SCHOOL MATHEMATICAL DISCOURSE IN A LEARNING LANDSCAPE: BROADENING THE PERSPECTIVE FOR UNDERSTANDING MATHEMATICS EDUCATION IN MULTICULTURAL SETTINGS

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*By bringing our research work together, we are able to discuss the potential of combining the notions of the **learning landscape** and **school mathematical discourse**. We do so in a search for concepts and methodological tools to challenge the simplification of issues in regard to mathematics learning in multicultural settings, when adopting restricted perspectives on issues of bilingualism. In the paper we discuss the relationship between the learning landscape and school mathematical discourse. We then use these notions to analyse two case studies in Danish and New Zealand schools. Our conclusion raises possibilities about how these notions can be used when researching mathematics education in multicultural settings.*

INTRODUCTION

Generating theoretical and methodological tools to study and understand the complexity of mathematics education practices in multicultural settings is a current challenge for researchers in many countries. Multicultural classrooms include cultural, ethnic, religious, linguistic and socio-economic diversity among students and teachers (Hodge 2006; Nieto, 2002). Bringing together our research in multicultural classrooms in Denmark and New Zealand has enabled us to explore socio-cultural-political perspectives on language and school mathematics practices. In this paper, we discuss two central notions emerging from our research, and the relationships between them. *Learning landscape* interprets (mathematics) education as a complex network of social practice that is constituted by different interrelated dimensions (Alrø, Skovsmose & Valero, 2007). *School mathematical discourse* is understood to be anything that affects the communication of meaning within the mathematics classroom and can include the mathematics register, the interaction patterns between teachers and students and between students and students, the textbook format, and the more general classroom language (Sfard, 2001; Meaney, 2005).

LANGUAGE AS A DOMINANT APPROACH TO DIFFICULTIES WITH MINORITY STUDENTS

For many teachers and researchers one of the most evident problems when meeting minority students in their classrooms is the students' lack of competence in the language of instruction. Consequently, there is a belief that if students cannot participate effectively in schooling and in school mathematics it is because they have not mastered the language of instruction. Such a view is challenged in reports such as "The teaching of bilingual students", on the instruction of minority students in Denmark, produced by the Danish Evaluation Institute:

> The prevailing view on language appears static in more of the schools. The schools focus primarily on the students linguistic skills as a condition for learning and do not sufficiently see language as something that is created in a context. As a consequence second language acquisition pedagogy recedes into the background, and a focus on pedagogy and learning is replaced by a focus on the students' deficiencies. Therefore, a central challenge to the schools is to develop their view on language. Another important challenge is to develop their view on inclusion. This evaluation emphasises that the education of bilingual students often requires a difficult reorganization from handling a group of students that require inclusion in one way to a group of students that as a whole require more kinds of inclusion. (Danmarks Evalueringsinstitut, 2007, p. 8, Troels Lange's translation)

The report emphasises the need for schools to increase their views of both language and inclusion. Language can be simultaneously a wide and also a very narrow construct. It is wide because it can be considered as the vehicle for communicating meaning between people (or with one's self). Therefore, it can include a variety of forms of communication (words, symbols, gestures) as well as all the meanings that can be conveyed (Bower, 2005). This broad construct can easily be related to beliefs about language as a thinking tool (Sfard, 2001; Langer, Applebee, & Nystrand, 1995; Vygotsky, 1962). A wide view of language includes the contexts in which it is used and learnt, such as the society as a whole, but also the immediate school and classroom contexts. However, as a cultural artefact, language has been developed with and in response to cultural demands. It is not a simple, unproblematic construct that can be learnt without also absorbing cultural baggage (Holmen, 2008).

If language is restricted to just having fluency in the language of instruction, then language can become a narrow construct. When this is the case, students who are learning the language of instruction as an additional language are labelled as learning disabled (Naudé, Pretorius, Vandeyar, 2002). Contrary to this belief, research has shown that students who are taught bilingually not only achieve better results in content areas such as mathematics but that they also achieve higher in the second language (Saunders, 2001).

In the same way that language can be interpreted in multiple ways, inclusion also has equally distinct definitions, depending on the kinds of relationships established among

minority groups and the dominant culture with which they interact. These can be seen in the Table 1.

