This paper reports pre-service teachers’ Specialized Content Knowledge (SCK) on multiplication of fractions. The responses of ninety-two first year Bachelor of Education (Primary) pre-service teachers, enrolled in a regional university in New South Wales, Australia, in a multiplication of fractions mastery test item were analyzed using an analytical tool. This tool, designed by the authors, consisted of four components, of which three are elements of SCK suggested by Lin, Chin and Chiu (2011): correctness of answer, justification, explanation, and representation. Preliminary findings suggest that mathematics pedagogy subjects in pre-service teacher education programmes should expose pre-service teachers to the various explanations and representations of concepts in mathematics.

INTRODUCTION

Pre-service teachers’ mathematics knowledge and pedagogical content knowledge have been an international concern since the last two decades, resulting in a growing number of research in this area (e.g., Ball, 1990; Isiksal and Cakiroglu, 2011; Fennema & Franke; 1992, Goulding, Rowland & Barber, 2002; Hill, Ball & Schilling, 2008, Ball et. al., 2009). The introduction of Mathematical Knowledge for Teaching (MKT) – the mathematical knowledge that teacher need to carry out their work as teachers of mathematics – helped clarify the various types of teacher knowledge involved in a teacher’s work repertoire. Studies on MKT have indicated that teachers’ Specialized Content Knowledge (SCK) is a possible predictor of students’ achievement of mathematics (Hill, Ball & Schilling, 2008). This paper reports preliminary findings on the SCK of pre-service teachers from a regional university in New South Wales, Australia.

MATHEMATICS TEACHERS’ SPECIALIZED CONTENT KNOWLEDGE

In Shulman’s seminal work, he (1986) suggested that teacher knowledge consisted of subject knowledge, pedagogical content knowledge and curricular knowledge. Ball, Thames and Phelps (2008) extended Shulman’s definition of teacher knowledge by coming up with a working definition of MKT. Specialized Content Knowledge (SCK), a component of MKT, is knowledge that involves both the knowledge and skill unique to teaching mathematics, that is, SCK involves conceptual understanding of mathematics concepts and knowledge of students’ errors in mathematics (Ball, Thames, & Phelps, 2008). It should be noted that SCK is a form of knowing mathematics needed by teachers (to explain mathematical concepts and ideas to students) and not needed by those who do not teach (Ball, Thames & Phelps, 2008;
Ball et al., 2009). In a recent study, Lin, Chin and Chiu (2011) suggested that there are three elements of SCK. They are: Justification (how to explain and justify one’s mathematical ideas by rigorous arguments based on mathematical definitions and theorems), Explanation (how to provide mathematical explanations for common rule and procedures), and Representation (how to choose, make and use mathematical representations). Hill, Ball & Schilling (2008) indicated that teachers’ SCK is a possible predictor of students’ achievement. They also noticed that there has been a lack of measures to assess how teacher knowledge is related to student achievement.

Before we could address this concern, we need a measure to assess teacher’s SCK first. In this study, an analytical tool was developed to capture pre-service teachers’ SCK. Preliminary analysis of data in this study was conducted using this analytical tool.

DESIGN AND METHODS

Background of the study

The study involved ninety-two first year Bachelor of Education (Primary) pre-service teachers at a regional university in New South Wales, Australia. One of the objectives of the first year Bachelor of Education (Primary) mathematics pedagogy subject was designed to enhance pre-service teachers’ SCK. As part of the assessment criteria, all the pre-service teachers had to sit for a ten-item paper-and-pencil mathematics mastery test. This mastery test required pre-service teachers to demonstrate their conceptual knowledge of the mathematics topics stipulated in the New South Wales K-6 mathematics syllabus. Each item in the mastery test was four marks. Difficulty was found in deciding how many marks (i.e., 1, 2 or 3) should be awarded for incomplete or partly correct answers. The scores of this mastery test could also not reflect exactly the pre-service teachers’ SCK, that is, which mathematics concepts the pre-service teachers understood or did not understand. As a consequence, the lecturers and tutors could neither give feedback nor provide much help to the pre-service teachers on their performance in the mastery test.

The test item

This paper focuses on one test item in the mastery test which involved multiplication of fractions, that is, $\frac{1}{3} \times \frac{3}{4}$:

$\frac{3}{4}$ of a pizza is left after a party. $\frac{1}{3}$ of the left-overs are given to Sarah to take home. What fraction of the pizza does Sarah take home?

