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YOUNG CHILDREN'S[1] ACCESS TO POWERFUL MATHEMATICAL IDEAS

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Introduction

Since the publication of the first edition of this chapter, there has been a surge in interest and research in early childhood mathematics education across the world. There have been a number of important special issues of mathematics education research and more general educational research journals (*British Education Research Journal, Early Childhood Research Quarterly, Mathematics Education Research Journal)*, as well as many other reports and articles, devoted to early childhood mathematics education. In, for example, the United States of America (National Council for Teachers of Mathematics (NCTM) / National Association National Association for the Education of Young Children (NAEYC)) and Australia (Australian Association of Mathematics Teachers (AAMT) / Early Childhood Australia (ECA)), the national mathematics education and early childhood associations have joined together to produce position statements on early childhood mathematics education. The 'readiness to learn' prerogative of the United States of America and its *No Child Left Behind* legislation have also stimulated a great deal of investigation in the early childhood years, particularly in the areas of literacy and numeracy. The importance of 'brain research' and its impact on and recognition of early intervention programs is also pertinent here as are large scale research programs such as the *Effective Provision of Pre-school Education* project in the United Kingdom and the *Early*

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[1] In this chapter, we shall use the term 'young children' to designate people in the internationally accepted

Childhood Longitudinal Study in the United States of America. Further, many research-based systemic early childhood mathematics education programs have been established in prior-to-school[2] and school settings. Examples include the *National Literacy and Numeracy Strategies* in the United Kingdom, *Count Me In Too* in Australia, the New Zealand *Numeracy Development Project* and programs such as *Building Blocks* and *Big Math for Little Kids* in the United States of America. A further emphasis has been placed on the prior-to-school years and early years of school through more general transition to school research across the world which highlights the importance of pedagogical, if not structural, continuity across the time of children starting school (Dockett & Perry, in press). The time is right for a revised consideration of the state of early childhood mathematics education research.

Chapter overview

The chapter begins with a brief section on the characteristics of the early childhood years and children's learning in these years. This general discussion of early childhood is linked with ideas about mathematics learning and teaching in these years through an historical perspective. The second section of the chapter analyses and categorizes these powerful mathematical ideas and illustrates the children's access to these ideas with examples from the literature. Particular emphasis is given to issues around mathematics learning as children start school.

Although much mathematics learning in the early childhood years appears to take place without the direct intervention of adults, there is a great deal that adults can do to facilitate and enhance this learning. The third section of the chapter considers the roles of children, early

period of early childhood, between the ages of 0 and 8 years.

childhood educators and families in the learning and teaching of mathematics to children in their early childhood years. The development of research-based learning and teaching approaches such as those adopted by systemic numeracy programs and the use of information and communication technology in learning and teaching programs are considered, as are innovative approaches to the assessment of children's mathematical ideas, including the powerful mathematical ideas introduced earlier in the chapter. The roles and professional attributes of teachers of early childhood mathematics are also considered.

The chapter will conclude with our attempt to map future research needs in the early childhood mathematics education field and to offer a challenge to ourselves and our colleagues across the world to continue with and expand their work in helping to ensure that every young child is fulfilled in terms of her/his mathematics learning.

Learning in the early years

The internationally accepted definition of 'early childhood' covers the period from birth to eight years (OECD, 2001). This is a time of rapid change, as children grow, develop and learn a great deal about themselves, other people and the world in which they live.

Many theoretical perspectives are reflected in modern early childhood education. These include elements of maturational theory, evidenced by beliefs about children's perceived readiness for particular experiences (Gessell, 1925); behaviorist theory, seen in the strong focus on reinforcement in promoting positive behavior (Weber, 1984); Piagetian theory, with the emphasis on the stage-wise emergence of children's competencies (Piaget, 1952); and psycho-

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[2] The term 'prior-to-school' is used here to encompass all possible educational settings that might be met by young children before they commence formal schooling. These setting include preschools, day care centres, homes, museums, parks and so on.

social theory, focusing on the social and emotional challenges encountered as children grow and develop (Erikson, 1950). Most recently, early childhood education has been influenced by socio-cultural theories which emphasize the significance of children as active and interactive participants in a variety of social and cultural contexts (Rogoff, 2003; Vygotsky, 1978).

The focus on the social and cultural contexts of children highlights a growing awareness of the impact of these areas not only on what children learn, but also on how it is learned and how it is taught. For example, Rogoff (1998) has emphasized that "learning involves not just increasing knowledge of content but also incorporation of values and cultural assumptions that underlie views about how material should be taught and how the task of learning should be approached" (Siegler, 2000, p. 27). A shift towards a consideration of Vygotskian principles relating to the social mediation of knowledge has prompted a focus on not only what it is that children are capable of on their own (for example, as assessed through Piagetian tasks), but also, what they are capable of achieving with the assistance of more knowledgeable others through scaffolding, and through teachers developing and implementing tasks that target the zone of proximal development (Berk & Winsler, 1995; Bodrova & Leong, 1996).

Recent research developments in the field of early childhood education have emphasized the significance of children's early experiences for their future and ongoing development, describing the "powerful capabilities, complex emotions, and essential social skills that develop during the earliest years of life" (Shonkoff & Phillips, 2000, p. 2). The early years of life, from birth to about age five years, are described as a time when development "proceeds at a pace exceeding that of any subsequent stage of life" (Shonkoff & Phillips, 2000, p. 4). One consequence of this is that the early childhood years are being recognized as a prime time for both education and intervention.

Part of the rationalization for this relates to growing awareness of the significance of brain growth and development in the early years. The 'new brain research' does indicate that children's physical, social and psychological environments have a crucial role in promoting brain development – including impacting upon the hard-wiring of the brain and the neural connections established and retained (Shonkoff & Phillips, 2000; Shore, 1997). Some researchers and educators have taken this to mean that formalized instruction in specific context areas is needed for young children, with the warning that if such instruction is not provided within the 'critical periods' in which children are likely to acquire specific skills, potential capabilities will be limited. Yet, a great deal of research evidence suggests that this is not the case. It *is* important to recognize the significance of learning in the early years. However, it is also important to recall that a great deal of this learning occurs in naturalistic situations, in the context of relationships and through the processes of play. The environment *is* an important contributor to young children's learning and development. Within this, the establishment of strong, sensitive, and trusting relationships, rather than specific curriculum content, appears to be the crucial factor. Further, while it is possible to identify sensitive periods during which children's receptiveness to some aspects of learning and development are heightened, it is a misapprehension to conclude that any such 'missed opportunities' are irretrievable (Shonkoff & Phillips, 2000).

One of the challenges of early childhood education rests with an apparent contradiction: that "development in the early years is both highly robust and highly vulnerable" (Shonkoff & Phillips, 2000, pp. 4-5). While recognizing the competencies demonstrated by children, this apparent contradiction requires that we consider the contextually bound nature of some of these competencies, as well as the fragility of others. For example, in a familiar, comfortable family context, children are likely to demonstrate different competencies than they would in an

unfamiliar environment, where others do not expect them to be competent. Similarly, children's competence may well be challenged by changing contexts, such as the unavailability of a trusted adult and strained or conflictual relationships.

The recognition of children as competent participants in a range of social and cultural worlds represents a major shift in the ways in which children are viewed. Rather than focusing on the perceptual and conceptual limitations of children, it presents children as experts on their own lives, demonstrating multiple competencies in contexts that are familiar and significant for them, when engaging with adults who care for and respect them (Clark & Moss, 2001; James, Jenks, & Prout, 1998; Lansdown, 2005).

There are many challenges for educators who adopt the perspective of children as competent and capable. One of these involves the ability to relate to and with children in ways that respect their competence: in other words, a commitment to listening to children and to taking their views seriously. Listening to children is an active process that requires openness to the unexpected, the unpredictable and a suspension of prejudice (Rinaldi, 2005). In their interpretation of what is required for active listening, Clark, McQuail, and Moss (2003) describe "an active process of communication involving hearing, interpreting and constructing meanings" that is "not limited to the spoken word" (p. 13).

Regarding children as competent highlights the importance of children's social and cultural contexts. Rogoff (2003) locates development within contexts, noting that "humans develop through their changing participation in the socio-cultural activities of their communities, which also change" (p. 11). As children engage within their social and cultural contexts, they develop competence in areas that are of importance within those contexts.