¿Se promueven las relaciones intergrupales?	¿Se facilita el mantenimiento de la identidad y las características culturales y lingüísticas minoritarias?	
	SÍ	NO
SÍ	INTEGRACIÓN	ASIMILACIÓN
NO	SEGREGACIÓN	MARGINACIÓN

Table 1: Linguistic and cultural ideologies (from Martin-Rojo, 2003, p. 33)

In describing different linguistic and cultural ideologies, Martin-Rojo brings together two dimensions: whether there is a promotion of relationships between groups, and whether the maintenance of minorities' cultural identity and cultural characteristics is promoted. Different intersections of these two variables produce four types of ideologies: *Integration*, promoting inter-group relationships and allowing the maintenance of identity and cultural characteristics of minorities; *assimilation*, promoting inter-group relationships but denying the maintenance of cultural and linguistic identity; *segregation,* not promoting inter-group relationships but allowing minorities' identities; and *marginalisation*, denying both dimensions. The way in which a society deals with minorities has clear consequences for schools and mathematics classrooms. In a parallel way to Martin-Rojo's typology, one could ask whether minority students' identity and cultural characteristics have been recognised and their retention supported in relation to the students' understandings of schooling and school mathematics. In this way we could talk about inclusion not only in terms of having minority students succeeding in mathematics performance tests —which unfortunately is not the case in many countries (OECD, 2006)— but also in terms of bringing diversity to the way in which school mathematics is done and understood in each national context. As a result of attending mathematics classes, students should neither be alienated from their home culture or from the dominant school culture (Cantoni, 1991).

SCHOOL MATHEMATICAL DISCOURSE IN A LEARNING LANDSCAPE: OPENING THE PERSPECTIVE

Understanding the relationship between language and mathematics education cannot be narrowed because then the complexity of the situation is lost. In order to overcome the shortcomings of a restricted understanding of school mathematics practices in situations of diversity, we have devised the following model (See figure 1).