The analytical tool

To address the above stated issue, an analytical tool was designed. This tool was used to analyse the pre-service teachers’ written responses to this test item. This analytical tool (See Figure 1) consisted of four components – Correctness of answer,
Justification, Explanation, and Representation. Among the four components, three were elements of SCK, as suggested in a recent study by Lin, Chin and Chiu (2011). In Figure 1, the number in the parentheses represents the marks awarded for displaying the corresponding response or understanding. Each pre-service teacher’s solution to the item was first checked for whether the answer to the item was correct or incorrect [Correctness of answer]. If the answer was incorrect, zero mark was awarded. If the answer was correct, 1 mark was awarded. Next, the solution was checked for completeness of the justification given to the correct answer given. Solutions that had ‘correct and complete justifications’, and ‘correct answer and incomplete justification’, were further checked for explanation. That is, whether it was a procedural explanation, a conceptual explanation, or both procedural and conceptual explanation were given. An explanation that was procedural, in this case, is explanation based on how to execute the multiplication of fractions algorithm. A conceptual explanation involves knowing why the algorithm works. A conceptual explanation was then further analysed whether it was a mathematically-based explanation or a practically-based explanation (see Levenson, Tsamir & Tirosh, 2010). A mathematically-based explanation is “based on mathematically definitions or previously learned mathematical properties, and often use mathematical reasoning” (Levenson, Tsamir & Tirosh, 2010, p. 346). A practically-based explanation is one which uses “daily contexts and/or manipulatives to “give meaning” to mathematical expressions” (Levenson, Tsamir & Tirosh, 2010, p. 345).

Both authors of this paper coded and scored each of the ninety-two pre-service teachers’ written responses. The coding and scoring by each author was checked by the other author. Any discrepancy in coding was discussed and a final coding and score were agreed by both authors.

![Figure 1: The analytical tool](image)

**The research questions**

This study addresses the following research questions:
1. Were the pre-service teachers successful in solving the test item?
2. Were the pre-service teachers able to justify their answer to the test item?
3. Which type of explanation did the pre-service teachers use to explain their answer?
4. Did the pre-service teachers make use of representations to justify and explain their answer to the test item? If so, which type of representations did they use?

RESULTS

1. *Were the pre-service teachers successful in solving the test item?*
   Majority of the pre-service teachers (76 out of 92, or 82.6%) gave the correct answer to the test item. Slightly less than a fifth of the pre-service teachers (16 out of 92 or 17.4%) were unsuccessful in getting the correct answer to the test item. Analysis of the overall score for the test item showed that majority (about 38%) of the pre-service teachers scored 3 marks out of a possible maximum of 8 marks. This means that majority of the pre-service teachers in the study were able to provide a correct answer to the test item, but unable to provide a complete justification of their answer.

2. *Were the pre-service teachers able to justify their answer to the test item?*
   Table 1 shows the frequency and percentage of the pre-service teachers’ (who gave correct answer to the test item) level of justification of their answer to the test item. Recall that majority of the pre-service teachers was able to provide the correct answer to the test item. However, majority of these pre-service teachers were not able to justify their answer successfully. Only 46.1% of the 76 pre-service teachers (or 38.0% of 92 pre-service teachers) who gave the correct answer to the test item was able to provide a complete justification to their answer.

<table>
<thead>
<tr>
<th>Justification</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incorrect</td>
<td>11</td>
<td>14.5</td>
</tr>
<tr>
<td>Incomplete</td>
<td>30</td>
<td>39.5</td>
</tr>
<tr>
<td>Complete</td>
<td>35</td>
<td>46.1</td>
</tr>
<tr>
<td>Total</td>
<td>76</td>
<td>100.1</td>
</tr>
</tbody>
</table>

Table 1: Level of justification of the pre-service teachers.

3. *Which type of explanation did the pre-service teachers use to explain their answer?*
Table 2 shows the type of explanation given by the 76 pre-service teachers who provided the correct answer to the test item. Majority of these 76 pre-service teachers provided an explanation by showing the execution of the multiplication of fractions algorithm, that is, a procedural explanation. Although the test item had a daily context (pizza left over after a party), only about a third of these pre-service teachers (35 pre-service teachers) made use of this daily context to explain their answer. None of the pre-service teachers gave a mathematically-based explanation. None of the pre-service teachers provided all three types of explanations – procedural, practically-based (conceptual), and mathematically-based (conceptual).

<table>
<thead>
<tr>
<th>Type of explanation</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Procedural</td>
<td>51</td>
<td>67.1</td>
</tr>
<tr>
<td>Practically-based</td>
<td>11</td>
<td>14.5</td>
</tr>
<tr>
<td>Procedural &amp; Practically-based</td>
<td>14</td>
<td>18.4</td>
</tr>
</tbody>
</table>

Table 2: Type of explanation given by pre-service teachers.

