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**Title:** Powerful mathematical ideas, numeracy matrix and learning stories: Assessing and celebrating the mathematical capabilities of preschool children and their teachers  
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Young children can be powerful mathematical learners (Perry & Dockett, 2002, in press; Thomson, Rowe, Underwood, & Peck, 2005), even before they start formal schooling. This paper reports work done with preschool teachers in South Australia in which powerful ideas in mathematics similar to the NCTM Standards were identified, linked to the Developmental Learning Outcomes in the mandated South Australian curriculum through an extensive numeracy matrix, and celebrated and extended through narrative assessment (Carr, 2001). It emphasises the processes involved in building the teachers’ confidence and competence in the observation, development, implementation and assessment of meaningful mathematical learning for young children and suggests ways in which this approach can improve the mathematics education of these children without weakening the strongly held traditional principles of sound early childhood practice. The paper also provides details of the numeracy matrix, how it was developed and how it has been revised through its use in preschools.

Key words: Preschool, mathematics, assessment, powerful ideas, Australia

Introduction

The Southern Numeracy Initiative (SNI) was established in 2004 among high schools, primary schools and preschools in two districts south of Adelaide, South Australia. The preschools in SNI had some concerns about the direction being taken by the schools involved, especially in terms of apparent tension between the formality of instruction and the methods of assessment chosen by the schools and the child-centred, play-based approaches that characterised their early childhood programs. As a consequence, two of the authors of this paper were invited to work with the preschool educators in SNI to develop a program aimed at improving teaching, learning and assessment practices in the numeracy development of young children. The key research question for the overall early childhood project was: How can the powerful mathematical ideas that are displayed by young children before they start school be recognised and celebrated in a valid manner within the context of a mandated reporting regime and a child-centred, play-based approach to learning?

Powerful Mathematical Ideas

There is reasonably general agreement that all children in their early childhood years are capable of accessing powerful mathematical ideas that are both relevant to their current lives and form a critical foundation for their future mathematical learning. Moreover, children should be given opportunities to access these ideas through high quality child-centred activities in their homes, communities, and prior-to-school settings (Kilpatrick, Swafford, & Findell, 2001; Perry & Dockett, 2002; in press; Thomson et al., 2005).

CSU Research Output
http://researchoutput.csu.edu.au
Two of the authors of this paper have constructed a list of powerful mathematical ideas that they have used for some time to plan, observe, facilitate and assess young children’s mathematical learning (Perry & Dockett, 2002, 2005). Their list bears many similarities to other such lists (see, for example, Greenes, Ginsburg, & Balfanz, 2004; National Council of Teachers of Mathematics, 2000) and consists of the following powerful mathematical ideas (for details, see Perry & Dockett, 2002, in press):

Mathematisation; Connections; Argumentation; Number sense and mental computation; Algebraic reasoning; Spatial and geometric reasoning; Data and probability sense;

**Developmental Learning Outcomes**

The South Australian Department of Education and Children’s Services is responsible for the education of children in preschools, primary and secondary schools throughout the state. A key curriculum document across this broad span is *The South Australian Curriculum, Standards and Accountability (SACSA) Framework* (Department of Education, Training and Employment, 2001). Educators in South Australian preschools and schools are accountable to this framework. In the preschool year, this accountability for children’s learning is assessed against eight Developmental Learning Outcomes (DLOs)—broad, observable and assessable consequences of the curriculum that reflect the integration of learning and development and allow for the different developmental pathways of individual children. The Developmental Learning Outcomes are:

- Children develop trust and confidence;
- Children develop a positive sense of self and a confident personal and group identity;
- Children develop a sense of being connected with others and their world;
- Children are intellectually inquisitive;
- Children develop a range of thinking skills;
- Children are effective communicators;
- Children demonstrate a sense of physical wellbeing; and
- Children develop a range of physical competencies.

This paper reports how the powerful mathematical ideas and the DLOs were brought together by a group of practising early childhood educators into a numeracy matrix which encouraged the educators to plan, implement and assess their practices. It also considers the use of learning stories by the early childhood educators to assess the mathematics learning of preschool children.