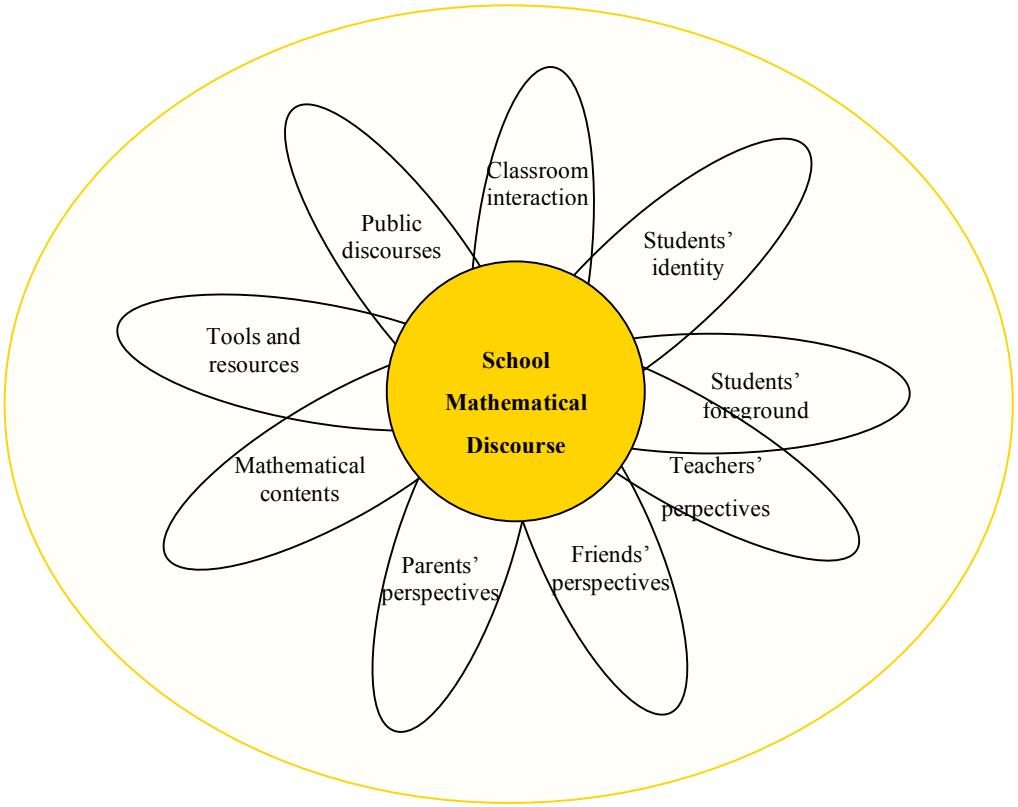

Figure 1: School Mathematical Discourse in a Learning Landscape

Rather than talking about language, we use the term school mathematical discourse to refer to the use of a natural language and the mathematics register as a part of the practices of the teaching and learning of mathematics. This includes the interaction among the participants in that practice and the establishment of patterns of communication for expressing mathematical meaning. The discourse is therefore the set of uses of language that have to do with all the implicated actors and factors impacting school mathematics. School mathematical discourse is at the centre of the learning landscape and permeates all aspects of it.

Based on recent research literature on multiculturalism and mathematics education, we have decided to select nine dimensions, which have been considered to be influential for learning possibilities (Alrø, Skovsmose & Valero, 2007). We bring the nine dimensions together in what we call a learning landscape. The notion of learning landscape is a tool to guide us in exploring the empirical field, and it has a double meaning. First, it represents an interpretation of (mathematics) education as a complex network of social practice that is constituted by different interrelated dimensions (Valero, 2007). Second, it makes it possible to identify specific –but correlated – dimensions of an empirical field to do research. Thus, it brings together a research perspective and a research field. The nine dimensions selected are: (a) Students' foregrounds as an experienced socio-political reality (Skovsmose, 2005); (b) Students' construction of identity and of cultural diversities (Sfard & Prusak, 2005); (c) Teacher's perspectives, opinions and priorities of teaching (César &

Favilli, 2005); (d) The content of learning, in our case the mathematical content for classroom interaction (Powel, 2002); (e) Tools or resources for learning that students might have available; (f) Classroom interaction among students and between students and teacher (Alrø & Skovsmose, 2002); (g) Family and parents who influence students' priorities (Gutstein, 2003); (h) Friends, who are reference groups for the students (Bishop, 2002); and (i) public discourses about immigrants, schooling and multiculturalism (Martin-Rojo, 2003).

We contend that the combination of school mathematical discourse within the learning landscape provides a powerful tool for researching the complexity of school mathematics practices in settings of diversity. In what follows, we illustrate how we have used this tool in researching schools and classrooms in our countries. We will use two case studies from our research, one in Denmark and one in New Zealand. After briefly describing each case, we analyse them using our theoretical tool to illustrate the types of insight that can emerge from bringing it into operation. Finally, we draw conclusions by using our tool to analyse the two cases together.

WHEN BEING GOOD AT MATHS IS NOT AN ADVANTAGE[1]

Minh is 15 and like many of his 9th grade school mates at Mælkevejen Skole[2], he seems to be more interested in anything except school. Showing signs of manhood, his body has changed dramatically in the last two years: man's voice, long hair and one of the tallest bodies in the class. From being an annoying trouble-maker together with some of his Vietnamese friends, Minh has now become a popular boy in the class. He is not only good at sports —and is therefore ready to take all kinds of physical challenges— he is also good at mathematics and gives a helping hand to those classmates in need of a push for finishing the set of exercises for homework.

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[1] This story is based on our empirical investigations in the project "Learning From Diversity" (Alrø, Skovsmose and Valero, 2003, 2005). In the study, the research team (Helle Alrø, Ole Skovsmose and Paola Valero) followed for a period of two school years, two classes of students and their teachers. We have carried out empirical foregrounds investigations (see Alrø, Skovsmose & Valero, 2007), a participant observation study in the school, and a study based on a punctual classroom intervention run by the researchers. The case of the student presented here has emerged from an analysis of a combination of information from the different studies in the two years period.

[2] Mælkevejen Skole is a primary and lower-secondary school situated in a low-income, difficult suburb of a bigger Danish city. This school hosts students from 29 countries from all over the world. We established collaboration with two 8th grade classes of 24 and 21 students respectively. Half of them come from countries such as Iceland, Ireland, Iraq, Greenland, Lebanon, Malaysia, Somalia, Turkey and Vietnam. The two teachers in the classes we followed are a young man and a very experienced man, both Danish.