Bredekamp & Copple (1997, p. 97) note that the prior-to-school years are "recognized as a vitally important period of human development in its own right, not as a time to grow before 'real learning' begins in school". The developments which occur in the early childhood years are remarkable for their speed, comprehensiveness and complexity. While the focus of this chapter is young children's mathematical skills, abilities, understandings and dispositions, it is important to remember that all areas of development and learning undergo rapid change in the early years and each influences the other:

> ...as children develop physically ... the range of environments and opportunities for social interaction that they are capable of exploring expands greatly, thus influencing their cognitive and social development. ... children's vastly increased language abilities enhance the complexity of their social interactions with adults and other children, which in turn, influence their language and cognitive abilities. ... Their increasing language capacity enhances their ability to mentally represent their experiences (and thus, to think, reason and problem-solve), just as their improved fine-motor skill increases their ability to represent their thoughts graphically and visually. (Bredekamp & Copple, 1997, p. 98)

Some historical perspectives

Ideas about the importance of early childhood education are not new. For example, Comenius wrote in the 16th century that education should begin in the early years when children were most open to learning and change: "a young plant can be planted, transplanted, pruned and bent this way or that. When it has become a tree these processes are impossible" (Comenius, 1967, p. 58). The malleability referred to by Comenius is not too far removed from the

perspective underlying many approaches to early childhood education and early intervention – the notion that early experience has a critical influence on later development.

Following Comenius, both Froebel and Pestalozzi promoted the idea of young children's learning and development 'unfolding' in a natural pattern, with the role of the teacher being to "observe children's natural unfolding and provide activities that enable them to learn what they are ready to learn when they are ready to learn it" (Morrison, 2006, p. 98). The context of play was regarded as important in facilitating this process:

> Play is the purest, the most spiritual, product of man at this stage … it produces … joy, freedom, satisfaction, repose within and without, peace with the world. The springs of all good rest within it and go out from it. (Froebel, 1885, p. 30)

To facilitate children's learning and development, Froebel developed a systematic curriculum based on gifts (objects for children to handle and use according to teacher instruction, promoting learning about color, shape, counting, measuring, contrasting and comparison) and occupations (materials designed to teach specific skills, such as sewing, drawing, weaving, pasting and folding). The gifts, as objects of play, were not new. However the structured approach to education utilizing the gifts was new.

The second gift developed by Froebel consists of a sphere, cylinder and cube. Brosterman (1997, p. 46) describes it as

> the most profound of all … Alike in their perfection, the cube and the sphere are, in respect to form, pure opposites… Theorising that learning is accomplished only by way of comparison … the gifts and occupations incorporated opportunities for

expressing antitheses for every object and action. The sphere with no flat planes, the cube with no curves; the sphere an expression of motion, the cube of absolute rest …

Mathematics was regarded by Froebel as an essential element of the kindergarten curriculum:

Kindergarten's universal, perfect, alternative language of geometric form cultivated children's innate ability to observe, reason, express and create. Its ultimate aim was to instil in children an understanding of what an earlier generation would have called 'the music of the spheres' – the mathematically generated logic underlying the ebb and flow of creation … Froebel believed that learning the sacred language of geometry in youth would provide a common ground for all people, and advance each individual, and society in general, into a realm of fundamental unity. (Brosterman, 1997, pp. 12-13)

As in Froebel's kindergarten, programs developed by Montessori (1976) included a strong focus on concrete materials as a means of "isolat[ing] a general principle or concept. A child manipulates them, performing actions, and in the meantime, through this sensorimotoric experience, gets acquainted with the principle or concept involved" (p. 65). Many of the materials developed by Montessori involve comparisons of size and/or quantity, as well as the operations of addition, multiplication, division and subtraction. The Montessori approach and the materials had the aim of developing children's independent mastery of specific tasks. The materials, and their supposedly embodied concepts, were designed to match children's interests, to support their independent use and to be self correcting. Many of the materials remain in present day early childhood education settings, whether or not those in these settings espouse a

Montessori approach to education and whether or not educators understand the mathematical bases for the materials.

Educational programs for young children flourished in many countries during the twentieth century. Several influential programs adopted Piagetian theory as their basis. One of these, the High Scope program, implemented initially in Ypsilanti, Michigan, has exerted significant influence on early childhood education programs across the world. Within this program, great import is attached to young children's development of logico-mathematical knowledge. Hence, objectives and key experiences are outlined for classification, seriation, number, space and time (Hohmann, Banet, & Weikart, 1979; Kamii, 1973). High Scope incorporates principles of active learning and problem-solving, with teachers and children playing active and interactive parts in guiding the curriculum (High Scope Educational Research Foundation, 1989).

Much of the basis of the High Scope program has been reiterated within the concept of developmentally appropriate practice (DAP) (Bredekamp, 1987; Bredekamp & Copple, 1997), which has had a substantial impact on teaching and learning in early childhood settings in recent years. A focus on a predictable pattern of development, influenced by individual variability, characterizes this approach. The pattern of development used as a basis for DAP is essentially Piagetian. Despite this, DAP is not explicit about the nature of logico-mathematical experiences and practices appropriate for young children. In trying to avoid a 'push-down' academic curriculum (Elkind, 1987), there is a sense of avoiding 'hard topics' such as mathematics altogether. For example, appropriate practice for infants and toddlers makes no mention of mathematical interactions. Only when children reach 3-5 years of age, is there recognition of the need to

plan a variety of concrete learning experiences with materials and people relevant to children's own life experiences and that promote their interest, engagement in learning and conceptual development. Materials include, but are not limited to, blocks and other construction materials, books and other language-arts materials, dramatic-play themes and props, art and modeling materials, sand and water with tools for measuring, and tools for simple science activities. (Bredekamp & Copple, 1997, p. 126)

One of the most influential educational approaches of recent years–Reggio Emilia–derives from a distinctive philosophical base of promoting children's intellectual development through symbolic representation (Edwards, Gandini & Forman, 1993; Tarr & Okuno, 2000). A reliance on concrete materials is incorporated within this. However, in this approach, relationships are regarded as the key to a successful learning and teaching experience. Learning takes place in relationships–with adults and children each making appropriate adjustments if the interactions are to continue: "the way we get along with children influences what motivates them and what they learn" (Malaguzzi, 1993, p. 61). Relationships are seen as involving the

dynamic conjunction of forces and elements interacting toward a common purpose… We seek to support those social exchanges that better insure the flow of expectations, conflicts, cooperations, choices, and the explicit unfolding of problems tied to the cognitive, affective and expressive realms. (Malaguzzi, 1993, p. 62)

Children in Reggio Emilia settings are described as learning through communication as well as concrete materials, with "the system of relationships ha[ving] in and of itself, a virtually autonomous capacity to educate … it is a permanent living presence always on the scene, required all the more when progress becomes difficult" (Malaguzzi, 1993, p. 63). It is within

relationships that children make meaning. There is regard for children as autonomous meaning makers, with the emphasis that "meanings are never static, univocal or final; they are always generative of other meanings". Within relationships, the adult role is described as one of activating, "the meaning-making competencies of children as a basis of all learning" (Malaguzzi 1993, p. 75).

Relationships among family members, children and educators also have a substantial influence on learning, including the learning of mathematics. Studies have shown a positive relationship between parental involvement in their children's schooling and the achievement of these children (Civil, 1998; Young-Loveridge, Peters, & Carr, 1998). Similar connections have been described between levels of parent involvement in prior-to-school settings, children's academic attainments and social adjustment (Arthur, Beecher, Death, Dockett, & Farmer, 2005).

In highlighting the importance of relationships, we also draw attention to the significance of social and cultural contexts in children's learning and development and the active role of children within these. We regard children as capable and competent, knowledgeable about many of the people, places and things that make up their world. However, we also regard children as vulnerable in contexts that are unfamiliar and in relationships where they have little power. Within the context of early childhood education, there are many opportunities to acknowledge the understandings and competencies children already have, as well as to present challenges that encourage them to refine, extend, elaborate or change these.

What powerful mathematical ideas are accessible to young children?

In their *Principles and Standards for School Mathematics* (NCTM, 2000), the National Council of Teachers of Mathematics presents the following vision for school mathematics:

Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it. (p. 3)

While, in this chapter, we would like to include prior-to-school settings, homes and other physical spaces frequented by young children and, perhaps, lessen the emphasis on formal instruction, the overall notions expressed in the vision statement still hold for the learning of early childhood mathematics. In particular, the emphasis on "important mathematical concepts and procedures with understanding" is critical to the development of early childhood

mathematics. What are these important mathematical concepts? NCTM lists them as their *Standards*:

- Number and operations

- Algebra

- Geometry

- Measurement

- Data analysis and probability

- Problem solving

- Reasoning and proof

- Communication

- Connections

- Representation.

