YOUNG CHILDREN'S REPRESENTATIONS OF THEIR DEVELOPING MEASUREMENT UNDERSTANDINGS

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This paper explores the development of young children's understandings about measurement, and the ways in which children represent these understandings. This paper presents a selection of data gathered during a three-year study that examined young children's engagements with measurement in prior-to-school and out-of-school contexts. In this present investigation, children's representations in the form of drawings and narratives are analysed in relation to a framework of emergent measurement. Initially, this paper considers the understandings about measurement which children are demonstrating in alignment with the framework, before offering a selection of data which represents a disruption to the framework and contests existing ideas about young children's measurement understandings.

Background

Clements and Stephan (2004) have suggested that understandings of measurement begin to develop in the prior-to-school years. Young children know that continuous attributes such as mass and length exist, although they may not be able to quantify or measure them accurately (Clements & Stephan, 2004). However, by about four or five years of age, most children begin to make progress in reasoning about and measuring quantities by overcoming perceptual cues and learning to use words that represent quantity of a certain attribute (Clements & Stephan, 2004). Children then learn to compare two objects directly and recognise equality or inequality (Boulton-Lewis, Wilss & Mutch, 1996). At this point, children are ready to learn to measure by connecting number to quantity (Clements & Stephan, 2004). Typically, students first learn to measure using informal units before progressing to the use of formal units. Although researchers debate the order of the development of these concepts and the ages at which they are developed, they tend to agree that these ideas form the foundation for measurement understanding (Stephan & Clements, 2003).

There are many developmental sequences for measurement learning presented in the literature, but most are similar in their progression from identification of the attribute and use of informal measurement through to the use of formal units. Three examples sequences of measurement learning are that of Clements and Stephan (2004); Piaget, Inhelder, and Szeminska (1960); and Board of Studies NSW (2002). These three perspectives on the development of children’s measurement understandings bear noticeable similarities. Table 1 summarises the key points of each of the three developmental sequences.

Table 1. Measurement learning frameworks of Clements and Stephan (2004), Piaget et al. (1960), and Board of Studies NSW (2002).

Clements and Stephan (2004)	Piaget et al. (1960)	Board of Studies NSW (2002)
Awareness of continuous attributes, but unable to quantify or measure accurately.		
Use of words that represent quantity of an attribute, direct comparison, and recognition of equality or inequality.		Identifying the attribute and comparison.
	Not capable of measurement, construction of units is impossible.	Informal units.
Connect number to quantity, identify unit of measure, and measure through unit iteration.	Ability to use a common measure, use of unit iteration.	Formal units.
	Direct measurement is possible.	Applications and generalisations.

When we look at the summary of the measurement learning frameworks of Clements and Stephan (2004), Piaget et al. (1960), and Board of Studies NSW (2002), it could be said that the framework can effectively be divided into two levels, these being *emergent* measurement and *proficient* measurement. *Emergent* measurement encourages children to develop an understanding of measurement by using it for their own purposes, talking about their measurement ideas, representing measurement processes in ways which make sense to them, and becoming more aware of their own measurement thinking (Whitebread, 2005). By contrast, *proficient* measurement requires: comprehension of measurement concepts, operations and relations; skills in carrying out procedures flexibly, accurately, efficiently and appropriately; ability to formulate, represent and solve problems; and capacity for logical thought, reflection, explanation and justification (Kilpatrick, Swafford & Findell, 2001). This study explores children’s emergent understandings and how these are leading into more proficient understandings.

Research design and methods

Participants

The data were collected at two schools in regional New South Wales. The participant children had just commenced their first year of formal schooling, known as Kindergarten in NSW. Children in NSW commence Kindergarten in late January. They “must start school by the time they are 6 years old but they may start in the year that they turn 5, provided their fifth birthday is before July 31 of that year. Hence, it is possible for a new Kindergarten class to contain children aged between 4 years 6 months and 6 years” (Perry & Dockett, 2005, p. 65).

Data collection

Data collection took place during February and March 2009, at the start of which the children had been at school for approximately two weeks. It was confirmed by all of the Kindergarten teachers that no formal teaching about measurement had taken place in the classroom up to this point in time, or throughout the data gathering period. The children were asked to complete a series of six drawing tasks relating to different measurement concepts, and provide a description of each drawing. This description was annotated on the drawing, and both the drawing and annotation were considered as a whole. The tasks were designed to progress from an open-ended exploration of what children themselves considered measurement, through to investigations of specific content areas and concepts. The tasks, and the measurement content and concepts they address, are outlined in Table 2.