When Helle first met Minh, he was at 8th grade. In an inter-view[3] with him about how he saw his future life and the role of mathematics in it, he stated that he liked to go to school. He considered school and mathematics to be very important for his future life, as at that time he wanted to become a banker or estate agent because he considered being good at mathematics to be his major strength. Therefore, he could imagine in the future to do "something with mathematics — I love numbers". However, Minh's appreciation seemed not to coincide with his parents' views. He did not think that his parents considered him to be good at mathematics. They had never told him so explicitly. Consequently, sometimes he doubted his ability and felt that he was not good enough. He seemed to be upset about that. As Minh's teacher expressed when commenting on the boy's possibilities for the future, Minh's parents were very ambitious on behalf of Minh. The father had a grill-bar, and he wanted his son to get a well-paid job, so he wanted him to become a medical doctor, a very respected and socially valued profession in the eyes of Vietnamese immigrants. Minh himself would rather get a job that he was interested in and happy about. Nevertheless, he knew that his father would be disappointed if he chose "only" to become a bank clerk. Considerations about being a medical doctor had also passed through Minh's mind.

One year later, Paola met Minh again. At that time his engagement in school had changed and teenage priorities were more clearly present in his mind. In the setting of the computer room where the teacher had invited students to work on an open problem —imagining themselves in a profession and making a budget for monthly expenses—, Minh was quite engaged in the task but seemed very hesitant and troubled about what to choose. On the web, he looked for information about a career in one of the local banks. Hmm... were those the numbers he should enter in his Excel spreadsheet to calculate a budget? He also looked for information about the monthly income of medical doctors. But… what would it take him to be a doctor? "Paola, if I want to become a medical doctor, how many years do I have to study?", "Well, after you finish school you will have to go to high school which is 3 years, then you have to go to University for 5 years more, then you have an internship of around 1 year more, then you have to choose an specialization line of about…" "Stop, stop, stop. That is a lot. Many years. I cannot imagine that. I rather become a bank assistant". For his young age, 3+5+1+… extra years seemed like an eternity. He expressed his doubt about whether he could engage in studies for such a long time. Despite expressing his wish to become a medical doctor, he used the data for a life in a bank. Finances could not be that bad, after all…

In a later interview Minh expressed his confusion. His parents had great expectations and showed great appreciation of the path taken by his cousin. She was doing really well, they thought. Minh felt annoyed by being measured against one of his relatives.

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[3] Following Kvale (1996), we use inter-views as a form of inquiry that allows the inter-viewer and inter-viewed to "see together" and explore together the object of research, in this case, the students' foregrounds. For more details see Alrø, Skovsmose and Valero (In press).

As far as he was concerned, he would make a good career… in a bank. “And what about medical studies?” Paola asked, “Well, my teacher says that I can’t be a medical doctor because I am not good enough at Danish”, “What do you mean?”, “My Danish teacher says that since I am not very good at writing essays and that stuff, I won’t become a medical doctor”, “But you are pretty good at Danish!”, “She really doesn’t think so. I could do well studying something with mathematics. Banking is fine for me”.

The mathematics teacher appreciated Minh’s ability in mathematics and thought he could almost study whatever he wanted —if he overcame his laziness. In the class, Minh was a relatively active participant. He did the tasks that the teacher gave him and worked with the exercises in the textbook. The teacher’s style at that time of 9th grade was preparing kids for the National Test at the end of the academic year, through giving students open, word problems from the main textbook, from other newer textbooks with problems that seemed more attractive to 15 year-old youngsters, and activities that he himself had designed. Minh was also known for doing the weekly assignment consisting of a set of old national tests, which he responded to thoroughly and in an organised fashion. He would explain the problems to some of his mates that they could not understand, and sometimes even let them copy from his own work sheets. In an environment where there was great freedom for the students to work in groups or individually, Minh engaged in the tasks as much as he allowed himself to be distracted, just like all the other students in the class.