**Constructing the Numeracy Matrix**

Two of the authors of this paper worked with a small group of early childhood educators for two days in 2005 and two days during 2006. On the first day, background information was presented and discussed on the nature of powerful mathematical ideas and their relevance to early childhood. Participants agreed to use the powerful mathematical ideas presented in their planning and assessment of children’s learning outcomes. As well, participants were introduced to the *learning stories* (narrative assessment) methodology for assessment (Carr, 2001) and were invited to use this methodology in their settings. The second professional development day emphasised links between the DLOs from the SACSA Framework and the powerful mathematical ideas introduced on the first day. During the second day, participants were introduced to the notion of the numeracy matrix and provided with some exemplary cells in the matrix. Part of their task on this day was to complete other cells in the matrix. The third professional development day was held in March, 2006 and allowed the early childhood educators to share their experiences with the numeracy matrix and with the learning stories assessment approach. A further meeting in June, 2006 continued the refinement of the matrix and the development of the educators’ expertise in using it in their settings, particularly in terms of using it to analyse their learning stories.
The Numeracy Matrix

The theoretical basis for the numeracy matrix is that the key determinants of children’s successful outcomes are the pedagogical relationships and practices of educators (Laevers & Heylen, 2004). Hence, the elements of the matrix that bring the DLOs and the powerful mathematical ideas together are ‘pedagogical inquiry questions’—questions asked of early childhood educators as to what practices they are using to ensure that their children’s learning outcomes for both the powerful mathematical ideas and the DLOs are developing. The numeracy matrix constructed during the SNI professional development days consists of 56 cells (eight DLOs x seven powerful mathematical ideas) in which each cell provides examples of pedagogical questions early childhood educators can ask themselves as they teach towards, assess or report on the DLOs while, at the same time, take cognisance of the appropriate mathematical development of their children.

An example of one of the cells of the numeracy matrix is presented in Table 1.

Table 1: An example of a cell from the numeracy matrix

<table>
<thead>
<tr>
<th>Powerful mathematical idea</th>
<th>DLO: Children develop a range of thinking skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data and Probability Sense</td>
<td>How do we encourage children to develop a notion of fairness in their lives?</td>
</tr>
<tr>
<td></td>
<td>In what ways do we provide opportunities for children to monitor change over time?</td>
</tr>
</tbody>
</table>

In this cell are two pedagogical questions which challenge early childhood educators to inquire as to what they are doing to help children develop both the mathematical idea and the developmental learning outcome. The answers to these questions will affirm those educators who are working towards these goals, as well as suggest to them that more activities might be needed to help the children develop further. The questions will also stimulate educators who have not considered their practices in these areas to investigate the relevance of current activities and practices or the need for new practices. These pedagogical questions have relevance to other key learning areas as well as mathematics, thus emphasising the integration of mathematics learning with other learning areas.

The numeracy matrix is, by its very nature, a work in progress. As the early childhood educators using the matrix have become more confident and competent, they have suggested changes. Some mathematics educators who have studied the matrix have suggested possible changes on the basis of recent research in their field—research which is not normally available to practising early childhood educators. The matrix is a dynamic reflection of the knowledge of the educators using it and, as such, should be expected not only to be grounded in the contexts in which these educators work but to change as their knowledge changes.

It is not possible in the space available for this paper to present the entire numeracy matrix. Hence, only two segments will be provided. The first, in Table 2, provides the complete description of one powerful mathematical idea against the eight cells representing each of the DLOs. The second, in Table 3, provides the complete description of one DLO against the seven cells representing each of the powerful mathematical ideas.

<table>
<thead>
<tr>
<th>Developmental Learning Outcomes</th>
<th>Powerful mathematical idea: Argumentation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children develop trust and confidence</td>
<td>What opportunities and support do we give children to explore and take risks as they justify their mathematical thinking?</td>
</tr>
</tbody>
</table>
|                                 | How do we encourage children to demonstrate flexibility and to
manage different mathematical ideas as they are presented to them by peers?

| Children develop a positive sense of self and a confident and personal group identity | How do we encourage children to develop and maintain respectful relationships with adults and children even though they may not agree with their mathematical ideas? | How do we encourage children to help develop agreed values and sociomathematical norms of behaviour in their groups? |
| Children develop a sense of being connected with others and their worlds | How do we encourage children to contribute constructively to mathematical discussions and arguments? | How do we encourage children to question why their and other people’s mathematical ideas work? |
| Children are intellectually inquisitive | What opportunities do we provide for children to communicate their own mathematical ideas to a respectful group of peers? | How do we assist children to gain confidence in their ability to explore, hypothesise and make appropriate choices in their mathematics? |
| Children develop a range of thinking skills | How do we encourage children to participate in group discussion and justification about the solution of mathematical problems? | What opportunities do we provide for children to suggest alternative solutions to mathematical problems? |
| Children are effective communicators | How do we encourage children to interact with others to explore ideas, negotiate possible solutions and share their mathematical learning? | What opportunities do we provide for each child to use different communication strategies to help clarify their and their peers’ mathematical thinking? |
| Children develop a sense of physical well being | What opportunities do we provide for children to develop confidence in expressing their mathematical ideas? | How do we encourage children to celebrate their efforts and achievements in mathematics learning? |

Table 2: Numeracy matrix for powerful mathematical idea: Argumentation

The powerful mathematical idea of argumentation:

allows children … to justify not only their own mathematical thinking but also to distinguish between the strengths of arguments and whether the mathematics being constructed within the arguments is actually different from previous mathematical arguments that have been interactively constructed (Perry & Dockett, 2002, p. 92).