Table 2. Drawing tasks and the measurement content and concepts they address.

Drawing task	Measurement content	Measurement concept
Task 1: Draw yourself measuring	Open-ended	Awareness of attributes
Task 2: Draw something tall and something short	Length	Comparison
Task 3: Draw something heavy and something light	Mass	Comparison
Task 4: Draw something hot and something cold	Temperature	Comparison
Task 5: Draw a ruler	Length	Unit iteration
Task 6: Draw a clock	Time	Unit iteration

Data analysis

Analysis in this study was based on the common elements across the representative developmental progressions shown in Table 1. It can be seen in Table 1 that notions of *attributes* and *comparisons* are common to both the Clements and Stephan (2004) and Board of Studies NSW (2002) progressions, while *units* are common to all three. Table 3 provides the resultant framework for analysis of the children's measurement content knowledge. Decisions were made as to which, if any, of these elements and descriptors were represented by each drawing and its description, and the data was coded accordingly.

Table 3. Framework for analysis of emergent measurement understandings.

Element	Descriptors
Attributes	Understanding that objects have attributes which can be measured.
Comparisons	Understanding that the key idea is to compare like attributes. Comparing objects directly. Multiple comparisons of objects.
Units	Recognition of units. Sequencing of units. Equal partitioning of units.

Results and discussion

The three elements of *attributes*, *comparisons*, and *units*—and their corresponding descriptors—form the basis of this discussion. Woven throughout the discussion are the descriptions of the drawings given by the children.

Awareness of measurement concepts and attributes

This section explores the children's responses to Task 1, the "Draw yourself measuring" task, with discussion based on the "Attributes" descriptor of the measurement framework.

Understanding that objects have attributes which can be measured

With respect to Task 1, the drawings collectively represented the concepts of area, length, mass, and temperature. Length was the most commonly represented concept, followed by area, mass and temperature respectively. This was not surprising because length is the measurement concept most easily understood by young children due to it being the most concrete, visual measurement concept for children to perceive (Gifford, 2005). As a result, the process of length measurement is also the most tangible and direct measuring process for young children. Indeed, the majority of the children in this study described their drawings as them finding out how "tall" or "long" the object being measured was. For example, Imogen described her drawing as "I'm measuring a piece of paper. I'm getting a ruler. I'm finding out how long it is".

In addition to this notion of using measurement to "find out" an object's properties, some children described contextualised applications of measurement. These drawings were highly personalised, with rich accompanying narratives. For example, Zofi drew a picture of herself testing the temperature of the bath water in preparation for a bathing a baby: "I'm measuring the bath and the baby's helping me. I'm checking the water".

Despite the fact that the task asked children to "Draw *yourself* measuring", most children showed measuring being carried out by others (usually their parents). For example, Caitlin drew "A brick wall and my Mum. She's measuring it to find out how big it is, with a measuring tape".

Ability to compare measurable attributes

The comparison descriptions of the emergent measurement framework are organised to reflect a progression in understanding about comparison. These comparison skills will be discussed in relation to children's responses to Tasks 2, 3, and 4.

Understanding that the key idea is to compare like attributes

With regard to Task 2—"Draw something tall and something short"—almost all of the students were able to represent objects relating to the attribute of height and identify whether an object was "tall" or "short". For example, Brodie drew "a short box and a tall box", while Lachlan drew two cars, explaining "This car's short. This one's tall".

When considering Task 3, the "Draw something heavy and something light" task, children were required to identify the attribute of mass by describing objects as either "heavy" or "light". Most of the children were able to do so, such as Ella who drew "A big heavy bookshelf. A feather is light", and Annabelle, who said "A leaf is light. A rock is heavy".

Finally, Task 4 asked children to compare objects in relation to their temperature, describing the objects as either “hot” or “cold”. For example, Angel stated “The sun is hot. Snowballs are cold”, while Blake said “The pool is cold. Lava is hot”.