In contrast, Minh’s Danish teacher saw obstacles. Having been the class teacher for Minh all the way from 1st grade to 9th grade, she felt she knew well the boy’s strengths and weaknesses. He was clever at maths; that was true. But it was difficult to know whether studying medicine was the boy’s real wish or his parents’ desire for him —Vietnamese parents are always quite ambitious on behalf of their children … probably all too unrealistically ambitious sometimes, the Danish teacher thinks. Advising him —and his parents— about how to continue with his studies after compulsory school was a complicated issue. The teacher said:

> So when he shows hesitation, I think: If he takes an education in commerce, a three-year study, he will be in school three years and one in practical training in a bank, that is something he can complete. Then he can build on top of that if he wishes. But if we push him to go to high school and get a certification in mathematics, and later on six or seven years at the university, then I become more hesitant about whether he will be able to cope with that and succeed in finishing. And then he would drop out of school and will be left without any education at all. I would worry about him not having any certificate.

But why would he drop out? In reality, Minh’s weakness was his lack of mastery of the Danish language. “He is not very articulated, he … hum… he is not very good to formulate himself in written form. He is not a catastrophe, but he is not good. We have given him special support in the past but it has not really helped much.” the Danish teacher says. In general, the Danish teacher thinks that it is difficult for

students who do not speak Danish at home to get acquainted with many words and with the right syntactic structure of Danish, particularly if they use bad oral Danish or if they do not communicate in writing at home. And even if they wrote, their parents cannot really correct them. Minh is not an exception. "Students can get better, Minh has in fact improved. But he will never have a fully developed language like many other in the classroom", the Danish teacher adds. In a school such as Mælkevejen Skole, such a problem is common to many bilingual students, as well as to many students coming from "weak homes" or "homes without many resources", according to the Danish teacher.

A few months after, Ole met Minh again. Ole had agreed with the teachers to do a sequence of activities with the students to explore how they saw mathematics in their everyday activities. Minh was enthusiastic and, in the same way he had welcomed Helle and Paola, showed interest in Ole being in the school. He was ready to engage in Ole's proposal. Unfortunately, during the very first session of the project-work with Ole, Minh was called out of the classroom by the school doctor for a routine check. The second session, Minh was absent because he had to do some sports. The third session … well, then it was too late to get engaged. Sometimes other school priorities interfere with school mathematics.

Reading Minh's story from the perspective of school mathematics discourse in a learning landscape

There is nothing unusual about Minh when seen as a typical Danish youngster at 9th grade. But when seen as a Danish, second generation Vietnamese youngster, his story reveals interesting aspects of second-generation immigrants and their experiences in schools and in mathematics classrooms. Minh is in conflict about the choices he will have to make soon about his life. For many Danish students the end of 9th grade and the possibility of leaving the compulsory school represents the beginning of a life of further study, career and life choices, that may not represent problematic challenges. For Minh, this moment seems to be decisive in dealing with what he wants, what his teachers show as possibilities for him, and what his family expects from him. Minh's conflicts illustrate some of the conflicts that minority students may experience when giving meaning to the learning of mathematics in relation to their future life possibilities. They also reveal the role given to mathematical learning as a factor for success in life and how such a role is constructed in school practice, in the intersection between teachers' priorities and views of their immigrant students' futures and the students' views of their own wishes and hopes in life (and the parents' expectations). In the following analysis we show how bringing together the dimensions in Figure 1 can allow exploring the intersections of multiple factors impacting learning possibilities in multicultural mathematics classrooms.

One possibility to analyse Minh's case is to focus on his participation in the mathematics classroom and trust the mathematics teacher's appreciation about Minh's being good at mathematics and, therefore, being able to have a good future.

He could study almost whatever he wanted. His competence in mathematics opens the doors of an education in banking and finances, not at the university though, but as a middle vocational education. This possibility, although being expressed strongly by Minh when we first met him, seems to generate conflict one year later. Being good at mathematics may not necessarily be so advantageous since it will lead to a type of study and profession that is not so highly valued as medicine, at least from the point of view of Minh's parents. Mathematical competence is not enough to succeed in high school and to make it through university. For a possible future as a medical doctor, mathematical competence seems to be subordinated to his mastery of the Danish language, according to the Danish teacher, his class teacher, a person who plays an important role in guiding Minh in his further choices at the end of the compulsory school.

The mathematics teacher did not express any particular concern about Minh's competence in the Danish language. In the context of the mathematics classroom, this did not seem to be an issue that represented an obstacle for Minh. However, in a Danish context where there is a tradition of collaboration among teachers in interdisciplinary teams organised around a class teacher, the latter has a great influence on assessing students' strengths and guiding students and parents with advice about possible future study paths. This teacher's assessment of Minh's lack of mastery of the Danish language is decisive for how he is perceived in general in the school and how different school teachers' practices contribute to create an image of Minh's potential foregrounds.