These *Standards* have been used as the foundation for numerous mathematics programs and approaches (for example, Casey, Kersh, & Young, 2004; Clements, Sarama, & DiBiase, 2004; Sarama & Clements, 2004) and research endeavours (for example, Sophian, 2002, 2004; Starkey, Klein, & Wakeley, 2004; Young-Loveridge, 2004).

A related list of important mathematical concepts and processes were introduced in the original version of this chapter (Perry & Dockett, 2002c) and are reprised here as powerful mathematical ideas to which we believe most young children have access.

- mathematization;

- connections;

- argumentation;

- number sense and mental computation;

- algebraic reasoning;

- spatial and geometric thinking; and

- data and probability sense;

Each of these is begun and needs to be nurtured in the early childhood years.

Mathematization

Mathematization is a term coined by the eminent Dutch mathematics educator, Hans Freudenthal, in the 1960s to signify the process of generating mathematical problems, concepts and ideas from a real world situation and using mathematics to attempt a solution to the problems so derived. Two forms of mathematization are distinguished. The first is 'horizontal mathematization', where "students come up with mathematical tools which can help to organize and solve a problem set in a real-life situation" (Heuvel-Panhuizen, 1999, p. 4). The other is 'vertical mathematization' which "is the process of reorganization within the mathematical system itself" (Heuvel-Panhuizen, 1999, p. 4). De Lange (1996, p. 69) has expanded on these components of mathematization in the following way.

> First we can identify that part of mathematization aimed at transferring the problem to a mathematically stated problem. Via schematizing and visualizing we try to discover regularities and relations, for which it is necessary to identify the specific mathematics in a general context. …

As soon as the problem has been transferred to a more or less mathematical problem this problem can be attacked and treated with mathematical tools: the mathematical processing and refurbishing of the real world problem transformed into mathematics. …

Mathematization always goes together with reflection. This reflection must take place in all phases of mathematization. The students must reflect on their personal processes of mathematization, discuss their activities with other students, must evaluate the products of their mathematization, and interpret the result.

Horizontal and vertical mathematizing comes about through students' actions and their reflections on their actions. In this sense the activity 'mathematization' is essential for all students.

The critical and central role of mathematization is further expanded by Gravemeijer, Cobb, Bowers, and Whitenack (2000, p. 237):

… the goal for mathematics education should be to support a process of guided reinvention in which students can participate in negotiation processes that parallel (to some extent) the deliberations surrounding the historical development of mathematics itself.

The heart of this reinvention process involves mathematizing activity in problem situations that are experientially real to students.

Numerous studies with children in the first years of school (Clarke, 2004; English, 1999; English & Watters, 2005; Jones et al., 2000; Yackel & Cobb, 1996) have shown that young children can mathematize. The ability to mathematize has also been observed in a number of

studies with children who have not yet started school (Aubrey, 2004; Aubrey, Bottle, & Godfrey, 2003; Perry & Dockett, 2002b).

Connections

The question of connections—mathematics learning being related to learning in other areas; mathematics learning being relevant to the contexts in which the child is working or playing and learning in one area of mathematics being related to learning in another area of mathematics—is clearly pertinent in the early childhood years, both in prior-to-school and school settings, where children are beginning to develop their knowledge and skills in mathematics while applying them to their own contexts. In some instances, these connections are enhanced by integrated curriculum. For example, a child learning to count will use this to find the answers to questions of 'how many' in many meaningful situations. The development of the knowledge and skill go hand-in-hand with their application. Just as mathematics is learned 'in context' so it is used 'in context' to achieve some worthwhile purpose.

> When students can connect mathematical ideas, their understanding is deeper and more lasting. They can see mathematical connections in the rich interplay among mathematical topics, in contexts that relate mathematics to other subjects, and in their own interests and experience. ...
>
> Mathematics is not a collection of separate strands or standards, even though it is often partitioned and presented in this manner. Rather, mathematics is an integrated field of study. Viewing mathematics as a whole highlights the need for studying and thinking about the connections within the discipline. (NCTM, 2000, p. 64)

The notion of mathematical connections is strongly related to other concepts with labels such as numeracy, mathematical literacy or quantitative literacy (Askew & Brown, 2001; Department of Education, Training and Youth Affairs, 2000; Hughes, Desforges, & Mitchell, 2000). A succinct description of numeracy is that it involves using "some mathematics to achieve some purpose in a particular context" (AAMT, 1997, p. 13) while mathematical literacy has been described as having components including "thinking, talking, connecting and problem solving" (Liedtke, 1997, p. 13). At the early childhood level, numeracy, mathematical literacy, and mathematics go hand in hand (Liedtke, 1997; Munn, 2001; Perry, 2000; Perry & Dockett, 2002a; Van de Rijt et al., 2003), as children, for example, strive to satisfy all of their friends by sharing out their lollies evenly to thus avoid social turmoil. The application of mathematics to a contextual problem or challenge confronts young children throughout their day in, for example, prior-to-school settings, schools, homes, and shopping centers. To solve these problems and meet the challenges, young children need not only to have developed their mathematical skills and knowledge but their dispositions and self-confidence so that they are willing to apply these in novel situations. The contextual learning and integrated curriculum apparent in many early childhood—particularly prior-to-school—settings ensures that there is little distinction to be drawn between numeracy, mathematical literacy and aspects of mathematical connections with the children's real worlds.

One of the clearest links between mathematics learning and children's contexts occurs when we consider children's literature. For example, Ginsburg and Seo (2000) and Cutler, Gilkerson, Parrott, and Bowne (2003) highlight the many mathematical ideas which can be introduced to children in prior-to-school settings through the use of children's literature. Anderson and her colleagues have investigated reading children's books at home as a context for

mathematics learning and the development of strong relationships between the adults and children sharing the home (Anderson, Anderson, & Shapiro, 2004, 2005).

There are many fine examples of children's literature and numerous suggestions as to how these might be used by teachers (see for example, *Australian Primary Mathematics Classroom*; *Teaching Children Mathematics*; Thatcher, 2001; Whitin & Whitin, 2004). Literature can provide a very useful link between something which most children seem to enjoy and mathematics, although care does need to be taken to ensure that the joy of the literature is not lost through the overly zealous pursuit of the mathematics, or vice versa.

In both prior-to-school and school settings, one powerful way in which the mathematics children are learning can be connected to them and their knowledge is through consideration of cultural aspects of learning mathematics (see, for example, Barta & Schaelling, 1998; Guha, 2006; MacMillan, 2004; Perry & Howard, 2000; Perso, 2001, 2003; Vagi & Green, 2004). One activity which the authors have found quite useful in celebrating the diversity of cultures which occur in the classrooms in which they work is that of *Honest numbers* (Bezuska & Kenney, 1997). This activity encourages the celebration—in a fun way—of the cultures brought to the classroom by the children and shows that there is more to mathematics than the canonical western curriculum which is so dominant in schools around the world (Nebres, 1987). Guha (2006) reports a study based on an Indian method of finger counting as an example of 'culturally responsive' teaching in the early childhood years and concludes:

Using culture as bait to 'hook' students who tend to shy away from mathematics education, or students who are considered less interested or even weak in mathematics skills, the teacher can potentially engage all students regardless of race, ethnicity, caste, or socioeconomic status to learn mathematics in a playful way.

Although culturally unfamiliar manipulatives or tools may be more interesting for one group of children as compared to others, they can also generate curiosity among the entire group that can assist in their learning endeavor. (p. 32)

Maree and Erasmus (2006, p. 16) make a similar point slightly more succinctly when they suggest that "mathematics teaching cannot be divorced from the socio-economic context in which it is taught".

One example that shows the subtle links between mathematics learning – in this case, patterning – and children's social and cultural contexts comes from Taiwan.

In Taiwan, young children are taught to applaud success by clapping in socially appropriate ways. While it is clear that clapping in time involves some measurement skill, the patterns used are also mathematical. For instance, clapping in the following way: *clap-clap / clap-clap-clap / clap-clap-clap-clap / clap-clap* means 'cheering with love' in Taiwan. Desirable social attributes can be integrated into mathematics learning. (S. Leung, personal communication, November 3, 2000).

As well as these links with aspects of wider contexts of children, there are clear connections between different aspects of mathematics which need to be developed and understood. Young children have access to some of these links. For example, research (Cobb, Stephan, McClain, & Gravemeijer, 1998; McClain, Cobb, Gravemeijer, & Estes, 1999; Outhred & Mitchelmore, 2000; Outhred, Mitchelmore, McPhail, & Gould, 2003; Stephan & Clements, 2003) suggests that both measurement and fraction ideas are dependent on the notions of unitizing and of composite units, thus linking the two mathematical areas in terms of their underlying processes.

