This paper reports preservice teachers’ Specialised Content Knowledge (SCK) on multiplication of decimals. Ninety first year Bachelor of Education (Primary) preservice teachers enrolled in a regional university in New South Wales, Australia sat for a Mastery test, which required them to demonstrate their understanding of mathematics concepts. This paper focuses on an item in the Mastery test that required demonstration of the knowledge of multiplication of decimals. An analytical tool, designed by the authors, was used to analyse the data collected. This tool was designed according to four criteria of which three, are elements of SCK suggested by Lin, Chin and Chiu (2011): Correctness of answer, Justification, Explanation, and Representation. The findings suggest that mathematics pedagogy subjects in preservice teacher education programme could focus on enhancing preservice teachers’ SCK through more emphasis on the three elements of SCK.

Specialized Content Knowledge, Multiplication of Decimals, Justification, Explanation, Representation

INTRODUCTION

This paper documents preservice teachers’ Specialised Content Knowledge (SCK) on the topic of multiplication of decimals through their responses to one test item in a ten-item mathematics mastery test. This mastery test forms part of the assessment criteria that preservice teachers need to fulfil in their first year Bachelor of Education (Primary) degree in a regional university in New South Wales, Australia. In this test, preservice teachers were not only required to answer basic mathematics skills questions, they also needed to demonstrate their knowledge on the conceptual understanding of mathematics topics stipulated in the New South Wales K-6 mathematics syllabus.

SPECIALIZED CONTENT KNOWLEDGE

Teacher’s mathematics content knowledge and pedagogical knowledge are crucial for providing high quality teaching in classrooms (Fennema & Franke, 1992; Ball, Thames & Phelps, 2008; Cheang, Yeo, Chan, Lim-Teo, Chua & Ng, 2007; Silverman & Thompson, 2008). In particular, preservice teachers’ mathematical knowledge has been an international concern since the last two decades. As a consequence, there has been a growing number of research that investigated preservice teachers’ mathematical knowledge (e.g., Ball, 1990; Goulding, Rowland & Barber, 2002; Levenson, Tsamir and Tirosh, 2010). Tobias and Itter (2007) found the mathematical knowledge of preservice teachers in regional Australia to be
lacking. In fact, the study found that not only do these preservice teachers have inadequate understanding of concepts and principles in the primary curriculum, they also made errors and misconceptions which were often identified in primary school students.

In Shulman’s seminal work, he (1986) suggested that teacher knowledge consisted of subject knowledge, pedagogical content knowledge and curricular knowledge. Ball, Thames and Phelps (2008) extended Shulman’s definition of teacher knowledge by coming up with a working definition of what “mathematical knowledge for teaching” (i.e., the mathematical knowledge that teacher need to carry out their work as teachers of mathematics) meant. In particular, they defined Specialised Content Knowledge (SCK) to be knowledge that involves “mathematical knowledge beyond that expected of any well-educated adult but not yet requiring knowledge of students and knowledge of teaching (Ball, Thames and Phelps, 2008, p. 9). In short, SCK is the mathematical knowledge and skill unique to teaching, such as conceptual understanding of mathematics and looking for patterns in students’ error (Ball, Thames and Phelps, 2008; Ball et. al., 2009). Conceptual understanding of mathematics consists of logical relationships constructed internally and existing in the mind as a part of a network of ideas (Van De Walle, 2004). It reflects an understanding of why a procedure works (Hiebert & Wearne, 1986) or of whether a procedure is legitimate (Bisanz & Lefevre, 1992). For example, most people “add a zero” to the number when multiplying by ten and “move the decimal point one digit to the left” when dividing by ten. An accountant, for example, does not need to give a mathematical reason for moving the decimal point but a teacher need to do so in classroom teaching. Not only do teachers need to know how to do the mathematics, they also need to unpack and explain to students why these procedural rules work (Ball, Thames and Phelps, 2008). In order to unpack this mathematical knowledge, teachers need to know the principle of decimal system:

The value of a digit is a function of its position in the numeral. The value of a particular position is determined by beginning with the unit and, if moving to the right, dividing the previous value by 10 and, if moving to the left, multiplying the previous value by 10. The ones position is marked with a decimal point on its immediate right. One consequence of this principle is that the relationship between any two adjacent places is that the one on the left is worth 10 times as much as the one on the right or, alternatively, the one on the right is worth one tenth as much as the one on the left.

(Hiebert, 1992; p.286)

Thus, specialized content knowledge refers to the knowledge that requires conceptual demands of mathematics different than the mathematical understandings needed by other practitioners of mathematics (Silverman & Thompson, 2008).