Moreover, the issue of the lack of competence in the Danish language seems to be connected not to the fact that Minh may lack linguistic competence, but to the fact that Minh and his family are lacking the cultural capital associated to adequate linguistic fluency. Here, we find how public discourses around multiculturality and immigrants' possibilities and lacks enter the school scene: second generation immigrants such as Minh and his family lack the cultural (and even also the social) capital needed to succeed in higher education. Competency and fluency in the language of mathematics do not seem to be enough to succeed and climb up the social ladder. The view expressed by the Danish teacher seems to resonate with the idea that many immigrant families have weak cultural resources and therefore are unable to help their children with the linguistic habits that are necessary to succeed in school as a whole (not only in mathematics). When such a perception is part of the teachers' views and perspectives, then the perception that immigrant parents have extremely high (and unrealistic) ambitions for their children emerges as a kind of explanation among teachers for the mismatch between the school expectations and children's and parents wishes for their future.

TE REO TĀTAITAI: DEVELOPING RICH MATHEMATICAL LANGUAGE

For many years, we, Tamsin, Tony and Uenuku, have been working with teachers and others at a Māori immersion school, *kura kaupapa Māori*, in New Zealand. In

the first project, 1998-1999, we worked on developing a culturally appropriate mathematics curriculum. School mathematical discourse was just one issue that was considered. In the second project, which is ongoing from 2006, we are investigating issues to do with using the Mäori language, *te reo Mäori*, in the teaching of mathematics.

Kura Kaupapa Mäori were instituted by Mäori communities, initially without support from the state education system, in order to maintain te reo Mäori and to improve educational outcomes for Mäori children (May, 2003). Consequently, issues to do with language are never far away from teachers' and parents' minds, but generally being multilingual is something to be celebrated. However, this does not mean that the issues being considered are simple.

In this school, the parents and teachers had decided that "language of instruction for mathematics is Maori, language adapts with every new situation that it meets, rich language leads to a fuller understanding and discussion" (Minutes from previous meeting read out at Meeting held 15/8/99). With any language of instruction, they felt that there was a need to specifically teach the features of the mathematics register, *te reo Tätaitai*. They also felt that these features may be different to the school mathematical discourse learnt by the parents. For example, the parents discussed how the way that multi-digit additions and subtractions had been done when they were at school was different to how their children were being taught.

> Teacher4: And we talked about the different languages that we use, Parent3 found out about the way that we were saying our times was totally different to how she was understanding it, you know. Yeah.
>
> Teacher3: Oh and then we got into decomposition as opposed to abbracadabra.
>
> Teacher4: Talking about that, yeah, how the language is different because we're talking about maths and that. (Meeting 15/8/99)

If the parents did not understand the mathematics including the school mathematical discourse, it was difficult for them to talk with their children and help them with their homework. In the following extract from a lesson in 2006, a Year 12 class discussed the r (x) and t (y) co-ordinates and the t-intercept (k) in regard to the equation for a straight line, $t = pr + k$, (y=mx + b). This discussion began with ideas about the terms for a particular line.

> Student: Äe, tërä mea i konei kei runga.
>
> T7: Nä, ko te kotinga 't' he törunga, he töraro ränei?
>
> Student: He törunga.
>
> T7: Törunga. Engari käore tätou i te möhio, nö reira, täpiri.
>
> Student: Täpiritia te 'r', oh, he 'k'.
>
> Student: He aha?

T7: Äe, täpiritia te ‘k’ i tënei wä.

Student: ‘W’, oh, rima e (?)

T7: Arohamai, i hoatu i tëtahi mea uaua ki a koutou. Kei te tere whakaatu pëhea te kimi i te ‘k’.

Student: Ka taea te whaka, ah, (?) atu i ngä rärangi mö te mahi tukutuku. I te mea he tino uaua, äe, ki ngä mätua.

Student: Äe. Nä, te mea käore e taea te kite.

When the students showed confusion over the meaning of the different terms, the teacher (T7) acknowledged that it was difficult to learn all the words. One student followed this up by saying that it would be very difficult to explain it to parents and this was confirmed by another student.