Comparing objects directly

As evidenced by the preceding examples, children demonstrate a basic understanding of comparison by applying dichotomous descriptors of an attribute to objects. However, moving beyond this simplistic understanding is the notion of *direct* comparison, and the use of more sophisticated comparative language. In the case of Task 2, direct comparison of height could be evidenced by the positioning of the objects along a common baseline. The majority of the children were able to represent objects in this manner and state which was the taller/shorter of the two, as did Sarah, who drew a tree next to a volcano and stated “the tree is taller”.

When considering Task 3, the direct comparison of mass was evidenced by the children’s descriptions of comparing the masses of objects by lifting them – a process known as *hefting*. For example, Blake explained “A cat is light and a motorbike is heavy. I tried to pick up my cat once and it was light. I couldn’t pick up my motorbike because it was too heavy”.

Similar to Task 3, direct comparison was evident in responses to Task 4 with the children describing “feeling” objects to compare their temperatures. For example, Jurre described his drawing as “This is a sun with lots of arms and it is melting lots of ice blocks. The sun is hot. The ice blocks are cold. I know that because I feeled them”.

Multiple comparisons of objects

When considering progression in understanding about comparison, at the most sophisticated level children demonstrate an ability to compare more than two objects. The three comparison tasks given to the children did not explicitly ask them to draw more than two objects, however many children in fact chose to do so.

Task 2 required children to make multiple comparisons of objects on the basis of height. It was expected that the children represent their chosen objects in order along a common baseline, identifying which was the tallest and/or shortest. For example, Chelsea drew four flowers in order of height, and in her description she identified which was the tallest and which was the shortest. Similarly, Caitlin drew three mermaids in order of height and identified which mermaid was the tallest.

When looking at the responses to Task 3, it is interesting to note that—unlike with Task 2—very few students extended the task to making multiple comparisons of objects. Some of these children drew multiple objects and classified each of them in regards to “heavy” or “light” descriptors, while others were able to use more sophisticated comparative language relating to the ordering of objects, for example Andrew, who drew a cat, a ladder and a person and stated “The ladder is the heaviest”.

Similar to the responses to Task 3, there were also very few responses to Task 4 which represented multiple comparisons of objects. The majority of these responses were limited to simply drawing several objects and classifying them as “hot” or “cold”, or in some cases, the additional terms of “very hot” and “very cold”. A small number of children showed a sense of ordering in their responses, such as Abby who explained “This person is hot. This person is hot and cold – autumny. This person is cold. They’re outside and they don’t know how to get home”.

Knowledge about unit structure and iteration

The units component of the emergent measurement framework consists of three descriptors which reflect a progression in understanding about units. This progression in understanding about units will be discussed in relation to children's responses to the two tasks focusing on units, Tasks 5 and 6.

Recognition of units

In relation to Task 5—"Draw a ruler"—at the most basic level of understanding the children were able to recognise the units on a ruler by making reference to either the numbers or the lines on a ruler. At this level, the children typically did not accurately represent the units and instead attempted to show the units by using dots or similar. Other children were able to describe their recognition of units but were unable to represent this in their drawing, such as Krystal who said "They have numbers. We can count them. I don't know how to make the numbers on a ruler".

Similarly with Task 6—"Draw a clock"—at the most basic level of understanding the children were able to describe either the numbers or the lines on the clock, but had some difficulty in representing these. For example, Mikayla explained "It has numbers but I'm not sure how to draw them", while Lilli used circles to represent the numbers on a clock, stating "It has numbers to see what the time is".

Sequencing of units

At the next level of understanding, the children showed evidence of units with a sense of sequencing, but not necessarily evenly partitioned or with numbers in the correct order. For example, in her response to the "Draw a ruler" task, Jade wrote numbers at one end of her ruler, stating "It has numbers on it to see how much it is. No, how *long* it is".

When looking at the responses to the "Draw a clock" task, the children typically attempted to write numbers around the outside of their clock face, but did not accurately represent the numbers 1 to 12 with even partitioning, or in some cases, continued numbering beyond 12. For example, Hannah wrote numbers halfway around her clock and stated "A clock has 10 numbers. The numbers are so you see what time it is", while Makaylee wrote the numbers 1 to 12, but upon discovering that they did not go all the way around the clock she continued the sequence of numbers, explaining "It has numbers that go all the way around. They don't stop, they have to go all the way around".