4. Did the pre-service teachers make use of representations to justify and explain their answer to the test item? If so, which type of representations did they use?

As mentioned above, 35 of the 76 pre-service teachers who gave the correct answer to the test item provided a practically-based explanation in their solutions. All of the 35 pre-service teachers provided pictorial representations of their explanations.

DISCUSSION

This study, unlike other studies on teachers’ mathematics knowledge, used an analytical tool to indicate the level of SCK of an individual based on each three components of SCK (Justification, Explanation and Representation). Such information will be helpful for lecturers and tutors to provide feedback to the pre-service teachers and also to design mathematics pedagogy courses in pre-service teacher education programmes.

Using the analytical tool, the preliminary findings are: (a) majority of the pre-service teachers were not able provide a complete justification of their answer, (b) majority of the pre-service teachers provided a procedural explanation, (c) only about a third of these pre-service teachers used a practically-based explanation, even though the test item itself involves a daily context of pizza left over after a party, (d) none of the pre-service teachers provided mathematically-based explanation, (e) none of the pre-service teachers provided all three types of explanations – procedural, practically-based (conceptual) and mathematically-based (conceptual), and (f) only
about 38.0% of the pre-service teachers used a pictorial representation in their explanation.

The above findings echoed the findings from Tobias and Itters (2007) study which found the SCK of pre-service teachers in regional Australia to be lacking. This study found the pre-service teachers’ SCK in the topic of multiplication of fractions to be lacking. The findings of this study were also consistent with those of previous studies (e.g., Isiksal and Cakiroglu, 2011) – that many pre-service teachers’ understanding of mathematics concepts is characterized by rote knowledge of the algorithm rather than by the concepts underlying procedures. Although the role of procedural learning should not be ignored, if a teacher’s knowledge of mathematics concepts is limited to only procedures, we cannot expect his or her classroom to be one of “developing knowledge, skills and understanding through inquiry” (Board of Studies NSW, p. 7), an objective stipulated in the current New South Wales mathematics K-6 Syllabus.

Generating representations for a mathematical concept is a common teaching task in mathematics classrooms. Using a pictorial representation to explain the multiplication of fractions concept was found to be missing in majority of the pre-service teachers in the study. Connecting mathematics concepts with representations from daily contexts or the “real world” may help students make more sense when learning mathematics concepts. Further, it is important to note that “[w]ithout a solid knowledge of what to represent, no matter how rich one’s knowledge of students’ lives and no matter how much one is motivated to connect mathematics with students’ lives, one cannot still produce a conceptually correct representation” (Ma, 1999, p. 82). Many studies have indicated that the real mathematical thinking that goes on in the classrooms is dependent on how the teachers’ mathematics content knowledge and pedagogical knowledge (Fennema & Franke, 1992; Ma, 1999; Ball, Thames, & Phelps, 2008). Hence the findings in this study suggest that mathematics pedagogy subjects in pre-service teacher education programmes could help improve pre-service teacher’s SCK by not only exposing them to the types of explanations of mathematics concepts (procedural and conceptual), it is also pertinent that they be made aware of the various representations of mathematics concepts (pictorial and abstract).

Ma’s (1999) seminal work suggested that mathematics teachers should possess the following teaching skills: (a) connectedness (making connections between mathematics concepts and procedures), (b) multiple perspectives (providing explanations of different facets of a mathematical idea and various approaches to a solution), (c) basic ideas (revisiting and reinforcing basic mathematics ideas and concepts), and (d) longitudinal coherence (having a fundamental understanding of the entire elementary mathematics curriculum). The Chinese teachers in Ma’s study regarded the meaning of multiplication of fractions as “a “knot” that ties a cluster of concepts (e.g., fraction concept, meaning of multiplication of whole numbers) that support the understanding of the meaning of division of fractions (p. 82). Hence, in a similar sense, teachers’ SCK is also like a “knot” that ties the various kinds of teaching
skills and knowledge that teachers need in order to carry out their work as teachers well.

The findings in this study cannot be generalized to the whole of New South Wales, Australia. Also, only the written answers of the pre-service teachers were analyzed. Another study could employ an interview method to further understand pre-service teachers’ SCK on the topic of multiplication of fractions. Yet another study could further analyze the types of pictorial representations for multiplication of fractions that the pre-service teachers gave in the mastery test.

References