THE PROBLEM

In order to facilitate preservice teachers’ SCK, the first year mathematics pedagogy subject was designed for enhancing their understanding of conceptual knowledge of mathematics and conceptual errors related to specific mathematics ideas. All the first year Bachelor of Education (Primary) preservice teachers who enrolled on this subject had to sit for a ten-item Mathematics Mastery Test in which they were required to demonstrate their competency.
Four marks were rewarded for complete answers for each item for ten items in this test and it made up forty marks the full mark. It was reflected by the markers that the marking rubric was unable to give clear ideas of and guidelines for what conceptual understanding the markers should look for in preservice teachers’ responses. It was also hard to decide how many marks (i.e., 1, 2 or 3) should be awarded for incomplete or partly correct answers. In addition, the lecturers found that mere the scores could not inform them how much and what kind of conceptual understandings the preservice teachers had learnt. As a result, not much feedback could be given to preservice teachers for improvement. To address this issue, this paper aims at (1) trialling an analytical tool for marking the test items; and (2) providing a framework for analysing preservice teachers’ conceptual understanding.

THE PARTICIPANTS

Ninety first year Bachelor of Education (Primary) preservice teachers in a regional university in New South Wales, Australia, sat for a Mathematics Mastery test in May of 2010. This mastery test forms part of the assessment criteria that these preservice teachers need to fulfil in their degree course. The mastery test consisted of ten questions which included both basic mathematics skills questions that require them to show their understanding of how to teach certain mathematics concepts. This paper focuses on one test item in the Mastery test, as shown in Table 1. This item pertains to the Number Strand in the New South Wales K-6 curriculum. Note that the instruction to the preservice teachers was to provide all possible explanations when answering the test items.

Table 1: Item in the Mastery Test

<table>
<thead>
<tr>
<th>Item number</th>
<th>Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Susan got the answer 0.9 to the question 0.3 × 0.3.</td>
</tr>
<tr>
<td></td>
<td>Is she correct?</td>
</tr>
<tr>
<td></td>
<td>Why do you think so?</td>
</tr>
</tbody>
</table>

METHOD

In a recent study, Lin, Chin and Chiu (2011) suggested that Specialized Content Knowledge (SCK) consisted of three elements, namely,

*Explanation* - how to provide mathematical explanations for common rule and procedures

*Representation* – how to choose, make and use mathematical representations effectively and accurately

*Justification* – how to explain and justify one’s mathematical ideas

To address the above stated issue, an analytical tool was designed. This tool was used to analyse the pre-service teachers’ written responses to this test item. This analytical tool (See...
Figure 1) consisted of four components – Correctness of answer, Justification, Explanation, and Representation. Among the four components, three were elements of SCK, as suggested in a recent study by Lin, Chin and Chiu (2011).

In Figure 1, the number in the parentheses represents the marks awarded for displaying the corresponding response or understanding. Each pre-service teacher’s solution to the item was first checked for whether the answer to the item was correct or incorrect [Correctness of answer]. If the answer was incorrect, zero mark was awarded. If the answer was correct, 1 mark was awarded. Next, the solution was checked for completeness of the justification given to the correct answer given. Solutions that had ‘correct and complete justifications’, and ‘correct answer and incomplete justification’, were further checked for explanation. That is, whether it was a procedural explanation, a conceptual explanation, or both procedural and conceptual explanation were given. An explanation that was procedural, in this case, is explanation based on how to execute the multiplication of fractions algorithm. A conceptual explanation involves knowing why the algorithm works. A conceptual explanation was then further analysed whether it was a mathematically-based explanation or a practically-based explanation (see Levenson, Tsamir & Tirosh, 2010). A mathematically-based explanation is “based on mathematically definitions or previously learned mathematical properties, and often use mathematical reasoning” (Levenson, Tsamir & Tirosh, 2010, p. 346). A practically-based explanation is one which uses “daily contexts and/or manipulatives to “give meaning” to mathematical expressions” (Levenson, Tsamir & Tirosh, 2010, p. 345).

Figure 1. Analytical tool

Both authors of this paper coded and scored each of the ninety-two pre-service teachers’ written responses. The coding and scoring by each author was checked by the other author. Any discrepancy in coding was discussed and a final coding and score were agreed by both authors.
RESULTS

This section discusses the results of using the analytical tool to code the preservice teachers' responses to the test item. Majority of preservice teachers were found to score three marks for this test item. The results indicate that this cohort of preservice teachers did not perform well in this test item.