In kura kaupapa Mäori, parents are an integral part of their children’s education process (Meaney & Fairhall, 2003). As a result, various activities have been instigated over the years to increase parents’ knowledge about using te reo Mäori to talk with their children about mathematics. This has been integrated with discussions about current trends in the teaching of mathematics. This provided parents with possibilities to discuss with children their mathematics learning, even if the parents do not speak te reo Mäori.

The mathematics symbolism was seen as one way to support students to switch between languages

T3: I think a really important issue in Kura Kaupapa is that mathematical symbolic language has to be really emphasised because it’s one of the things, that’s the crutch, which let’s them move from language to language. They’ve got to see the mathematics there and written in mathematical notation, if they don’t get on top of that then they are, they will have those problems when they shift over. (Meeting 15/8/99)

At this kura, English, the dominant language in mainstream New Zealand, is not taught formally until students enter high school in Year 9. However, Spanish is introduced as an additional language when children start at the kura at the age of five. In mathematics lessons, both English and Spanish can occur with te reo Mäori. Figure 2 shows posters of 3-D shapes, some giving the title in te reo Mäori, *Ahua 3-D*, and some in Spanish, *Formas de 3D*. It also shows a copy made by a student of his teacher’s notes about the features of a triangle. *Koki*, angle, has the English translation underneath.

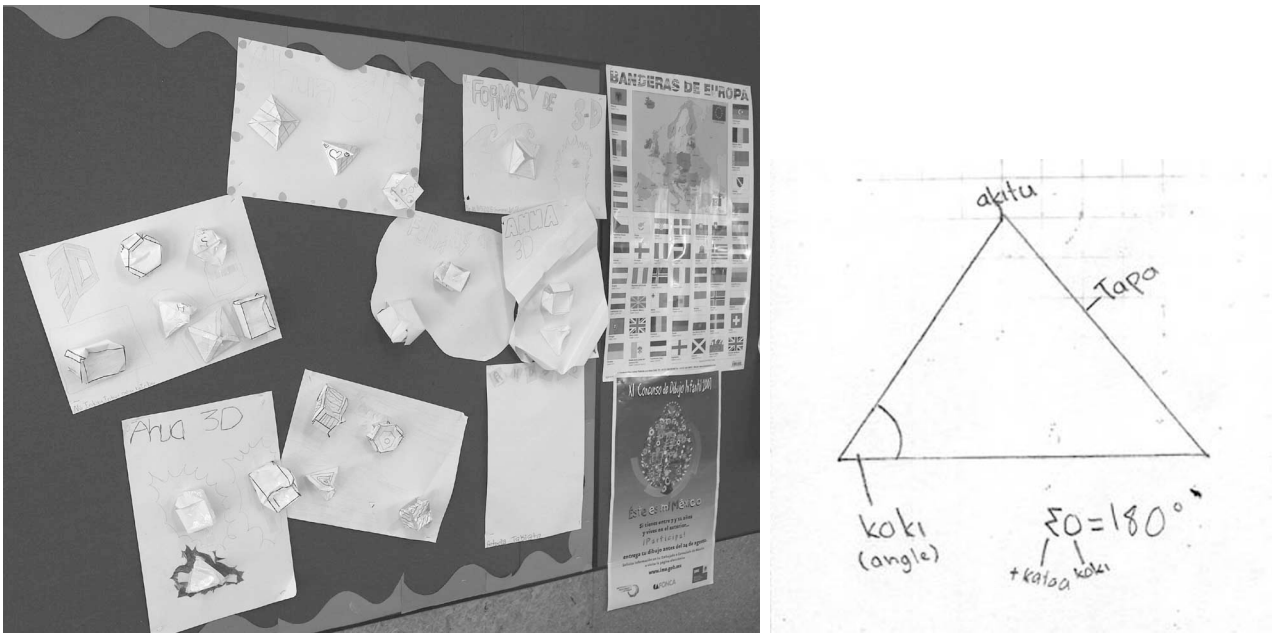

Figure 2: Student work from Year 7

In kura kaupapa Mäori, teaching mathematics is done in te reo Mäori. Although this is the second language of most students, the school community considers this essential if te reo Mäori is to be pulled back from the brink of extinction. Nevertheless, the choice to teach in te reo Mäori has not been without its challenges. However, through open discussion these challenges are being overcome.