Argumentation

Krummheuer (1995, p. 229) describes argumentation as a "social phenomenon, when cooperating individuals [try to] adjust their intentions and interpretations by verbally presenting the rationale of their actions". The process allows children, and other participants, to justify not only their own mathematical thinking but also to distinguish between the strengths of arguments and whether or not the mathematics being constructed within the arguments is actually different from previous mathematical arguments which have been interactively constructed (Voigt, 1995; Yackel, 1998; Yackel & Cobb, 1996). Recent work, in mathematics education, and in other areas of cognitive development, suggests that argumentation is central to the mathematics development of young children (Dockett & Perry, 2000; Epstein, 2003; Leitao, 2000; Perry & Dockett, 1998; Pontecorvo & Pirchio, 2000; Yackel, 1998; Yackel & Cobb, 1996). Given that argumentation will form the basis of mathematical proof in later years, it is important for us to realize the early genesis of this process.

Number sense and mental computation

While there is some debate as to the detailed components of 'number sense' (Howell & Kemp, 2005), it is a term that is used in many countries. While the details may differ, there is consensus that number sense includes "a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for dealing with numbers and operations" (McIntosh, Reys, & Reys, 1997, p. 322). Since almost all the mathematics which children will meet in the elementary school, and much of what they will meet beyond this, is firmly based in number, the importance of sound number sense cannot be overstated.

Number sense is evident in quite young children. Hughes (1986), for example, has shown that, before attending school, children understood concepts such as subtraction and could represent number and operations with these numbers when they were linked to concrete objects, even if these were hidden. More recent work from Worthington and Carruthers (2003) has expanded our knowledge of children's written mathematics. Bertelli, Joanni and Martlew (1998) have shown that 3-year-olds are able to reason about number—answering questions about more and less—even before they had mastered counting. Sophian and Vong (1995) and Young-Loveridge (2002) have noted the use of part-whole relationships in number by 4 and 5-year-olds. Aunio, Ee, Lim, Hautamäki, & Van Luit (2004) have suggested that while there may be some cross-national differences in children's performance on the Utrecht Early Numeracy Test of number sense (Van de Rijt, Van Luit, & Pennings, 1999), young children are able to demonstrate many features of number sense before they start school and in the first year of school.

Young children perceive and use numbers in almost every context they experience. Their play can provide many of these experiences. Encounters with stories, rhymes and other children's literature (Copley, 2000; Ginsburg & Seo, 2000; Whitin & Whitin, 2004) can also involve young children in meaningful number experiences. Many young children enjoy and are capable of talking about 'big' numbers and fractions such as 'half' (Hunting & Davis, 1991; Wing & Beal, 2004).

The importance of counting to young children's number development is well known (see, for example: Carraher & Schliemann, 1990; Steffe, Cobb, & von Glasersfeld, 1988; Steffe, von Glasersfeld, Richards, & Cobb, 1983; Verschaffel & De Corte, 1996). Many early number programs are now based on the enhancement of children's counting skills, including access to the forward and backward number word sequences, skip counting and counting in realistic

situations (Wright, Martland, & Stafford, 2000; Wright, Martland, Stafford, & Stanger, 2002). The need for facility in the use of the composite unit in base ten representations of number is seen to be a critical aspect of this approach to number (Cobb & Wheatley, 1988; Pengelly, 1990; Thomas & Mulligan, 1999; Wright, 1994). Certainly this facility is one well within the reach of children in the first years of school, if not earlier (Beishuizen, van Putten, & van Mulken, 1997; Menne, 2000; Tang & Ginsburg, 1999).

Mental computation is an integral part of young children's learning about number. It can be used as a tool to facilitate the meaningful development of mathematical concepts and skills, and to promote thinking, conjecturing and generalizing based upon conceptual understanding (Reys & Barger, 1994). Mental computation is closely linked to the development of number sense (McIntosh & Dole, 2000), and enables a "focus on strategies for computing with whole numbers so that students develop flexibility and computational fluency" (NCTM, 2000, p. 35).

One of the direct consequences – or, perhaps, one of the direct causes - of a stronger focus on number sense, mental computation and counting processes has been the development of systemic numeracy programs. These have been particularly popular in Australia and New Zealand (Bobis et al., 2005; Wright et al., 2002; Young-Loveridge & Peters, 2005). Programs such as *Count Me In Too* (Bobis, 2003; Bobis & Gould, 2000), *Early Numeracy Research Project* (Clarke & Clarke, 2004) in Australia and the *Numeracy Development Project* in New Zealand (Thomas, Tagg, & Ward, 2003) have been taken up by a large proportion of schools and have been exported to a number of other countries, including England and the USA. These programs are good examples of the power that mathematics education research is able to exert on systems that are willing to make substantial changes based on the research.

Algebraic reasoning

Algebraic reasoning in the early childhood years often comes in the guise of patterning activities, where relationships of equality, sequence and argument are developed (Fox, 2005; Mulligan, Mitchelmore, & Prescott, 2005; Papic & Mulligan, 2005; Warren, 2005; Waters, 2004).

Much of this patterning has to do with number and the development of a flexible, sound number sense. Many of the strategies developed by young children, including both inductive and deductive reasoning, will be useful in later years as the children work with number, especially in the development of their place value ideas and their facility with counting (Schifter, 1999; Tang & Ginsburg, 1999) and their representation and understanding of arithmetic formulae (Zhou, Wang, Wang, & Wang, 2006). In particular, Schifter (1999, p. 80) has made

> the case for an emphasis on the development of *operation sense* as crucial to this preparation [for algebra instruction]. … once the teaching of elementary school arithmetic is aligned with reform principles—when classrooms are organised to build on students' mathematical ideas and keep students connected to their own sense-making abilities—then children so taught will be ready for algebra.

Carraher, Schliemann, Brizuela, and Earnest (2006, p. 89) reinforce the centrality of algebraic thinking to the learning of arithmetic in the early years:

> The fact that algebra emerged historically after, and as a generalization of, arithmetic suggests to many people that algebra ought to follow arithmetic in the curriculum. However obvious this claim may seem, we believe there are good reasons for thinking otherwise. …

We are suggesting that arithmetic has an inherently algebraic character in that it contains general cases and structures that can be succinctly captured in algebraic notation. We would argue that the algebraic meaning of arithmetical operations is not optional 'icing on the cake' but rather an essential ingredient. In this sense, we believe that algebraic concepts and notation need to be regarded as integral to elementary mathematics.

Algebraic reasoning is a burgeoning field of early childhood mathematics education research and while much of the recent work has considered children older than the early childhood years, there are strong suggestions that many children in these years can be capable algebra learners (Warren, 2001). However, there is warning that the current emphases on "exploring numbers for the purpose of developing number sense and exploring numbers for the purpose of early algebraic understanding may indeed be in conflict with each other" (Warren, 2003, p. 718). Clearly, further research is needed in this area.

Spatial and geometric thinking and data and probability sense

The areas of data and probability, space and measurement all feature in the early childhood years both before and during elementary school. Data plays a critical role in our modern society. Much information uses statistical ideas and is transmitted through graphs and tables. Children at all levels of schooling need to be able to deal with these data in sensible ways. That is, they need a sense about data. They need to be able to treat reports of data critically and to establish the veracity of claims for themselves—or, at least, to test this veracity when claims are made. The work of Watson and Jones and their teams (see, for example, Jones et al., 2000; Watson & Moritz, 2000) has established in Australian contexts the need for children to develop such a data sense from an early age. Complementary work in other parts of the world has reinforced this

notion of building data sense (Cobb, McClain & Gravemeijer, 2000; McClain, Cobb, & Gravemeijer, 2000; Shaughnessy, 1997).

Chance (probability) experiences are had by almost everyone every day. The language of probability is met regularly—we have heard 2-year-olds talk about the chance that it will rain, or that they will receive a lollypop as a result of good behavior, for example. Early introduction of probability language and experiences can assist in the avoidance of misconceptions in problems where intuition alone is insufficient to solve them (Bright & Hoeffner, 1993; Jones, 2005; Shaughnessy, 1992; Way, 1997, 2003). There is a need to give children the opportunity to develop their thinking about chance and its quantification so that they are able to build on the informal chance experiences they will have in their lives and are in a position to make sensible decisions in situations of uncertainty (Borovcnik & Peard, 1996; Peard, 1996).