Providing such justification, while clearly important as children develop their mathematics, is also important in many other areas of learning and certainly contributes in numerous ways to the DLOs. On the other hand, the development of this powerful mathematical idea depends on early childhood educators’ pedagogical practices, some of which are presented in the form of questions or challenges within the numeracy matrix. For example, in answer to the pedagogical question “How do we encourage children to contribute constructively to mathematical discussions and arguments?” one of the SNI early childhood educators suggested:
We would firstly need to make sure that children felt safe in talking up about their solutions and those of others. We want them to say what they think but in ways that will not hurt anyone. That will depend a lot on the atmosphere in the group but it will also need the kids to know the maths that they are talking about.

<table>
<thead>
<tr>
<th>Powerful mathematical idea</th>
<th>DLO: Children are intellectually inquisitive</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematisation</strong></td>
<td>What opportunities do we give children to experiment with mathematical concepts and representations in problem solving and investigation?</td>
</tr>
<tr>
<td></td>
<td>How do we encourage children to gather information and ask questions that might be answered by this information?</td>
</tr>
<tr>
<td><strong>Connections</strong></td>
<td>What opportunities do we give children to investigate mathematical ideas that are part of the local natural and constructed environment?</td>
</tr>
<tr>
<td></td>
<td>How do we encourage children to use mathematics to be a critical consumer of everyday products?</td>
</tr>
<tr>
<td></td>
<td>How do we assist children to find connections between different mathematical concepts and representations?</td>
</tr>
<tr>
<td><strong>Argumentation</strong></td>
<td>What opportunities do we give children to put forward a mathematical argument and to justify it?</td>
</tr>
<tr>
<td></td>
<td>How do we assist children to gain confidence in their ability to explore, hypothesise and make appropriate choices in their mathematics?</td>
</tr>
<tr>
<td><strong>Number sense and mental computation</strong></td>
<td>What opportunities do we give children to explore, hypothesise, take risks and engage in symbolic and dramatic play with confidence?</td>
</tr>
<tr>
<td><strong>Algebraic reasoning</strong></td>
<td>What opportunities do we give children to experiment with word, language, number and shape patterns?</td>
</tr>
<tr>
<td></td>
<td>How do we encourage children to explore patterns using their senses?</td>
</tr>
<tr>
<td></td>
<td>How do we assist children to use pattern making and pattern continuation for problem solving and investigation?</td>
</tr>
<tr>
<td><strong>Spatial and geometric thinking</strong></td>
<td>What opportunities do we give children to explore their local environment and record what they see using visual means?</td>
</tr>
<tr>
<td></td>
<td>How do we encourage children to analyse critically the shapes found on the supermarket shelves?</td>
</tr>
<tr>
<td></td>
<td>How do we assist children to compare and classify shapes?</td>
</tr>
<tr>
<td><strong>Data and probability sense</strong></td>
<td>What opportunities do we give children to investigate different forms of data representation?</td>
</tr>
<tr>
<td></td>
<td>How do we encourage children to interpret data arising from the use of everyday products?</td>
</tr>
<tr>
<td></td>
<td>How do we assist children to gather information, ask questions, seek clarification and consider possibilities about their own lives?</td>
</tr>
</tbody>
</table>

Table 3: Numeracy matrix for the DLO: Children are intellectually inquisitive

The contribution of each powerful mathematical idea to the DLO is clearly illustrated through this part of the overall matrix. Very few of the early childhood educators involved in SNI would have argued against mathematics contributing to the DLO of children being...
intellectually inquisitive. However, very few of these educators were able to articulate how that might occur in a learning area such as mathematics with its perceived underlying (and constraining) structure. Through their use of the numeracy matrix, the educators are now able to see how each of the powerful ideas contributes to the DLO. One of them was able to suggest that the work with the numeracy matrix had helped them see how the DLOs were the capstones to all that they were trying to do in all learning areas.

When I thought about shapes and geometry, I thought all that was needed was for the children to know the names of some regular shapes. It was really not something I thought they would be inquisitive about. By using the matrix, I can see that they can develop their inquisitiveness by asking lots of questions about lots of different shapes in their environment – not just triangles and circles – and can investigate why things are the way they are. This will take them into asking about how things are used, where they come from, whether some shapes are better than others for a particular job and why some shapes look better than others. It is exciting for the children – and for me!