School Mathematics in a Learning landscape in a kura kaupapa Mäori

Learning is considered as "the process in which persons make the decision of engaging in getting to know" (Alrø, Skovsmose & Valero, 2007, p. 2). The learning landscape therefore provides information about the influences on this decision making act and the model provides an important tool in this analysis. This is because it allows the complexity of the situation to be acknowledged without overwhelming the analysis of it.

In the case study discussed above, the act of learning that students participate in is heavily influenced by the historical background to the setting up of kura kaupapa Mäori and the subsequent consequences for mathematics teaching. The recognition of the loss of te reo Mäori and the poor academic outcomes for Mäori students in mainstream schools was very much part of the public discourse for the Mäori community in the 1980s (Bishop & Glynn, 1999). Public discourse, therefore, contributed to the identification of alternatives for Mäori students. In this situation, public discourse did not constrain the discussion about what was normal/possible rather it broadened many parents' perceptions.

However, the decision to teach mathematics in a language where a mathematics register had to be engineered did have implications for the learning that could occur in classrooms (see Meaney, Fairhall & Trinick, 2008 forthcoming). The classroom dialogue identified how difficult students found learning new vocabulary and

grammatical expressions of the mathematics register. Although the earlier discussion between parents and teachers showed an awareness that this might be the case, they had believed that this was little different to the difficulties that students would face in learning the mathematics register in any language. They also felt that because of the changes that occurred from generation to generation in approaches to teaching mathematics that the mathematics register that their children learnt was likely to be different to that they had learnt at school. The parents' and the teachers' perceptions showed an awareness of the potential difficulties faced by students in using the relevant school mathematical discourse. However, no one believed that these were insurmountable difficulties.

It could be said that the strong beliefs in their students' capabilities contributed to the ways that the teachers found to support their learning. The strategies that teachers felt were valuable to their students in acquiring the mathematics register, te reo Tätaitai, included ones that supported their students becoming self-regulatory in their learning. Over the course of our 2005-2006 project, teachers adopted strategies where they "provid[ed] students with and ha[d] them describe a rationale for their learning" (Meaney, Fairhall & Trinick, 2007, p. 78).

In this kura being multilingual was also seen as something to be valued and the teachers made use of the linguistic resources that the students had available through their knowledge of te reo Mäori, English and Spanish. Resources can only be aids to learning if they are perceived as being valuable. The teachers also considered mastery of mathematical symbolism as something that would contribute to students' foregrounds. Although 'x' and 'y' as algebraic terms were not used in te reo Tätaitai because these letters are not part of the Mäori alphabet, the mathematical reasons behind using letters in algebraic reasoning was discussed. The concept of using letters as variables was therefore seen as a transferable concept that students could make use of in any language that they operated in.

The model in Figure 1 can be used to show how different aspects of a learning landscape can be built up and interact. This means that particular aspects that connect with school mathematical discourse can be identified without isolating them away from other aspects that they interact with.

BRINGING THE CASES TOGETHER

An analysis of these case studies using the model presented in Figure 1 highlights the interconnections between different aspects of the stories. School mathematical discourse is a thread that runs through any learning landscape. However, in the situation where the students are learning mathematics in a second language, the linkages that it provides between other aspects of the learning landscape are more obvious than when the students are using their first language as the language of instruction. By using aspects of the learning landscape as an analysis tool it is possible to not only identify challenges but also to see how these challenges can be successfully managed.

May, Hill, Tiakiwai (2004) made the following statement about different approaches to bilingualism:

> Māori-medium contexts foster *additive* bilingualism – that is, their goal is to acknowledge and promote bilingualism, and its attendant advantages. In contrast, English-only submersion programmes are invariably subtractive bilingual contexts – that is, they specifically devalue and exclude the child's bilingualism, and thus also the potential resource such bilingualism is to learning (p. 49).

The two case studies described in this paper show how beliefs about being bilingual can contribute to perceptions of success or failure within a learning landscape by students, teachers and family members. When bilingualism is seen as something valuable a range of opportunities are presented to support students in making decisions to engage in getting to know. When bilingualism is seen as problematic, then opportunities can be restricted to gaining fluency in the language of instruction.

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