Children in the early childhood years begin to reason about shapes by considering certain features of the shapes. Spatial thinking plays a role in making sense of problems and in representing mathematics in different forms such as diagrams and graphs. The use of manipulatives in the development of mathematical ideas can require some spatial awareness. Spatial ideas—usually then called geometry—was one of the first areas of mathematics to be systematically taught to young children. Many of Froebel's 'gifts' mentioned earlier in this chapter were based in geometry. More recently, in a study designed to see whether preschoolers could think analytically about space, Feeney and Stiles (1996) showed that by four and a half, children were able to distinguish wholes and parts of simple designs such as plus or cross signs. They could do this by construction, by perception, selecting from a picture, and by drawing. Clearly, young children have access to many spatial ideas. For example, in a class of 6-year-olds, Perry and Dockett (2001) describe a play session in which a group of children are using large

wooden shapes designed to assist teachers to draw on the board to create patterns, construct images of local buildings, make roads and maps. The children found that two semicircles could be put together to make a circle and that triangles could fit together to cover an area.

One of the key approaches in the development of spatial ideas in the early childhood years has been the work of Clements and Sarama and their colleagues using the *Building Blocks* approach (Clements, 2004). For example, Clements, Wilson, and Sarama (2004) report on the development of a hypothetical learning trajectory for young children's composition of geometric figures and effective assessment processes for the learning in the trajectory. Such research will provide much-needed structures for the learning and teaching of spatial ideas.

Much of the number research, particularly that dealing with the concept of fractions and the notion of iterable composite units, is pertinent to measurement (McClain et al., 1999; Stephan & Clements, 2003). For example, measurement of length has been taught traditionally through a sequence of activities described by Clements (1999b, p. 5) as: "gross comparisons of length, measurement with nonstandard units such as paper clips, measurement with manipulative standard units, and finally measurement with standard instruments such as rulers". There is some evidence to suggest that using informal units in early measurement lessons may make the activity one of counting, with little concept of what is being measured, and why counting results in a measure rather than a number (Bragg & Outhred, 2000; Clements, 1999b; Owens & Outhred, 1998). As well, there is evidence (Boulton-Lewis, Wilss, & Mutch, 1996; Clements, 1999b) that the use of rulers may facilitate the development of length measurement ideas and may be preferred by many children. Clements (1999b, p. 7) suggests that:

using non-standard units early so that students understand the need for standardization may not be the best way to teach. If introduced early, children often

use unproductive and misleading strategies that may *interfere* with their development of measurement concepts.

Stephan and Clements (2003, p. 4) have explained the complexity of learning about measurement in the following way:

There are several important concepts, or big ideas, that underpin much of learning to measure. It is important to understand these concepts so we can use them to understand how students are thinking about space as they go through the physical activity of measuring. These concepts are: (1) partitioning, (2) unit iteration, (3) transitivity, (4) conservation, (5) accumulation of distance, and (6) relation to number.

Utilizing the potential of computer technology in the learning and teaching of measurement ideas is a strong feature of two major research programs in the USA and Australia. Clements and his colleagues have developed their *Building Blocks* technology as a comprehensive pre-Kindergarten to Grade 2 package. Further details of *Building Blocks* will be canvassed later in this chapter. In Australia, Yelland (2002) developed a computer microworld based on the use of *Geo Logo* (Clements & Sarama, 1995) which encouraged young children mean age 7 years and 4 months) to explore concepts of length measurement. She found that not only did the microworld assist in the learning of the expected curriculum outcomes in length measurement but that it also afforded opportunities that would not have been available without the technology. She concluded that the computer microworld contexts:

afforded the children with the opportunity to engage in mathematical discussions that were significant and important to them. The computer based microworlds not only enabled them to extend their thinking about occurrences in the real world and

play with them to see what was possible but they also facilitated playing with units

of measurement in ways that were not possible without the technology. (Yelland,

2002a, pp. 90-91)

Mathematics learning before children start school

Many mathematics education researchers have reported on the vast array of mathematical

knowledge, skills and dispositions young children bring to school (Aubrey, 1993, 2004; Aubrey

et al., 2003; Baroody, 2000; Bobis, 2002; Ginsburg, 2000; Greenes, Ginsburg, & Balfanz, 2004;

Perry & Dockett, 2002a, 2005; Suggate, Aubrey, & Pettitt, 1997; Tang & Ginsburg, 1999). This

mathematics can be analyzed according to the powerful mathematical ideas discussed earlier in

this chapter and can be seen to be important for its own right as well as for the children's future

mathematical development.

However, many parents and prior-to-school educators seem to miss recognition of much of

this mathematical power in their children (Perry & Dockett, 2005; Tang & Ginsburg, 1999) with

the consequence that the potential of many early childhood activities to contribute to the

children's mathematical development is being missed. There are many publications that provide

support to parents and prior-to-school educators as they try to facilitate their children's

mathematical learning (for example, Charlesworth, 2004; Copley, 2004; Geist, 2003). It is not

necessarily simple for these adults to assist in their children's mathematical development but it

certainly is possible for them to do so. When such an effort is made it will be rewarded strongly.

Baroody (2000, p. 66) summarizes these thoughts in the following way.

Preschoolers are capable of mathematical thinking and knowledge that may be

surprising to many adults. Teachers can support and build on this informal

mathematical competence by engaging them in purposeful, meaningful, and inquiry-based instruction. Although using the investigative approach requires imagination, alertness, and patience by teachers, its reward can be increasing significantly the mathematical power of children.

There is ample evidence that at the prior-to-school level, children's cultural and socio-economic contexts can influence their performance on regularly used assessments of their mathematical knowledge and skills (Starkey et al., 2004; Thomson, Rowe, Underwood, & Peck, 2005). For example, Starkey et al. (2004, p. 114) suggest that, on their assessment instrument, "the scores of middle-income children at age 4 years, 3 months were significantly higher than the scores of low-income children at age of 4 years, 10 months, suggesting a gap of at least 7 months prior to 5 years of age". However, the same authors are clear that well-planned mathematical programs in prior-to-school settings can work towards reducing this gap.

It is critical that early childhood educators hold high expectations of all their children in terms of their potential as mathematical learners. Quality prior-to-school programs can make a difference for everyone but seem to make more of a difference for children from low socio-economic contexts than those from higher socio-economic groups (Thomson et al., 2005). We should celebrate these programs while warning of the dangers of assuming that young children, from whatever background, are not capable learners.

[D]espite some opinion to the contrary, low-income minority children are capable of complex mathematical reasoning. They arrive in school with considerable capability for abstract thought and potential for learning mathematics. Indeed, potential for learning mathematics may well be universal. Virtually all young children may well be capable of the kinds of reasoning we have described. Yet educators often fail to

recognize, nourish, and promote mathematical abilities, particularly those of the disadvantaged. As a result, poor children's subsequent inferior performance in later school mathematics should be attributed more to our failures in educating them than to their initial lack of ability. (Tang & Ginsburg, 1999, p. 60)

Transition to school and mathematics learning

Both 'continuity' and 'change' are important aspects of children's transition to school. The inclusion of familiar routines, activities and people in children's lives as they start school seem to be positive. On the other hand, children want school to be different from their prior-to-school settings if for no other reason than it is a clear sign to them and all around them that they are 'getting big' (Dockett & Perry, 2004). Pianta (2004) has characterized the general changes that occur at transition to school as:

- there is a shift upwards in formal academic demands;

- the social environment becomes radically more complex for the children;

- there are many more peers at school than in prior-to-school settings;

- there is less parent support for both children and teachers; and

- children spend much less time interacting individually with their teacher.

In short, the demands go up and the support goes down.

In mathematics education, the transition to school is characterized by:

greater emphasis on whole class approaches to learning, less choice for children as to the activities in which they might involve themselves, less child control over these activities and their outcomes … less support from adults … greater emphasis on comparison of one child with another (Perry & Dockett, 2005, p. 32).

Such changes affect different children in different ways. Many children make successful transitions to school and simply 'take it in their stride'. Others, even some who appear to be both socially and academically 'ready' for school, struggle to make the change smoothly (Dockett & Perry, 2001; Einarsdóttir, 2003). Broström (2005) describes this situation as a paradox and asks the question: "How can active and independent children be transformed into people dominated by reserve and insecurity?" (p. 18). Such a transformation can lead to a (hopefully) temporary loss of self esteem and a perceived loss of competence. However, given that the first year of school

> is a context in which children make important conclusions about school as a place where they want to be and about themselves as learners … it is essential that the transition … occur in such a way that children and families have a positive view of the school and that children have a feeling of perceived competence as learners. (Bailey, 1999, p. xv)

Children's engagement with mathematics in prior-to-school settings is often through the medium of play. Broström (2005) suggests that play may be one means through which the transition to school may be made more successful for many children.