Table 2 shows the mean scores for each element of Specialized Content Knowledge (SCK). From Table 2, the mean score of the test item is only 2.63 out of 8, which is rather low. It indicates that preservice teachers’ general conceptual understanding of the multiplication of decimals is far from satisfactory. When analysing the three elements of SCK separately, the mean score was 1.72 out of 3, 0.73 out of 3 and 0.19 out of 2 for Justification, Explanation and Representation respectively.

![Table 2: Mean scores for the elements of SCK](image)

The results of the analysis will be presented and discussed according to these three elements of SCK - explanation, representation and justification.

**Justification**

Table 3 shows the frequency and percentage of preservice teachers who gave the correct response to the question in the test item, “Is she correct?” Table 4 shows the frequency and percentage of preservice teachers’ completeness in justification of their response to the test item. 74 out of 90 preservice teachers (see Table 3) responded that 0.9 is an incorrect answer for 0.3 x 0.3. However, only 28 of these 74 preservice teachers (see Table 4) gave a complete justification for their response. 25 of these 74 preservice teachers gave an incomplete justification. 21 of these 74 preservice teachers did not provide a correct justification for their response.

![Table 3: Frequency table of Justification component – Correctness of response](image)
Table 4: Frequency table of Justification component – Completeness of response

<table>
<thead>
<tr>
<th>Response</th>
<th>Number of good tables</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incorrect</td>
<td>37</td>
<td>41.1</td>
</tr>
<tr>
<td>Incomplete</td>
<td>25</td>
<td>27.8</td>
</tr>
<tr>
<td>Complete</td>
<td>28</td>
<td>31.1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>90</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>

**Explanation**

As already mentioned in the Justification subsection above, 74 preservice teachers gave the correct response that $0.3 \times 0.3$ is not equals to 0.9. 21 out of these 74 (or 28.4%) preservice teachers did not give a correct explanation to their response.

Table 5 shows the types of explanations that these 74 preservice teachers wrote in their test scripts. Majority of the 74 preservice teachers gave a procedural explanation, followed by a conceptual-mathematically based explanation. Only 3 preservice teachers provided a conceptual-practically based explanation. Two preservice teachers gave two types of explanations, procedural and conceptual-mathematically based. Four preservice teachers gave a procedural and conceptual-practically based explanations. The instruction given for the mastery test was to provide all possible explanations to the test items. Only 2 preservice teachers provided all three types of explanations.

Table 5: Types of explanations

<table>
<thead>
<tr>
<th>Types of explanations</th>
<th>Number of preservice teachers (out of 74 preservice teachers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Procedural</td>
<td>24 (32.4%)</td>
</tr>
<tr>
<td>Conceptual-Mathematically based</td>
<td>16 (21.6%)</td>
</tr>
<tr>
<td>Conceptual-Practically based</td>
<td>3 (4.05%)</td>
</tr>
<tr>
<td>Procedural and Conceptual-Mathematically based</td>
<td>2 (2.7%)</td>
</tr>
<tr>
<td>Procedural and Conceptual-Practically based</td>
<td>2 (2.7%)</td>
</tr>
<tr>
<td>Conceptual-Mathematically based and Conceptual-Practically based</td>
<td>4 (5.4%)</td>
</tr>
<tr>
<td>Procedural and Conceptual-Mathematically based and Conceptual-Practically based</td>
<td>2 (2.7%)</td>
</tr>
</tbody>
</table>
Incorrect explanations 21 (28.4%)

**Representation**

Table 6 shows the types of representations used by preservice teachers who gave conceptual-practically based explanation(s) in their test scripts. As already mentioned in the above Explanation section, 3 preservice teachers provided conceptual-practically based explanations. All of them provided pictorial representations only.

Table 6: Types of Representations used by preservice teachers who gave conceptual-practically based explanation(s) in their test scripts

<table>
<thead>
<tr>
<th>Types of explanations</th>
<th>Number of preservice teachers (out of 74 preservice teachers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptual-Practically based, Pictorial Representation</td>
<td>3 (4.05%)</td>
</tr>
<tr>
<td>Procedural and Conceptual-Practically based, Pictorial and Abstract Representation</td>
<td>2 (2.7%)</td>
</tr>
<tr>
<td>Procedural and Conceptual-Mathematically based and Conceptual-Practically based (Pictorial and Abstract Representation)</td>
<td>2 (2.7%)</td>
</tr>
<tr>
<td>Conceptual-Mathematically based and Conceptual-Practically based (Pictorial and Abstract Representation)</td>
<td>4 (5.4%)</td>
</tr>
</tbody>
</table>