Equal partitioning of units

At the most sophisticated level of understanding, the children were able to demonstrate equal partitioning of units and represent this is in a spatially appropriate manner, i.e. along the full length of the ruler, or appropriately positioned around the clock face. Additionally, when the children included numbers as representations of units, typically the numbers were placed in the correct order. In response to Task 5, Blake chose to represent the units on a ruler by using equally partitioned lines, alternating between long and short lines as is often seen on a standard ruler, and he was able to explain that "The lines are for the numbers that tell how big the paper is".

With regard to Task 6, many children were able to represent a stereotypical clock face with some accuracy by evenly partitioning the numbers 1 to 12 around the clock face in a spatially appropriate manner. Many children also used lines to demarcate the units on a clock face, such as Blake who described his drawing as "This is a clock at my home. It doesn't have numbers – it has little lines".

Disruptions to the framework

As the previous data has shown, many of the children in this study were able to demonstrate understandings about measurement that aligned to the framework of understanding for young children. However, there was a significant body of data from both of the participant schools which represented "disruptions" to this framework – that is, the children were presenting knowledge and skills which were not in alignment with the expected development for children of their age.

One such disruption evident in the data was the integration of measurement concepts. Rather than understanding measurement concepts such as length, mass, etc in isolation, many of the children actually integrated concepts and in this way, used one concept to understand another. For example, in her response to the "Draw something heavy and something light" task, Annabelle used her understanding of area to contribute to her understanding of mass, explaining "Big blocks are heavy. Little blocks are light". Lachlan blended both area and mass in his "heavy and light" drawing, explaining "When things are big, they are heavy. The bigger they are, the heavier they are".

Another significant disruption was the children demonstrating an understanding of the measurement process at the start of school, far sooner than would generally be expected. When the children first completed the "Draw yourself measuring" task at the beginning of Kindergarten, it was evident from the children's drawings and descriptions that despite the fact that the children had not engaged in any *formal* measurement learning experiences at this point in time, their personalised engagements with various people and in various contexts had contributed to an emerging understanding of the measurement process. To begin, many of the children showed an understanding of using direct comparison when measuring. Abby (Figure 1) compared the lengths of two pencils by placing them side by side with the ends aligned: "I am measuring two pencils. I put them beside each other to see which is the longest".

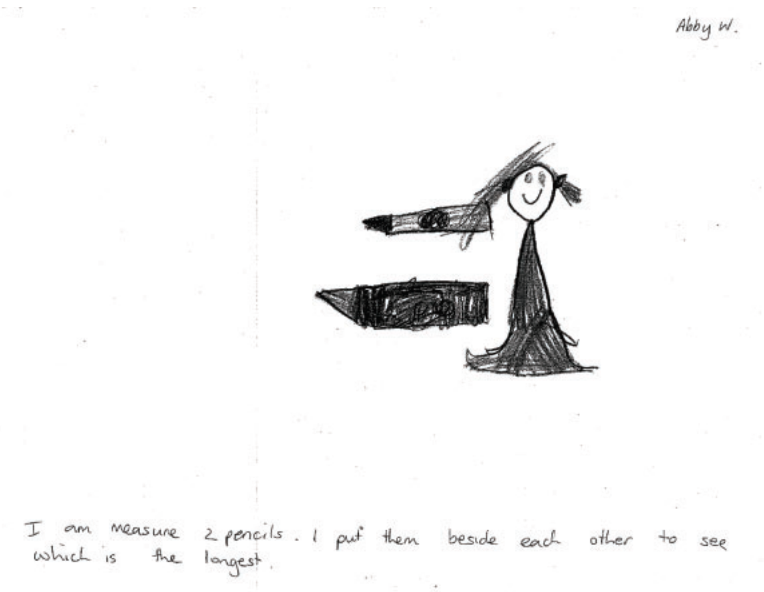

Figure 1. Abby's drawing.

Similarly, Kody was able to compare himself to a big rock, explaining "This is me against a big rock. The rock is bigger".