> Through play, new knowledge, skills and actions often emerge, so it can be assumed that play can serve as a transition tool which contributes to children's thinking. In this way, play is seen as an activity which leads the development of higher mental functions such as language, thinking and memory.

> Because play has the potential to contribute to children's metacognitive development, it seems to hold double benefits. On the one hand, play enables

children to achieve new competences which help them to make a successful transition; on the other hand, play can be a bridging tool to school. (p. 19)

Young children's play can be complex - in terms of theme, content, social interactions and the nature of the understandings displayed and generated. As well, children can have many mathematical experiences during play (Ginsburg, 2000; Seo, 2003; Wolfgang, Stannard, & Jones, 2003). Perry and Dockett (2004) suggest that play is important in the mathematics development of young children because:

1. Play both reflects children's current states of learning and provides opportunities to refine these.

2. Play is one context where children can be confronted with different perspectives and understandings.

3. Play is a context where children can interact with more skilled or experienced members of the social group who guide and scaffold the experience.

4. Learning occurs much more readily when there is a clear purpose and such a purpose exists in child-initiated play.

5. Play provides a context in which abstract mathematical concepts can be linked to concrete aspects.

6. Play is controlled and directed by the players who confront issues which are problems for them and employ a range of strategies to work their way through these.

7. Play provides time for children to consolidate understandings or to explore challenges. (p. 107)

Clearly, play has an important role in mathematics learning as children start school. Equally, that role can continue throughout the early childhood years (and beyond).

Teachers who are effective in promoting their children's learning through play often adopt the role of *provacateur* (Edwards et al., 1993) through which they observe and assess the understandings demonstrated by individual children and then generate situations which challenge these. This may involve asking questions, introducing elements of surprise, requiring the children to explain their position to others and working with children to consider the logical consequences of the positions they adopt. Teachers who use play in their classrooms have opportunities to observe what it is that children know and then to plan learning experiences which follow. In order for this to occur, the children need to feel comfortable in their classroom. They must feel free to interact with their peers about their mathematical ideas and they must feel comfortable in taking risks with their learning. This process can begin in early childhood—both prior-to-school and the first years of school—when teachers recognize the importance of play as one context in which children can safely explore understandings, make and test conjectures and communicate these to others.

Mathematics learning in the first years of elementary school

Many changes occur as children move into and through the first years of their formal schooling experiences (Dockett & Perry, 2004). Some of these have been outlined above and many will be canvassed by Langrall, Mooney, and Nisbet in Chapter 6 of this volume. In mathematics classrooms, there are efforts made to recognize the individual and idiosyncratic approaches to learning displayed by the children and, in at least some cases, curricula have been constructed in such ways that these differences can be exploited and celebrated to the ultimate benefit of the children as learners. Syllabus statements and curriculum documents reflect the

strength of these efforts and systemic numeracy programs exhort teachers to ascertain and use children's own approaches to the solution of problems or their own strategies for calculation (Clarke & Clarke, 2004). For example, in Australia and New Zealand, such systemic numeracy programs have raised teacher expectations of what children can do in the early years of school, have provided integrated individual assessment regimes and have dealt with differences among children's mathematical knowledge through innovative activities and grouping strategies (Bobis et al., 2005). The basic approach is to find out what the children already know, and how they know it, and to extend and deepen this knowledge through investigation. This approach is succinctly summarized in the following syllabus statement:

> Teachers need to acknowledge the learning that children bring to school, and plan appropriate learning experiences that make connections with existing mathematical understanding. … Teachers need to become familiar with children's existing mathematical understanding as they commence school to ensure that programming is designed to meet the needs of individual students. (Board of Studies, NSW, 2002, p. 14)

The importance of recognizing and using what the children already know has also been noted in the NAEYC / NCTM joint position statement:

> In high quality mathematics education for 3-to-6-year-old children, teachers and other key professionals should … build on children's experience and knowledge, including their family, linguistic, cultural, and community backgrounds; their individual approaches to learning; and their informal knowledge. (Copley, 2004, p. 159)

There is clearly much more to the development of excellent mathematical experiences than the recognition of the mathematical power that young children bring to these experiences. However, such recognition does treat children as capable learners who can be expected to learn a great deal of mathematics provided they are presented with learning that is interesting, relevant and challenging, and that they are supported in appropriate ways. As a consequence, early childhood mathematics educators need to engage not only with current syllabuses and approaches but also with their own children and their families to ensure that the best possible opportunities are provided. The nature of these opportunities will vary from context to context but their aim will always be the same – to ensure, as much as is possible, that all children experience what is needed for each of them to achieve to their full potential.

Particular considerations

In this section of the chapter, we gather together a collection of topics that have an overarching relevance to the development of young children's mathematical knowledge and skills and discuss the research pertinent to each of them. The collection is idiosyncratic and is not at all meant to be either extensive or exhaustive. Rather, it is chosen to allow particular points on the provision of quality early childhood mathematics education, particularly at the school level, to be made.

Models and modeling

While the use of manipulatives in mathematics education is well-established, particularly in the early childhood years, and they have been shown to have great value in many aspects of mathematics, particularly in the development of place value ideas and written algorithms with whole numbers (Cobb & Bauersfeld, 1995; Cobb, Wood, & Yackel, 1991; National Council of

Teachers of Mathematics, 2000; Sherman & Richardson, 1995), there is a deal of evidence to suggest that such manipulatives are not automatically helpful in the development of children's mathematical ideas (Clements, 1999a; Howard & Perry, 1999; Perry & Howard, 1994, 2004; Price, 1999; Thompson, 1992). Part of this problem stems from the children's inabilities to argue cogently from the analogies which are formed through the manipulatives or to be overcome by these analogies to such an extent that it is the manipulatives, not the mathematics which becomes most important (English, 1999). The Realistic Mathematics Education (RME) approach, emanating from the Netherlands, has suggested an alternative way of thinking about models. It is suggested, in contrast to the common approach where

> the students are to discover the mathematics that is concretized by the designer, … in the RME approach, the models are not derived from the intended mathematics. Instead, the models are grounded in the contextual problems that are to be solved by the students. The models in RME are related to modeling; the starting point is in the contextual situation of the problem that has to be solved. … The premise here is that students who work with these models will be encouraged to (re)invent the more formal mathematics. (Gravemeijer, 1999, p. 159)

This approach to modeling allows a development of the notion of 'model of' mathematical activity becoming a 'model for' mathematical reasoning. For example,

> problems about sharing pizzas were modeled by the students by drawing partitioning of circles that signify pizzas (model of). Later, the students used similar drawings to support their reasoning about relations between fractions (model for). (Gravemeijer, 1999, p. 161)

More recently, English and Watters (2005) have suggested that mathematical modeling in which young children can "identify the underlying mathematical structure of complex phenomena" (p. 59) should be introduced into the early years of elementary school. They suggest that mathematical modeling is not only accessible to young children but that it provides support for much of their mathematics learning. The pedagogical basis of mathematical modeling provides young children with a way of accessing and demonstrating their powerful mathematical ideas:

> Key mathematical constructs are embedded within the problem context and are elicited by the children as they work the problem. The generative nature of these problems means that children can access mathematical ideas at varying levels of sophistication. (English & Watters, 2005, p. 60)

Approaches such as the mathematical modeling described by English and Watters fit well with what Zheng (2006) has described as the heuristic nature of teaching that is commonly employed by Chinese teachers.

> Teaching in China is not regarded as a process of conveying well-developed knowledge. On the contrary, by paying more attention to the process of creation or discovery, and by making their own re-creation of the content, teachers in China are doing their best to make the content of their lessons really understandable to the students, and to disclose the underlying ways of thought. As a result, the students can not only grasp the related concrete mathematical knowledge or skills, but also learn to think mathematically. (p. 387)

Many would claim that such results are appropriate in all early childhood mathematics education experiences.

Language

The importance of *language* in the development of mathematical ideas is well documented (see, for example, Ellerton, Clarkson & Clements, 2000). Without sufficient language to communicate the ideas being developed, children will be at a loss to interact with their peers and their teachers and, therefore, will have the opportunities for mathematical development seriously curtailed (Cobb et al., 2000). In short, children need sufficient language to allow them to understand their peers and their teachers as explanations are presented, and to allow them to give their own explanations. This has particular ramifications for those children for whom the language of instruction is not their first language. Examples abound of children starting school and not understanding even the most basic instructions when they are given in a language other than their home language. This situation is often exacerbated in the development of mathematical ideas because of its specialized vocabulary and its use of 'common' words to have specialized meanings.

