretically a streamribbon can be created by drawing constant-length normal vectors at all corresponding points on streamlines if the normal vector at the starting point is given. The streamline constructed using the algorithm in Section 2 consists of a series points. The density of the points of a segment on a streamline is big when they are close to a singular point or asymptote line. The algorithm for generating streamribbon is as follows.

1. Draw the constant-length normal vector at the second point on the streamline after the constant-length normal vector at the first point is chosen using the angular calculated from Eq. (4).

2. Draw a quadrilateral patch using the four points (two on streamlines and two end points of the constant-length normal vectors at the two points on the streamline).

3. Take the last point on the streamline and the constant-length vector in Step 2 as the initial point and vector, and find the next point on the streamline and the corresponding normal vector using Eq. (4).

4. Repeat Steps 2 and 3 until finish.

V. EXAMPLES

The following two examples show the adaptively tracked streamline by the method in [4] and the streamribbon constructed based on the streamlines. The tracked streamlines in red in Fig. 1 and Fig. 5 are drawn for the threshold numbers \( T = 15 \).

Example 1 Saddle-spiral flow

\[
V = \left( xz - y, yz + x, -z^2 \right)
\]

with seed point \((-0.8, 0.8, 1)\).

Fig. 1 shows the streamribbon in three dimensions. Fig. 2 shows the projection of the streamribbon in Fig. 1 on yz-plane. It is shown that the normal vector at the seed point is perpendicular to yz-plane and then it rotates gradually up to almost perpendicular to xy-plane. Fig. 3 shows the projection of the streamribbon in Fig. 1 on xy-plane. This figure indicates that the variation of rotations are very small from the point of view on z-axis. Fig. 4 shows the projection of the streamribbon in Fig. 1 on xz-plane.
Example 2 Toroidal flow velocity field

\[
V = \left( \frac{2x(z-1)}{r^2}, \frac{-2y(z-1)}{r^2}, \frac{2x}{5r}, \frac{r}{r} \right)
\]

with seed point \((6.0006, 6.7076, 1)\), where \(r = \sqrt{x^2 + y^2}\).

Fig. 5 shows the streamribbon in three-dimensions. The rotation for this example varies not significantly from Fig. 5. Fig. 6 shows the projections of Fig. 5 on yz-plane. This figure shows the rotations clearly on yz-plane. Fig. 7 shows the projection of the streamribbon xy-plane and it again shows the rotation varies not significantly. Fig. 8 shows the projection of the streamribbon xz-plane in which we can see clearly the density of normal vectors.
VI. CONCLUSION

This paper has introduced a streamribbon construction method based on the streamlines tracked by the mass conservative, adaptive streamline tracking method in [4]. As shown in [4], the streamlines are very accurate. We can conclude that the streamribbons constructed in this paper are also accurate.

REFERENCES