The children also showed more sophisticated understandings of the measurement process by moving beyond direct comparison to notions of using measuring tools. Most of the children made reference to a generic "measurer", for example, Emily-Rose explained "I'm measuring a house with a measurer. You just put it on something and see how long it is". However, some of the children described using standardised measuring tools including tape measurers and rulers, such as Chloe, who described her drawing as "I'm measuring a dog with a measuring tape to find out how big it is". The children were also attempting to use notions of quantity and units at this early stage. In Chloe's drawing, it can be seen that her measuring tape has been represented as a sequence of numbers, which are equally partitioned. Lara similarly showed a sequence of equally partitioned numbers on her measuring tape, used to measure a tree.

A number of students were able to show more sophisticated understandings of measuring than would be expected by attempting to identify a quantity in relation to formal units. For example, Kyra used her understanding of the measurement process to articulate a measurement of height: "I am measuring Mrs M. I use a pencil to draw a line against a measurer with a giraffe on it – at my house. She is 6 metres tall!"

Importantly, there were some children who could demonstrate how their measurements were reached. As shown in Figure 2, Jurre was able to line up the end of his measuring tape with the end of the car in order to determine a measurement of 15, while Brodie was able to articulate a process of counting units, saying that "There are 33 spaces around my car. I counted the spaces".

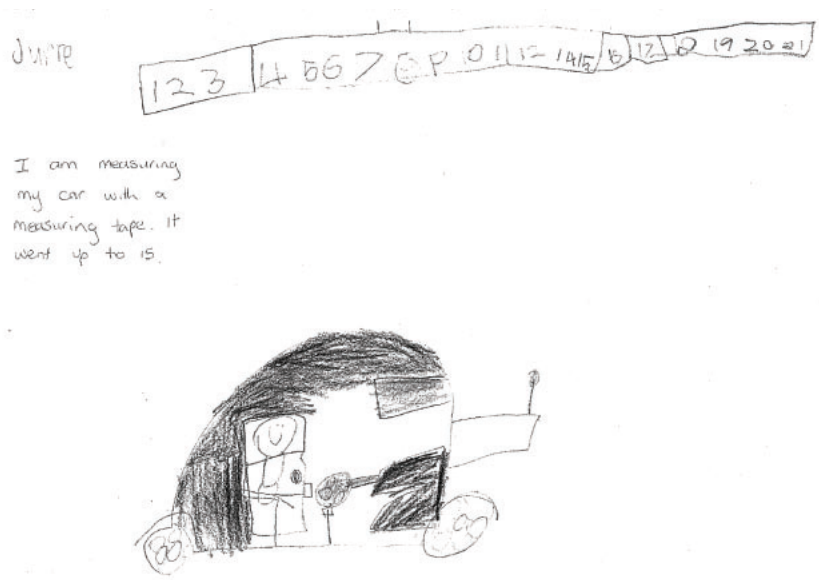

Figure 2. Jurre's drawing

Interesting applications of formal units included Willis who explained "I am measuring a clock tower with a long ruler. It is 16 kilometres"; and William, who described his drawing as "I am measuring a clock with a measurer. It is 12 megalitres". While these formal units were, for the most part, used inappropriately, it is important to note that even at this early stage of schooling the children could see a need for formal units.

Conclusions and implications

Results in this study have shown that young children have highly sophisticated understandings of measurement. These understandings both align with, and challenge, extant frameworks for the development of measurement knowledge. Within an emergent measurement context, these children have shown understandings about the measureable attributes of objects, comparisons of attributes, and the application of units. With particular regard to units, it is important to note that the children show a remarkable awareness of a range of formal units, including some that they would not normally be expected to have an awareness of, i.e. megalitres. It is also important to highlight that children have individualised, idiosyncratic ways of understanding measurement concepts, such as using one attribute to understand another, i.e. comparing areas in order to compare masses. Of crucial significance is that these are the understandings which children have developed *for themselves* in prior-to-school and out-of-school contexts, and educators must recognise and build on these existing understandings so as to make the in-school measurement learning relevant and meaningful.

The notion of representing measurement understandings in a visual form has widespread classroom applications beyond simply as a data gathering technique. The drawing activities described in this paper could easily be adapted to classroom practice, and such an adaptation would allow teachers to both recognise and extend the understandings about measurement which children possess; assess children's understandings about particular measurement concepts; discover information about the contexts and experiences that influence children's developing understandings; and gain insight into the personalised ways in which children construct measurement understanding.

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