Mathematical symbols are another important form of language which needs to be considered. There has been a great deal of work done on symbolization which has particularly important ramifications for early childhood mathematics learning and teaching (see, for example: Cobb, Yackel, & McClain, 2000; Kieran & Sfard, 1999; Worthington & Carruthers, 2003). There is no doubt that, eventually, children should be able to express their mathematical ideas using the standard mathematical symbols which have become socially accepted and there is some evidence that this can be achieved by quite young children from a variety of socio-economic backgrounds (Zhou et al., 2006). However, it is unnecessary, and even counterproductive, to expect this level of symbol use among many young children who often have developed their own system of symbols and can use this consistently until another, more

standardized system can be taken on board (Hughes, 1986; Worthington & Carruthers, 2003). Children can be encouraged to use their own symbols, and, in fact, their own names for mathematical entities, and teachers should become familiar with these. Just as we would want to encourage teachers at all levels of early childhood education to encourage the use of the children's own strategies and methods, we would also want to encourage the use of their own language, at least in the stages where their concepts are being formed.

Technology

Information and communication technology are important components of the learning and teaching of mathematics at all levels, including early childhood, at least in most of the world's developed countries. There has been much recent work on developing appropriate ways of using this technology in early childhood mathematics learning and teaching.

Despite the extensive literature (see summaries of studies in Groves, 1996, 1997; Groves & Stacey, 1998; Hembree & Dessart, 1992; Stacey & Groves, 1996) on the value of using calculators from early schooling, their use is still not as frequent as it could be (for example, Anderson, 1997; Sparrow & Swan, 1997a, 1997b; Swan & Sparrow, 2004). Groves (1996) illustrates well how calculators expand children's knowledge of number when they are used in a range of different ways. Marley, Skinner, and Kenny (1998) and Huinker (2002) also emphasize the value of calculators in the first years of school. In spite of the success of a number of calculator projects in helping to develop number ideas in young children (see, for example: Groves, 1996, 1997; Ruthven, 1996), there does not seem to have been a great enthusiasm for them in the early years of school and almost no recognition of their value in prior-to-school settings. One way this could be rectified at least in part, is through the introduction of calculators into young children's play.

Huinker (2002) extols the virtues of young children using calculators in their mathematics learning in the following way.

Calculators in the hands of young children are exciting tools for exploring aspects of number, such as numerals, counting, number magnitude, and number relationships. These topics are only the beginning of children's explorations when they use calculators to investigate mathematical ideas. The ongoing use of calculators will continue to enhance learning by presenting opportunities and possibilities for children. (p. 321)

Computer technology is also seen to have great value in young children's learning through aspects such as:

• social and cognitive gains;

• children interacting within an individually appropriate learning environment over which they have some control;

• a sense of mastery;

• the development of representational competence; and

• encouraging children to create and explore in a variety of ways not otherwise possible. (Dockett, Perry, & Nanlohy, 2000, p. 50)

Clements (2002) provides extensive evidence for the value of using computers in early childhood mathematics learning and teaching. His paper concludes with the following summation.

Some criticize computer use, arguing that computers, by their nature, are mechanistic and algorithmic and support only uncreative thinking and production. However, adults increasingly view computers as valuable tools of creative

production. Educational research indicates that there is no single 'effect' of the computer on mathematics achievement, higher-order thinking and creativity. Technology can support either drill or the highest-order thinking. Research also provides strong evidence that certain computer environments such as word processing, art and design tools, computer manipulatives, and turtle graphics hold the potential for the computer's facilitation of these educational goals. There is equally strong evidence that the curriculum in which computer programs are embedded, and the teacher who chooses, uses, and infuses these programs, are essential elements in realizing the full potential of technology. (p. 174)

Clements (1999c; Sarama & Clements, 2002, 2004) describes a "PreK to grade 2 software-based mathematics curriculum development project, designed to comprehensively address the *Principles and Standards for School Mathematics*" (Sarama & Clements, 2002). This package—*Building Blocks* is designed to assist young children in their construction of mathematical knowledge and, in particular, develop higher-order thinking.

Clements claims that *Building Blocks* models an appropriate way in which computers might be used by young children because it provides:

a manageable, clean manipulative; offering flexibility; changing arrangement or representation; storing and later retrieving configurations; recording and replaying students' actions; linking the concrete and the symbolic with feedback; dynamically linking multiple representations; changing the very nature of the manipulative; linking the specific to the general; encouraging problem posing and conjecturing; scaffolding problem solving; focusing attention and increasing motivation; and

encouraging and facilitating complete, precise, explanations. (Clements, 1999c, p. 100)

Building Blocks has shown great potential to enhance children's mathematical development and continues to grow in its extent and influence. Sarama and Clements (2004) reported such benefits: "The initial field test results indicate that such an approach can result in significant assessed learning gains consistent with the new *Principles and Standards for School Mathematics*" (p. 188).

In Australia, Yelland and her colleagues (Kilderry & Yelland, 2005; Kilderry, Yelland, Lazaridis, & Dragicevic, 2003; Yelland, 2002) have investigated the impact of computer technology on issues such as what numeracy means in the information age and how technology can not only facilitate "the development of mathematical understandings but also suggest new sequences of learning which should fundamentally change how we organize mathematics curricula in the early childhood years" (Yelland, 2002, p. 198).

The potential for the use of computer technology by young children is enormous and ever increasing. It seems that the constraints to the use of this technology lie not with mathematics, nor with the learner but, most often, with the adults interacting with the young children involved. Both parents and early childhood educators—in both prior-to-school and school settings—need to develop the knowledge and confidence to allow their children to run with the technology, even if they run well beyond the adults (Dockett et al., 2000).

Assessment of mathematics learning

While both the *Principles and Standards for School Mathematics* (NCTM, 2000) and the position statement *Early Childhood Mathematics: Promoting Good Beginnings* (NCTM/NAEYC, 2002) have a principle:

> Assessment should support the learning of important mathematics and furnish useful information to both teachers and students. (NCTM, 2000, p. 22)

and a recommendation:

> In high-quality mathematics education for 3- to 6-year-old children, teachers and other key professionals should support children's learning by thoughtfully and continually assessing all children's mathematical knowledge, skills, and strategies. (Copley, 2004, p. 159)

respectively on the assessment of young children's mathematical learning, there is surprisingly little research evidence on how these might be put into practice. However, there are a number of clear practice messages. It is generally recognized that in the early childhood years, multiple assessment approaches should be used and that "reliance on a single group administered test to document 3- to 6-year-old children's mathematical competence is counter to expert recommendations" (Copley, 2004, p. 167). Ginsburg and his colleagues (for example, Ginsburg, 2000; Seo, 2003) have shown the power of authentic observations of children's mathematical activity and how this can enhance the opportunities provided to children in prior-to-school years while Carr (2002) has reported on the potential of her *Learning Stories* approach to assessment in the early childhood years.

Systemic numeracy and mathematics programs such as those mentioned earlier in this chapter all have integrated assessment approaches and practices (for example, Bobis et al., 2005; Greenes et al., 2004; Sarama & Clements, 2004) that are designed not only to allow young children to show what mathematics they know and can do but also to assist teachers to plan future mathematical tasks and activities. Many of these programs utilize, among other approaches, one-on-one assessment interviews with the children.

> Such assessment interviews are recognized as a means of gathering rich data about children's numeracy understandings and strategies. As well as providing opportunities for children to demonstrate the power of their mathematical understanding, interviews provide opportunities for teachers to listen to, and learn from, children's explanations. (Perry & Dockett, 2004, p. 109)

While there have been some criticisms of one-on-one assessment interviews—in particular, the cost of adopting the approach on a large scale and the unrealistic situation in which most of these interviews are undertaken—they are becoming widespread in their use and in their impact on both children's and teachers' learning (Bobis, 2003; Bobis & Gould, 2000; Mitchelmore & White, 2003).

Earlier in this chapter, we have mentioned the use of the Utrecht early numeracy assessment (Van de Rijt et al., 1999) in a number of research studies. Another group of assessment instruments which has been developed specifically for the early childhood years by the Australian Council for Educational Research are *Who Am I?* and *I Can Do Maths* (Doig, 2005). These assessment tools were designed specifically to provide standardized early years mathematics assessments and have been used effectively internationally. Doig (2005, p. 116) recognizes that

While not all early years professionals need or want such tools for their particular contexts, there are others whose interests lie in mapping children's mathematical abilities. Quality assessment tools provide a means of achieving a mapping over time, place or culture. Further, tools such as these provide a language for discussion about contemporary issues, such as the pre-school to school transition, which benefits practitioners and researchers and which, in turn, should benefit the children we serve.