Learning Stories and the Assessment of Powerful Mathematics Ideas

The approach to assessment known as Learning Stories has been pioneered by Carr (2001). Learning Stories are qualitative snapshots, recorded as structured written narratives, often with accompanying photographs that document and communicate the context and complexity of children’s learning (Carr, 2001). They include relationships, dispositions and an interpretation by someone who knows the child well. They are “structured observations in everyday or ‘authentic’ settings, designed to provide a cumulative series of snapshots” (Carr & Claxton, 2002, p. 22). Learning stories acknowledge the multiple intelligences and holistic nature of young children’s learning, educators’ pedagogy and the context in which the learning takes place. Educators use their evaluation of the learning story to plan for future, ongoing learning. In South Australia, learning stories have been used by preschool educators for some time, especially in the area of literacy learning. However, they tended not to be used in the area of mathematics, partly because the preschool educators did not have sufficient confidence in their ability to link what they were observing with mathematical learning outcomes. The introduction of the numeracy matrix has given this confidence to the group of educators working with the authors and has produced some outstanding results. A mathematical learning story from the project illustrates this point. The child who is the subject of this learning story is almost 5 years old and attends one of the preschools in the project. The learning story was recorded by his teacher in this setting.

Figure 1: Learning Story

*What can you do with wood?*

Harrison discovered the activity we had set up outside, specifically set up with him and his friends in mind. On one table I’d placed a collection of wood; on another, a selection of natural materials and the glue guns. Harrison’s first instinct was to immediately begin gluing the larger pieces together. I asked him to slow down and to begin playing with wood first, to think about what he might do…which pieces was he going to use? How/where would he place them?

Harrison played for a few minutes and then said he was ready to begin joining the first few pieces which formed the base. Once he’d done this he began to add corks, pop sticks and interestingly shaped pieces of wood. Some shapes didn’t stick very well so he had to problem solve what he might do differently to ensure they stuck. I talked to Harrison and the children working alongside him about perspective – that is...
standing back to look at their work before continuing. Harrison worked on his structure for most of the session. It was then placed on the “work in progress shelf” inside so that he could return to it the following day. Other children looked at the structure and commented on how “good” it was.

Harrison returned to his structure over a number of days, each time adding more pieces to it, often creating structures within structures. Even the strings of glue from the glue gun were part of his project- they were Spider Man’s webs - his favourite Superhero!

Harrison said “It’s a Spider man fort. He sleeps on top. He gets there by stairs”

**Review:**

Harrison worked very hard on his “Spiderman’s house”. He showed persistence, the ability to choose a project and to keep working at it until it was completed to his satisfaction even when he faced difficulty. He was able to problem solve, use trial and error, seek help from others as required and as a consequence he became more confident and his expertise grew. Harrison’s structure also shows mathematical knowledge such as shape, symmetry and balance.

In this learning story, there is a clear link into the numeracy matrix through the powerful mathematical idea Spatial and geometric thinking. The play episode recorded shows Harrison’s developing spatial and geometric sense in a number of ways, including, as the educator has suggested, shape, symmetry and balance. The initial intervention of the educator after Harrison had rushed at the prepared activity without much apparent thought turned out to be inspirational. The educator considered pedagogical inquiry questions such as *What opportunities do we give children to explore their local environment and record what they see using visual means?* and *How do we assist children to compare and classify shapes?* in order to stimulate Harrison’s mathematical development. Without access to the numeracy matrix, it is likely that Harrison would have continued to play aimlessly and not had the opportunity to realise his mathematical potential.

**Conclusion**

The purpose of this paper was to introduce the numeracy matrix which has been developed as part of the Southern Numeracy Initiative in South Australia and to celebrate the
work of the early childhood educators who have been involved in its development. There is evidence that the use of the numeracy matrix and the thinking behind it have had positive effects on the pedagogical practices of the early childhood educators involved. This, coupled with the learning stories assessment methodologies which allows the preschool educators to meet their reporting obligations while at the same time remaining true to their early childhood philosophies suggest that the SNI preschool project will lead to improved practices and, consequently, improved learning outcomes for the children who are fortunate enough to be taught by this enthusiastic group of educators. In the words of one of the educators:

The numeracy matrix has been an extremely useful tool for helping us to maintain a focus on holistic learning while exploring mathematical concepts and processes in the context of our mandated curriculum framework. The development and use of a narrative form of assessment has been one of THE most useful tools in helping us to remain true to our underlying early childhood philosophy of learning and to assess children's learning without formal testing.

References


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