Also mentioned in the above Explanation section, two preservice teachers, gave two types of explanations, namely procedural and conceptual-practically based explanations. Both preservice teachers, Kristina and George, included a pictorial representation in their conceptual-practically based explanation. Kristina presented both pictorial and abstract representations. Figure 5 shows Kristina and George’s pictorial representations for their conceptual-practically based explanations. Kristina drew a pictorial representation for 0.3 x 0.3. George drew a pictorial representation for 0.1 x 0.1 and then stated that the method would work the same for 0.3 x 0.3 (see Figure 2).
Two preservice teachers gave both conceptual-mathematically based and conceptual-practically based explanations. The representations that they drew to explain their response were both pictorial and abstract representations. The two preservice teachers who included all three types of explanations procedural, conceptual-mathematically based and conceptual-practically based, provided both abstract and pictorial representations for their conceptual-practically based explanations.

DISCUSSION AND CONCLUSION

The findings in the study have important implications for teacher education programs. This study found that approximately 82% of the 90 preservice teachers in the Bachelor of Primary Education (Primary) course responded correctly that 0.9 is an incorrect answer for $0.3 \times 0.3$. Although majority of the preservice teachers gave the correct response, they were, in general, unable to provide complete explanations as to why 0.9 is an incorrect answer. Specifically, only approximately a third of these preservice teachers (28 preservice teachers or 31.1%) gave a complete justification to why 0.9 is an incorrect answer for $0.3 \times 0.3$, and slightly less than a third (25 preservice teachers or 27.8%) provided incomplete justifications. This finding echoes the finding from Tobias and Itter's (2007) study who found the mathematical knowledge of preservice teachers in regional Australia to be lacking. Such lacking in mathematical knowledge is definitely a cause for concern for teacher education programs.

There are three types of explanations for $0.3 \times 0.3$ – procedural, conceptual-mathematically based and conceptual-practically based. The study found that majority of the preservice teachers (32.4% of the 74 preservice teachers who provided a correct response to the test item provided a procedural explanation compared to other types of explanations (mathematically based or practically based). This finding is consistent with those from previous research on preservice teachers’ misconceptions about the meaning of multiplication of rational numbers – that many preservice teachers’ understanding is characterized by knowledge of rote procedures rather than by the concepts underlying the procedures (Borko, Eisenhart, Brown, Underhill, Jones and Agard, 1992; Post, Harel, Behr and Lesh, 1991; Simon & Blume, 1994). Only about a third of the preservice teachers (24 preservice teachers) demonstrated the execution of the multiplication of decimals rule (procedural explanation) successfully. This is both a surprising and worrying finding. The reason being these preservice teachers would
Lai and Ho

have already learnt how to execute the rule for multiplication of decimals during their primary schooling.

The study also found that among the preservice teachers who provided a conceptual explanation, more teachers gave a mathematically based explanation than those who gave a practically based explanation. In a recent study, Levenson, Tsamir and Tirosh (2010) found that there were more mathematically based explanations than practically based explanations in teacher-generated explanations. However they preferred using practically based explanations in their classrooms as such explanations would be convincing to their students. As it is part-and-parcel of a teacher’s everyday routine to provide mathematical explanations in their teaching, mathematics pedagogy subjects in teacher education programs could focus on enhancing preservice teachers’ Specialized Content Knowledge (SCK) through more emphasis on the three components of SCK: Justification, Explanation and Representation. By raising preservice teachers’ awareness of these three components of SCK, they would have a better idea of the teaching expectations on them as future classroom teachers, and at the same time be able to know which SCK component they should spend their time (during their teacher-training) to improve on.

The findings in this study cannot be generalised to the whole of regional universities in New South Wales, Australia. Firstly, the analysis using the analytical tool was confined to only one item (multiplication of decimals). Further work is required to test the workability of this analytical tool on other mathematics strands in the New South Wales mathematics curriculum. Also, only the written answers in the preservice teachers’ answer booklets were analysed. More work could be done to reach a greater understanding of preservice teachers SCK on multiplication of decimals. Another study could employ an interview method to investigate preservice teachers’ SCK in the same topic.

As discussed earlier, another objective of designing the analytical tool was to enable preservice teachers have clearer learning goals regarding the specialised content knowledge (SCK) about the topic. A study could investigate the effectiveness of the tool to preservice teachers’ learning.

References


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