In contrast to standardized assessments is the approach to assessment known as *Learning Stories* pioneered by Carr (2002). These are qualitative snapshots, recorded as structured written narratives, often with accompanying photographs, documenting and communicating the context and complexity of children's learning (Carr, 2002). They include relationships, dispositions and an interpretation by someone who knows the child well. They are "structured observations in everyday or 'authentic' settings, designed to provide a cumulative series of snapshots" (Carr & Claxton, 2002, p. 22). Learning stories acknowledge the multiple intelligences and holistic nature of young children's learning, educators' pedagogy and the context in which the learning takes place. Educators use their evaluation of the learning story to plan for future, ongoing learning.

Role of the adult

Adults—prior-to-school educators, school teachers, parents, and others—have an important role to play in young children's mathematics learning. Through their actions and words, adults can encourage children to persevere with a problem, think about it in different ways and share possible solutions with peers and other adults. They can challenge children to extend their thinking or the scope of their investigations. They can also hinder any or all of these. It is difficult to know when to intervene in a child's activity and to know when 'support' looks like

becoming 'domination'. This is a delicate balance and one which can be learned only through experience, education and by getting to know well the children with whom one is working.

In their discussion of how the NCTM *Principles and Standards for School Mathematics* could be made accessible for early childhood educators, Skipper and Collins (2003) have highlighted a dilemma that has faced early childhood education for many years.

Teachers whose formal educations did not cover specific strategies to build on young children's intuitive understandings of mathematical concepts may make two kinds of mistakes. Some may fall back on a general concept that play is the only important and developmentally appropriate approach for young children. These teachers may favor a completely unstructured approach in which mathematical learning is believed to occur incidentally during play, with little teacher participation. Other teachers with less formal training may rely on their own understandings of what it means to be a teacher, perhaps by imitating the way they were taught in elementary school. This group may be more comfortable with a highly structured or scripted approach. Neither of these approaches maximizes opportunities for young children to connect mathematical concepts to the real world in meaningful ways. (pp. 421-422)

This is not a phenomenon unique to the USA. For example, Diezmann and Yelland (2000) have suggested that

Our work with preservice teacher education students has revealed that many of them need to develop their own mathematical literacy and review their attitudes toward mathematics before they embark on planning and implementing a program for the young children in their care. We have observed in students initial negative

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attitudes towards mathematics, as well as gaps in their knowledge of basic mathematical conceptual understandings. (p. 54)

Many teachers of young children do not have a sound understanding of the mathematics that they are expected to teach nor of where this mathematics might lead. This not only makes it difficult for them to provide necessary scaffolding for young learners but it may even have led to negative attitudes about the subject – attitudes which will be transferred to the children in their care. In the case of child care teachers, Skipper and Collins (2003) make the following observations:

> … child care teachers with limited educations may feel incompetent or even fearful about mathematics. These insecurities may lead them to avoid planning activities that are explicitly related to mathematics. Of particular concern is that teachers also may fail to recognize the mathematical concepts that are embedded in everyday preschool materials and activities, or they may lack the mathematical vocabulary to make the necessary connections for young children. (p. 422)

It is well known that teacher quality is one of the key components of quality early childhood education. There are many very able early childhood educators who have enhanced many children's mathematical learning and who continue to do so. However, with the unrelenting pressure on universities to streamline their teacher education programs, particularly in the curriculum areas it remains a challenge to engender a culture of informed, interested early childhood educators having responsibility for the mathematics education of young children. This presents a major challenge for mathematics educators, teacher educators, and mathematics education researchers if we are to support our young children in their development of powerful mathematical ideas.

Future research

We began this chapter noting the recent surge in research in early childhood mathematics education. Hopefully, the intervening words have illustrated the strength and diversity of this research and its impact on the learning and teaching of mathematics in early childhood settings, both prior-to-school and the early years of school.

However, this strength and diversity has also served to show very clearly that there is still much to be done if we are to achieve the aim of all young children being able to show their mathematical power to its full potential. For example, further work is needed on the development of appropriate, research-based curricula built around sets of powerful mathematical ideas for young children such as the set introduced in this chapter or the NCTM *Standards for School Mathematics* while there is a clear need for further work on ways in which early childhood teacher education students and practicing teachers can successfully implement these curricula. This is not all, however.

In a recent review of early childhood mathematics education research in Australasia, we suggested the following areas for future research:

1. approaches to assessment and teaching / learning in numeracy and possible mismatches between these;

2. successful approaches to the mathematics education of young Indigenous students;

3. successful approaches to the mathematics education of young children from culturally and linguistically diverse backgrounds;

4. technology in the mathematics education of young children;

5. play in the mathematics education of young children;

6. development of mathematics concepts among children before they start school;

7. continuities and discontinuities of learning in children as they move from prior-to-school to school settings; and

8. recognition of young children as capable learners of mathematics and the results of such recognition in their mathematical outcomes in the first years of school. (Perry & Dockett, 2004, pp. 119-120)

In the special issue of the *Early Childhood Research Quarterly* on early learning in mathematics and science, Ginsburg and Golbeck (2004) have offered their thoughts on the future of research on mathematics and science learning and education. Ginsburg and Golbeck (2004, p. 190)

argue that researchers and practitioners should examine carefully not only the possibility of unexpected competence in young children, but also its complexity and the limits on it; investigate the socio-emotional context of learning and teaching; attend closely to those children in need of extra help, including low-socio-economic status (SES) children, children with disabilities, and children who receive schooling in an unfamiliar language; create sensitive evaluation strategies that examine program quality, the effectiveness of teachers and administrators, and children's achievement; develop creative and enjoyable curricula that stress thinking as well as content and integrate an organized subject matter with projects and the thoughtful use of manipulatives; investigate the complex processes of teaching in various contexts; and investigate the possible benefits and disadvantages of parental involvement in early mathematics and science education.

Not surprisingly, there are similarities and some differences between the Perry and Dockett and the Ginsburg and Golbeck lists but the key issue of enhancing the mathematical power of all young children to its full potential remains central to both. Much has been done but there remains much to do.

Conclusion

Early childhood education, especially at the prior-to-school level has had a long history of attempting to provide "purposeful, meaningful, and inquiry-based instruction" (Baroody, 2000, p. 66) for young children. Influenced by the nurturance of strong and positive relationships among all concerned, some of the approaches used in early childhood education provide models for what mathematics education for young children might look like in a wide range of educational settings. In this chapter, we have argued that young children have access to powerful mathematical ideas and can use these to solve many of the real world and mathematical problems they meet. These children are capable of much more than they are often given credit for by their families and teachers. Programs such as that emanating from Reggio Emilia have shown the power of the young mind and what can be achieved when children are placed in a supportive, challenging environment. The biggest challenge for mathematics educators and mathematics education researchers is to find ways to utilize the powerful mathematical ideas developed in early childhood as a springboard to even greater mathematical power for these children as they grow older and more experienced.

The powerful mathematical ideas highlighted in this chapter are all processes used by young children in their everyday lives. They are processes that will be used in later mathematics education but that have a real purpose for the children, even when they are young. While the developments in the prior-to-school years have been the province of many researchers over the

years, only a small proportion of these have been mathematics education researchers. If we are to understand how young children develop their mathematical ideas and to use this effectively in the teaching of mathematics, there is a need for a lot more mathematics education research at the prior-to-school level.

One of the tensions in mathematics teaching and learning in the early childhood years is that while children demonstrate remarkable facility with many aspects of mathematics, many early childhood teachers do not have a strong mathematical background. At this time when children's mathematical potential is great, it is imperative that early childhood teachers have the competence and confidence to engage meaningfully with both the children and their mathematics. Until early childhood teaching is seen to be as prestigious a career as elementary teaching—and it is in some countries—young children will be affected by teachers who may have neither a positive attitude towards mathematics nor a deep understanding of elementary mathematics.

Young children are capable of dealing with great complexity in their mathematics learning. Teachers are capable of dealing with great complexity in their facilitation of children's learning. These complexities can be harmoniously linked if teachers build relationships with the children in their class, ascertain what mathematics they know, how they know this and how they can use it to solve realistic problems. Using this and the children's interests as a basis, teachers can plan challenging and complex experiences for their young children, with the aim of helping them reach their potential in mathematics learning.

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